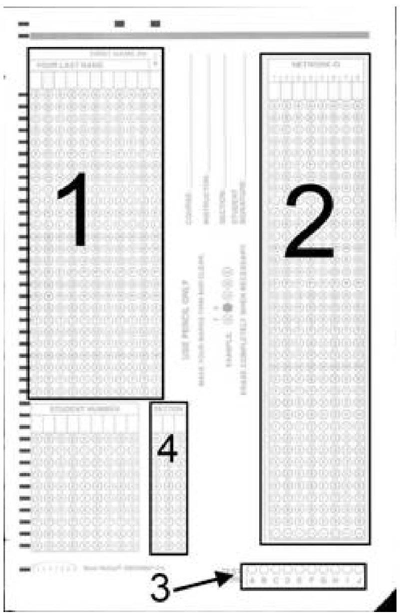
CS357 **Spring 2012**

Exam #1 A Thursday, March 1st

READ and complete the following:

- Bubble your Scantron only with a No. 2 pencil.
- On your Scantron (shown in the figure below), bubble :

  1. Your Name
  2. Your NetID
  3. Form letter "**A**"



- No electronic devices or books are allowed while taking this exam. However, you may use a "cheat sheet" - a single sheet of size 8.5" x 11" or smaller.
- Please fill in the most correct answer on the provided Scantron sheet.
- We will not answer any questions during the exam.
- Each question has only ONE correct answer.
- You must stop writing when time is called by the proctors.
- Hand in both these exam pages and the Scantron.
- DO NOT turn this page UNTIL the proctor instructs you to.

1. (True/False) Double precision positive floating point numbers in IEEE-754 are equally spaced.

(a) True

(b) False (*)

2. (True/False) If a matrix is singular then it cannot have an LU factorization.

(a) True

(b) False (*)

3. Which of the following is **NOT** true about **machine epsilon** (ϵ_m)?

(a) $(3 + \epsilon_{\mathtt{m}}) - 2 = 1$ in floating point arithmetic .

(b) $\frac{\epsilon_{\mathtt{m}}}{2}$ is the maximum relative error in representing nonzero real numbers.

(c) $\epsilon_{\mathtt{m}}$ is the smallest representable positive normalized machine number. (*)

(d) $\epsilon_{\mathtt{m}}$ is the smallest floating point number for which $(1 + \epsilon_{\mathtt{m}}) > 1$.

4. What is the output of the following Matlab script?

```
i = 3.0;
while i ~= 2.0
   i = i - 0.1;
end
i
```

(a) 2.0

(b) 0.0

(c) machine epsilon (ϵ_m)

(d) non-terminating loop (*)

5. Consider computing the LU factorization (in IEEE double-precision) of the following matrix

$$A = \begin{bmatrix} k & 1 \\ -1 & 1 \end{bmatrix}$$

When is pivoting necessary for accuracy?

(a) When $|k|$ is larger than $\frac{1}{\epsilon_m}$.

(b) When $|k|$ is smaller than ϵ_m. (*)

(c) When $|k|$ is near 1.

(d) The LU factorization will be relatively accurate for all machine numbers k.

6. Statement A: The digits lost in cancellation are the most significant digits.
Statement B: The digits lost in rounding are the least significant digits.

(a) Both A and B are true (*)

(b) A is true, B is false

(c) A is false, B is true

(d) Both A and B are false

7. What is the Jacobian of the function

$$\mathbf{f}(x_1, x_2, x_3) = \begin{bmatrix} 3x_1^2 x_2 + x_3^2 \\ 2x_1^2 + x_1 x_3 \\ 3x_2^2 + x_3^3 \end{bmatrix}$$

at the point $(1, 2, 3)$?

(a) $\begin{pmatrix} 3 & 4 & 9 \\ 3 & 0 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} 12 & 3 & 6 \\ 7 & 0 & 1 \\ 0 & 12 & 27 \end{pmatrix}$ (*)

(c) $\begin{pmatrix} 12 & 7 & 0 \\ 3 & 0 & 12 \\ 6 & 1 & 27 \end{pmatrix}$

(d) $\begin{pmatrix} 15 \\ 5 \end{pmatrix}$

8. Suppose at the n^{th} step of finding the roots of the function using the secant method,

$$f(x) = x - x^2 - 1,$$

we get $x_n = 1$ and $x_{n-1} = -1$. What is the next value by this method?

(a) $x_{n+1} = 0$

(b) $x_{n+1} = 1$

(c) $x_{n+1} = 2$ (*)

(d) $x_{n+1} = 3$

9. Which of the following is NOT TRUE about the secant method?

(a) Its converging rate is normally slower than of the Newton's method.

(b) It might not converge if $f'(x) \approx 0$ is encountered.

