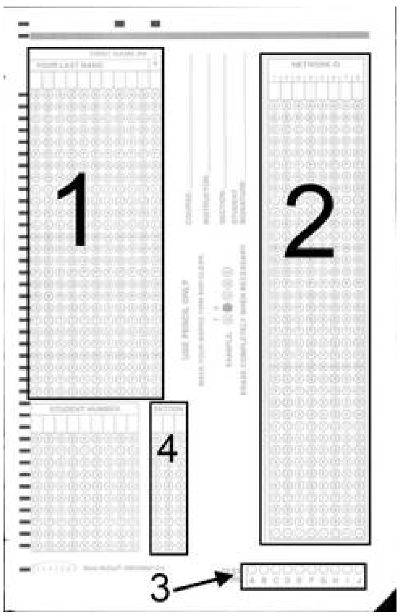
CS357 **Summer 2014**

Midterm Exam
Thursday, July 10^{th}

READ and complete the following:

- Bubble your Scantron only with a No. 2 pencil.
- On your Scantron (shown in the figure below), bubble :
  1. Your Name
  2. Your NetID
  3. Form letter "**A**"



- No electronic devices or books are allowed while taking this exam. However, you may use a "cheat sheet" - a single sheet of size 8.5" x 11" or smaller.
- Please fill in the most correct answer on the provided Scantron sheet.
- We will not answer any questions during the exam.
- Each question has only ONE correct answer.
- You must stop writing when time is called by the proctors.
  No extra time will be given after the exam ends to fill in bubble sheets with answers.
- Hand in both these exam pages and the Scantron.
- DO NOT turn this page UNTIL the proctor instructs you to.

1. Using floating point arithmetic, $2 + 2 * \epsilon_m = 2 * (1 + \epsilon_m)$ where ϵ_m is the machine epsilon.

(a) True

(b) False

2. IEEE 754 double precision floating point numbers are **not** equispaced, i.e., the difference between any two consecutive floating point numbers is **not** a constant.

(a) True

(b) False

3. When given both the approximate value x_a and the true value x_t and assuming that the true value is not equal to zero then the absolute value of absolute error is always greater than or equal to the absolute value of relative error.

(a) True

(b) False

4. Newton's method always converges at a quadratic rate, assuming that the first derivative of the function at the root is identically zero, i.e., $f'(x_r) = 0$ where x_r is a root of $f(x)$.

(a) True

(b) False

5. The following statements are equivalent for an $n \times n$ matrix A.

- A is singular.
- $\det(A) = 0$.
- A^{-1} does not exist.

(a) True

(b) False

6. Convert 0.53125 to binary.

(a) 1.01010

(b) 0.11001

(c) 0.1001

(d) 0.10001

7. Given the approximation from a Taylor series expansion, $f(x) = \frac{1}{1-x} \approx 1 + x + x^2$, what is the absolute value of the absolute error of computing $f(\frac{1}{2})$?

(a) $\frac{1}{4}$

(b) $\frac{3}{4}$

(c) $\frac{1}{2}$

(d) 0

8. Using Horner's method, how many multiplications and additions are required to compute $f(x) = 6x^4 + 5x^2 + 2x + 4$?

- **(a)** 4 multiplications and 4 additions
- **(b)** 4 multiplications and 3 additions
- **(c)** 3 multiplications and 4 additions
- **(d)** 3 multiplications and 3 additions

9. Given a starting guess of $x_1 = 2$, what is an approximation to $\sqrt[3]{7}$ using one step of Newtons method (that is, compute x_2)?

- **(a)** 22/12
- **(b)** 23/12
- **(c)** 19/10
- **(d)** 48/25

10. For the functions (where $\log(n)$ is log base 10)

$$\begin{aligned} f(n) &= 4n^2 + 2\log(n) \\ g(n) &= 3\log(n) + 5 \\ h(n) &= 4n + 1, \end{aligned}$$

Which of the following rightly describes the asymptotical relationship? (You may assume that n takes on integer values only.)

- **(a)** $f(n) = \mathcal{O}(g(n))$
- **(b)** $g(n) = \mathcal{O}(h(n))$
- **(c)** $f(n) = \mathcal{O}(h(n))$
- **(d)** $h(n) = \mathcal{O}(g(n))$

11. What is the relative condition number in computing $G(x) = \cos(x)$ for $-\pi/2 < x < \pi/2$?

- **(a)** $|\tan(x)|$
- **(b)** $|x \cot(x)|$
- **(c)** $|x \sec(x) \tan(x)|$
- **(d)** $|x \tan(x)|$

12. Using bisection method to find the root of the function $f(x) = 4x^3 - 7x^2 - 21x + 18$ on the interval $[-1, 1]$, what would the interval be after 2 iterations?

