CS357 **Spring 2011**
Exam #2 A **Thursday, April 7th, 2011**

This is a 60 minute exam.
No electronic devices or books allowed.
Choose just one answer. Choose the *best* answer.
You may take with you a "cheat sheet" - a single sheet of size 8.5" x 11" or smaller.

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1. Which of the following is **not** an advantage of solving symmetric positive definite general linear systems as compared to general linear system?

(a) About half as much work is required
(b) About half as much storage is required
(c) They are always well conditioned (*)
(d) No pivoting is necessary for numerical stability

2. Consider

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 4 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

Using Jacobi's iterative method to approximate the solution $\mathbf{A}x = b$ with a starting guess of $x_0 = [0, 2, 0]$ what is x_1?

(a) $[0, 2, 0]^T$
(b) $[1/2, 1/3, 1/4]^T$
(c) $[1/2, 2/3, 1/4]^T$
(d) $[1, 2/3, 1/2]^T$ (*)

3. If **P** is a permutation matrix, which of the following properties do **not** always hold?

(a) $\mathbf{P} = \mathbf{P^T}$ (*)
(b) 2-norm condition number $\kappa(\mathbf{P}) = 1$
(c) $\mathbf{P^{-1}} = \mathbf{P^T}$
(d) $\|\mathbf{P}\|_\mathbf{2} = 1$

4. What is the 2-norm of the matrix **A**?

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

(a) 1
(b) 2
(c) 3 (*)
(d) 9

5. Which of the following transformations doesn't change the eigenvalues of the matrix $\mathbf{A}$?

(a) $c\mathbf{A}$ for any scalar c

(b) $\mathbf{T^{-1}AT}$ for any nonsingular matrix $\mathbf{T}$ (*)

(c) $\mathbf{A}^n$ for any integer $n > 0$

(d) $\mathbf{A} - k\mathbf{I}$ for any scalar k

6. Apply two iterations of normalized power method to the matrix $\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}$ using $x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ as starting vector. After normalization, one entry of the resulting vector will be one. What is the value of the other entry?

(a) 7/13 (*)

(b) 8/13

(c) 9/13

(d) 10/13

For next 2 questions use following matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$$

7. Consider MSR (Modified CSR) (Please note we are asking MSR not CSR), what is $\mathbf{AA}$?

(a) $\begin{bmatrix} 1.0 & 3.0 & * & 2.0 & 4.0 \end{bmatrix}$

(b) $\begin{bmatrix} 1.0 & 2.0 & * & 3.0 & 4.0 \end{bmatrix}$(*)

(c) $\begin{bmatrix} 1.0 & 4.0 & * & 3.0 & 2.0 \end{bmatrix}$

(d) $\begin{bmatrix} 1.0 & 3.0 & * & 4.0 & 2.0 \end{bmatrix}$

8. Consider MSR (Modified CSR) (Please note we are asking MSR not CSR), what is $\mathbf{JA}$?

(a) $\begin{bmatrix} 4 & 5 & 6 & 2 & 1 \end{bmatrix}$(*)

(b) $\begin{bmatrix} 1 & 2 & 2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$

9. Given $\hat{x}$ is the computed solution and x^* is the true solution of $\mathbf{A}x = \mathbf{B}$, and error $= x^* - \hat{x}$, which of the following is true?

(a) $\mathbf{A} \times$ Residual $=$ error

(b) Residual $=$ error

(c) Residual $= \mathbf{A} \times$ error(*)

(d) Residual $= x^*$ - $\hat{x}$

10. Given a non-singular symmetric positive definite $n \times n$ matrix $\mathbf{A}$, then the best bound on the number of iterations needed for the solution to $\mathbf{A}x = b$ obtained by the conjugate gradient method using infinite precision arithmetic is given by,

(a) $O(n^2)$

(b) $O(n)$ (*)

(c) $O(n^3)$

(d) $O(2^n)$

11. True or False (*) In the singular value decomposition of a matrix $\mathbf{A} = \mathbf{USV^T}$ the $\mathbf{U}$, $\mathbf{S}$ and $\mathbf{V}$, are unique.

