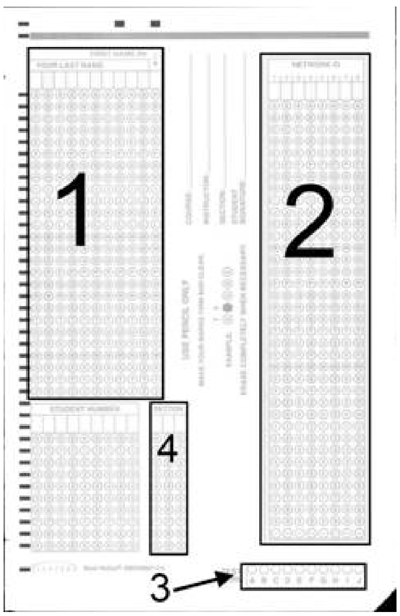
CS357 **Summer 2013**

Midterm Exam
Wednesday, July 3^{rd}

READ and complete the following:

- Bubble your Scantron only with a No. 2 pencil.
- On your Scantron (shown in the figure below), bubble :
  1. Your Name
  2. Your NetID
  3. Form letter "**A**"



- No electronic devices or books are allowed while taking this exam. However, you may use a "cheat sheet" - a single sheet of size 8.5" x 11" or smaller.
- Please fill in the most correct answer on the provided Scantron sheet.
- We will not answer any questions during the exam.
- Each question has only ONE correct answer.
- You must stop writing when time is called by the proctors.
  No extra time will be given after the exam ends to fill in bubble sheets with answers.
- Hand in both these exam pages and the Scantron.
- DO NOT turn this page UNTIL the proctor instructs you to.

1. **(True/False)** $2 + \epsilon_m = 2$? (ϵ_m is machine epsilon)

   (a) True

   (b) False

2. **(True/False)** $1 + \frac{1}{\epsilon_m} = \frac{1}{\epsilon_m}$? (ϵ_m is machine epsilon)

   (a) True

   (b) False

3. **(True/False)** Overflow error is considered more severe than underflow.

   (a) True

   (b) False

4. **(True/False)** If catastrophic cancellation occurs when subtracting two floating point numbers, it means the relative error in the result is high.

   (a) True

   (b) False

5. **(True/False)** The linear rate of convergence of the bisection method is only obtained when the first derivative at the root is not equal to 0.

   (a) True

   (b) False

6. Convert the value 0.6875 to binary.

   (a) 0.01011
   (b) 1.011
   (c) 0.1011
   (d) 0.11011

7. Given a starting guess of $x_1 = 2$, what is an approximation to a root of $f(x) = x^2 - 3$ using one step of Newtons method?

(a) 0

(b) -2

(c) 7/4

(d) 9/4

8. Given $f(x) = x^3 - x^2 - 1$ and an interval $[0, 2]$ that contains a root, if the first iteration of the bisection method produces $x_1 = 1$ what is x_3 (3-rd iteration)?

(a) 0

(b) 11/8

(c) 5/4

(d) 3/2

9. Consider the matrix

$$A = \begin{bmatrix} 1 & 3 & 2 & 4 \\ 2 & 2 & 4 & 5 \\ 4 & 0 & 0 & 16 \\ 2 & 6 & 4 & 8 \end{bmatrix}$$

How many solutions will the linear system $Ax = b$ have?

(a) 1

(b) None

(c) An infinite number

(d) It depends on b

10. What is the determinant of the matrix A?

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 3 & 1 \\ 0 & -3 & 5 \\ 0 & 0 & -1 \end{bmatrix}$$

(a) 0
(b) 1
(c) 6
(d) 12

11. Given the values for the two matrices shown below,

$$M_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 1 \end{bmatrix}$$

and for some matrices A and U (values not given) we can write,

$$M_2 * M_1 * A = U$$

then we can write,

$$A = L * U$$

What are the values of L?

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 4 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ -3 & 4 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 0 & 0 \\ -2 & 0 & 0 \\ 3 & -4 & 0 \end{bmatrix}$

12. What is the relative condition number in computing $G(x) = \frac{1}{x}$ for $x \neq 0$?

- **(a)** 0
- **(b)** 1
- **(c)** x
- **(d)** x^2

13. What is the Taylor expansion of the function $f(x, y) = xe^{x+y}$ about $(x, y, z) = (0, 0, 0)$ for terms $x^{k_1}y^{k_2}z^{k_3}$ where $|k| = 0, 1, 2, 3$ and $|k| = k_1 + k_2 + k_3$?