- **(a)** $[-1, -1/2]$
- **(b)** $[1/2, 1]$
- **(c)** $[0, 1/2]$
- **(d)** $[-1/2, 0]$

13. What is the rate of convergence of the following sequence? (ln is the natural logarithm)

$$e_n = 10^{-\ln(n)}$$

(a) Quadratic
(b) Linear
(c) Sublinear
(d) Superlinear (but not Quadratic)

14. Evaluate the interval arithmetic

$$[-1, 2] * ([-3, 4] + [-4, 2])$$

(a) $[7, 12]$
(b) $[-6, 7]$
(c) $[-6, 12]$
(d) $[-14, 12]$

15. Using Newton's method to find the root of the function $f(x) = x^3 - 2x + 2$, starting with $x_1 = -2$, what is x_2, i.e. x after 1 step?

(a) -3
(b) $4/3$
(c) $-11/5$
(d) $-9/5$

16. What is the Jacobian of the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ where,

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} x_1^2 + 2x_2 + 3 \\ 4x_1 + x_2^2 - 4 \end{bmatrix}$$

and $\mathbf{x} = [x_1, x_2]^T$?

(a) $\begin{bmatrix} 2x_1 & 2 \\ 4 & 2x_2 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 2 \\ 4 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} x_1 & 2x_2 \\ 4 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 2x_1 \\ 4x_2 & 2 \end{bmatrix}$

17. Use the secant method to find the root of the function $f(x) = 2x^3 + 4x - 3$ with $x_0 = -1$, $x_1 = 1$. What is x_2?

(a) $3/4$

(b) $-1/2$

(c) $3/2$

(d) $1/2$

18. Using IEEE 754 double precision floating point arithmetic, as described in our lecture notes, if you compute the sequence $a_{n+1} = a_n/2$, $a_0 = 1$ for $n = 0, 1, 2, ...$, what is the smallest **non-zero** number you will encounter?

(a) ϵ_m (machine epsilon)

(b) 2^{-53}

(c) 2^{-1074}

(d) 2^{-1023}

19. Solving $Ax = b$ given an upper or lower n x n triangular matrix A has what BEST asymptotical bound on cost?

(a) $\mathcal{O}(n)$ flops

(b) $\mathcal{O}(n^2)$ flops

(c) $\mathcal{O}(n^3)$ flops

(d) $\mathcal{O}(\log n)$ flops

20. Compute the determinant of the matrix named `M`.

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 3 & 3 & 3 & 0 \\ 4 & 4 & 4 & 4 \end{bmatrix} * \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

(a) 0

(b) 24

(c) -24

(d) -10

21. Using Gaussian elimination on the matrix A, what is the first elimination matrix M_1?

$$A = \begin{bmatrix} 2 & 3 & -4 & 5 \\ 4 & 6 & 11 & -2 \\ 2 & 1 & -12 & 1 \\ -6 & -2 & 4 & 12 \end{bmatrix}$$

(a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ -1/3 & 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ -3 & 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{bmatrix}$

22. What is the matrix product of C*A*B ?

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 7 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -9 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$

(a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 9 & 1 & 0 & 0 \\ -3 & -2 & 1 & 0 \\ 1 & 2 & -7 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -9 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ -1 & -2 & 7 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -9 & 1 & 0 & 0 \\ -15 & 2 & 1 & 0 \\ 38 & -2 & 7 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{bmatrix}$

23. Determine whether the following vectors are linearly independent or linearly dependent.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 4 \\ 5 \end{bmatrix}$$

(a) linearly independent

(b) linearly dependent

24. Given the code below, compute the BEST asymptotic bound for the number of floating point operations needed to execute the code below, based on the value n.

```
given A (matrix n x n with elements a_{i,j}), and
b (vector n x 1 with elements b_i)

x_n = b_n / a_{n,n}
for i = n-1...1
   s = b_i
   for j = i+1...n
      s = s - a_{i,j} x_j
   end
   x_i = s / a_{i,i}
end
```

(a) $\mathcal{O}(n)$

(b) $\mathcal{O}(n^2)$

(c) $\mathcal{O}(n^3)$

(d) $\mathcal{O}(n^4)$

25. What is the inverse of the following matrix named A?

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ -6 & 0 & 0 & 1 \end{bmatrix}$$

(a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ -1/3 & 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 6 & 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{bmatrix}$

Extra Credit

Answering the question below correctly will add points to your exam total. Answering incorrectly or not answering will not add points to your exam total.

26. What is the `U` in the `LU` factorization (without pivoting) of the following matrix named A?

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 6 & 9 & 12 \end{bmatrix}$$

(a) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1/2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 1 \\ -2 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$