12. Consider a full rank matrix $\mathbf{A}$, and SVD of $\mathbf{A} = \mathbf{USV^T}$. Which of the following is true?

(a) $\mathbf{V}$ is orthogonal

(b) $\mathbf{U}$ is the matrix of eigenvectors of $\mathbf{AA^T}$

(c) condition number of $\mathbf{A} = \sigma_{max}/\sigma_{min}$

(d) All of above (*)

13. What is the residual vector?

(a) $\mathbf{A^{-1}}b - x$

(b) $b - \mathbf{A}x$ (*)

(c) $\mathbf{A^T A}x - \mathbf{A^T}b$

(d) $\mathbf{A} - b$

14. The least square solution is given by which of the following conditions?

(a) maximizing the square of the 2-norm of the residual

(b) minimizing the square of the 2-norm of the residual (*)

(c) maximizing the eigenvalues in the 2-norm

(d) minimizing the eigenvectors in the 2-norm

15. True or False(*) The normal equation of a linear least squares problem always has a unique solution.

16. Which factorization of $\mathbf{A^T A}$ should be used to solve the normal equation $\mathbf{A^T A}x = \mathbf{A^T}b$, assuming the matrix $\mathbf{A}$ has full rank?

(a) LU factorization

(b) Cholesky factorization (*)

(c) Gaussian elimination

(d) Gaussian elimination with pivoting

17. True (*) or False An orthogonal matrix $\mathbf{Q}$ always preserve 2-norm of any vector v, i.e. $\|\mathbf{Q}v\|_2^2 = \|v\|_2^2$.

18. Consider $m \times n$ matrix $\mathbf{A}$ of rank r, if the singular value decomposition $\mathbf{A} = \mathbf{USV^T}$ is given, what is x of the least square $\mathbf{A}x \cong b$'s solution?

(a) $\sum_{i=1}^{r} (u_i^T b)\sigma_i^{-1} v_i$ (*)

(b) $\sum_{i=1}^{m} (u_i^T b)\sigma_i v_i$

(c) $\sum_{i=1}^{r} (u_i b)\sigma_i^{-1} v_i$

(d) $\sum_{j=1}^{n} \sum_{i=1}^{r} (u_i^j b_j)\sigma_i^{-1} v_i$

19. What is the condition number induced by the 2-norm of the orthogonal matrix shown below?

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

(a) 0

(b) $\sqrt{2}$

(c) 1 (*)

(d) 2

20. A matrix with a large condition number is said to be

(a) stable

(b) unstable

(c) well-conditioned

(d) ill-conditioned (*)

21. A backwards-stable algorithm gives _________ solution to _________ problem.

(a) the exact; a well-conditioned

(b) an approximate; an unstable

(c) the exact; a nearby (*)

(d) the exact; an ill-conditioned

22. What is the equation for the least-squares line ($y = ax + b$) for the data $x = -1, 0, 1$ and $y = 5, 1, 0$?

(a) $y = -5/2x + 2$ (*)

(b) $y = -5/2x - 2$

(c) $y = -3/2x + 5/3$

(d) $y = -5/2x$

The next three problems are about the structure of the matrix $\mathbf{A}$ discussed below. Assume that we desire to interpolate a function $f(x)$ at the points $\{x_1, \ldots, x_n\}$ using the basis $\{\phi_1, \ldots, \phi_n\}$. That is, we wish to construct a function $g(x)$, where

$$g(x) = \sum_{j=1}^{n} a_j \phi_j(x),$$

the a_j are real numbers, and $g(x_i) = f(x_i)$ for $i = 1 \ldots n$. To do this we create the linear system

$$\mathbf{A} \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} f(x_1) \\ \vdots \\ f(x_n) \end{bmatrix},$$

where $\mathbf{A}$ is an $n \times n$ matrix.

23. If the ϕ_i are the Lagrange polynomial basis, what is the form of the matrix $\mathbf{A}$?

(a) Identity (*)
(b) Diagonal, but not the identity
(c) Triangular, but not diagonal
(d) No special structure

24. If the ϕ_i are the Newton polynomial basis, what is the form of the matrix $\mathbf{A}$?

(a) Identity
(b) Diagonal, but not the identity
(c) Triangular, but not diagonal (*)
(d) No special structure

25. If the ϕ_i are the monomial polynomial basis, what is the form of the matrix $\mathbf{A}$?

(a) Identity
(b) Diagonal, but not the identity
(c) Triangular, but not diagonal
(d) No special structure (*)