- **(a)** $x + y + \frac{x^2}{2} + xy + \frac{y^2}{2} + \frac{x^3}{6} + \frac{x^2y}{2} + \frac{xy^2}{2} + \frac{y^3}{6}$
- **(b)** $x + x^2 + xy + \frac{x^3}{2} + x^2y + \frac{xy^2}{2}$
- **(c)** $2x + y + \frac{x^2}{2} + xy + \frac{y^2}{2} + \frac{x^3}{6} + \frac{x^2y}{2} + \frac{xy^2}{2} + \frac{y^3}{6}$
- **(d)** $x + x^2y$

14. What is the rate of convergence of the following sequence?

$$10^{-3}, 10^{-5}, 10^{-7}, 10^{-9}, \ldots$$

- **(a)** superlinear
- **(b)** sublinear
- **(c)** linear
- **(d)** quadratic

15. Given an n x n matrix A and n x 1 vector x, what is the BEST asymptotic upper bound for the number of floating point operations in computing,

$$x^T * A * x$$

based on n?

- **(a)** $\mathcal{O}(n)$
- **(b)** $\mathcal{O}(n^2)$
- **(c)** $\mathcal{O}(n^3)$
- **(d)** $\mathcal{O}(n^4)$

16. Given the code below, compute the BEST asymptotic bound for the number of floating point operations needed to execute the code below based on the value n.

```
for k = 1...n-1
    for i = k+1...n
        xmult = a_ik / a_kk
        a_ik = xmult
        for j = k+1...n
            a_ij = a_ij - (xmult) a_kj
        end
       b_i = b_i - (xmult) b_k
    end
 end
```

(a) $\mathcal{O}(n)$

(b) $\mathcal{O}(n^2)$

(c) $\mathcal{O}(n^3)$

(d) $\mathcal{O}(n^4)$

17. What is the product of the interval numbers $[-1, 3] * [-4, 9]$?

(a) $[4, 27]$

(b) $[-9, 27]$

(c) $[-12, 27]$

(d) $[-4, 9]$

18. In using Newton's Method for finding the root of $f(x, y) = 0$ where $f = [f_1(x, y), f_2(x, y)]^T$ as shown below, what would be the value of inverse of the Jacobian matrix, that is, J^{-1}?

$$f_1(x, y) = 3x + 2y + 3 = 0$$
$$f_2(x, y) = 6x + 4y + 6 = 0$$

(a) $\begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 2 & 0 \\ 6 & 4 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(d) Does not exist

19. Use the Secant Method to find a root of $f(x) = x^3 - 2x^2 + x + 1 = 0$. Given $x_1 = -0.5$, $x_2 = 0$ what is the value of x_3?

(a) -1

(b) $-3/2$

(c) $-4/9$

(d) $-5/6$

20. Given the matrix A shown below. What would be the first elementary elimination matrix M_1 in order to perform Gaussian Elimination?

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ -1 & 2 & 3 \end{bmatrix}$$

(a) $M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

(b) $M_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

(c) $M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

(d) $M_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

21. Which one of the following sets of vectors are linearly independent?

(a) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 9 \\ 10 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$

(c) $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

(d) $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

22. Consider the statements below.

(a) For an m x n matrix A with $m > n$ then A^TA is a square matrix.

(b) For an m x n matrix A with $m > n$, if $A * B = A$, then B will also have entries not equal to either 1 or 0.

(c) It is possible to multiply a square and a rectangular matrix.

Choose the correct statement below.

(a) All the statements are true.

(b) Only (a) and (c) are true.

(c) Only (a) and (b) are true.

(d) Only (b) and (c) are true.

23. The matrix A has the values shown below. After factoring (without permuting the rows) the matrix A as $A = L * U$ what is L?

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

(a) $L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & 2 & 1 \end{bmatrix}$

(b) $L = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ -7 & -2 & 1 \end{bmatrix}$

(c) $L = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix}$

(d) $L = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ -7 & 0 & 1 \end{bmatrix}$

24. For a system of equations $Ax = b$, where A is 5 x 5 matrix, which of the following is TRUE when converting $A = (a_{i,j}),\;\; i = 1 \ldots 5, j = 1 \ldots 5$ to an upper triangular matrix? (Assume that no rows are permuted and $a_{i,j}$ is the element in the i-th row and j-th column of A.)

(a) $a_{1,1}$ is changed once.

(b) $a_{1,5}$ is changed once.

(c) $a_{3,3}$ is changed twice.

(d) $a_{3,5}$ is changed four times.

25. Compute the determinant of the matrix P shown below.

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

(a) 0

(b) 1

(c) -1

(d) 4

Extra Credit

Answering the question below correctly will add points to your exam total. Answering incorrectly or not answering will not add points to your exam total.

26. If `A`, `B` and `C` are n x n non-singular matrices and `b` is an n x 1 vector , can the equation,

$$x = A^{-1} * (B^{-1} + C) * b$$

be solved without computing any matrix inverses?

(a) No.

(b) Yes, solve $A * y = b$ for y then solve $B * x = C * y$ for x.

(c) Yes, solve $B * y = b$ for y then solve $A * x = C * b + y$ for x.

(d) Yes, solve $(B * A) * x = C * b$ for x.