(c) It needs more function evaluations than Newton's method. (*)

(d) Iterates are not confined to the initial bracket.

10. What is the BEST asymptotic bound for the number of floating point operations in multiplying two $n \times n$ matrices based on the value n?

(a) $\mathcal{O}(n)$

(b) $\mathcal{O}(n^2)$

(c) $\mathcal{O}(n^3)$ $(*)$

(d) $\mathcal{O}(n!)$

11. What is true about the following equation?

$$Ax = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

(a) There is a unique solution. (*)

(b) There are infinitely many solutions.

(c) There is no solution.

(d) There are exactly two solutions.

12. Use the bisection method to find a root of the function $\texttt{f(x)} = \texttt{x}^2 - \texttt{1}$ starting from interval $[0.9, 1.3]$. What is the size(diameter) of the interval when a root is found? Note that the root will be the midpoint value of the interval.

(a) 0.05

(b) 0.2 (*)

(c) 0.4

(d) 1.0

13. Using Newton's method to find a root of the function $\texttt{f(x)} = \texttt{x}^2 - \texttt{1}$, which of the following starting values will converge to the root $x = 1$?

(a) 2.0 (*)

(b) 0.0

(c) -2.0

(d) all of above

14. What is the convergence rate of following sequence?

$$10^{-1},\ 10^{-3},\ 10^{-6},\ 10^{-10},\ 10^{-15}$$

(a) superlinear (*)

(b) quadratic

(c) linear

(d) sublinear

15. What is the next term in the Taylor expansion $e^x = 1 + x + x^2/2 + \ldots$?

(a) x^3

(b) $x^3/6$ (*)

(c) $x^4/24$

(d) x^2

16. What is the Taylor expansion of function $f(x, y, z) = x\sin(y) + 3z^2x$ about $(x, y, z) = (0, 0, 0)$ for terms $x^{k_1}y^{k_2}z^{k_3}$ where $|k| = 0, 1, 2, 3, 4, 5$ and $|k| = k_1 + k_2 + k_3$?

(a) $\mathtt{xy + 3z^2x}$

(b) $\mathtt{xy - xy^3/6 + 6zx}$

(c) $\mathtt{xy - xy^3/6 + 3z^2x}$ (*)

(d) $\mathtt{x + y + 6z}$

17. Mesopotamian culture had iterative formulas for computing algebraic quantities such as roots. In particular, they approximated $\sqrt{N}$ by repeatedly applying the formula obtained by Newtons method. The formula for n = 0,1,2,3,.. is :

(a) $\mathtt{x_{n+1} = \frac{1}{2}(x_n + N)}$

(b) $\mathtt{x_{n+1} = (x_n + \frac{N}{2x_n})}$

(c) $\mathtt{x_{n+1} = \frac{1}{2}(x_n + \frac{N}{x_n})}$ (*)

(d) `None of the‘above`

18. Given the $n \times 1$ vectors u and v, what is the cost in floating point operations of computing uv^T ?

(a) $\mathcal{O}(1)$
(b) $\mathcal{O}(n)$
(c) $\mathcal{O}(n^2)$ $(*)$
(d) $\mathcal{O}(n^3)$

19. What is the relative condition number (C) of f(x)= $\sqrt{x}$ for $x > 0$?

(a) $\texttt{C} = \frac{1}{4}$

(b) $\texttt{C} = \frac{1}{2}$ (*)

(c) $\texttt{C} = 1$

(d) $\texttt{C} = 2$

20. What is the inverse of the following 2 by 2 matrix?

$$A = \begin{bmatrix} 2 & 4 \\ -3 & 1 \end{bmatrix}$$

(a) $\begin{bmatrix} \frac{1}{14} & \frac{1}{7} \\ \frac{1}{2} & \frac{3}{14} \end{bmatrix}$

(b) $\begin{bmatrix} \frac{1}{14} & \frac{-2}{7} \\ \frac{3}{14} & \frac{1}{7} \end{bmatrix}$ (*)

(c) $\begin{bmatrix} \frac{3}{7} & \frac{1}{14} \\ \frac{2}{7} & \frac{5}{14} \end{bmatrix}$

(d) $\begin{bmatrix} \frac{3}{14} & \frac{1}{7} \\ \frac{1}{14} & \frac{-3}{14} \end{bmatrix}$

21. What is the interval value of [1, 2]/[5, 10] where / is interval divide?

(a) [0.1, 0.4] (*)
(b) [0.2, 0.4]
(c) [0.1, 0.2]
(d) [0.0, 0.4]

22. In the LU-decomposition of a matrix A we get

$$LU = PA,$$

where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0.25 & 1 & 0 \\ 0.5 & 0 & 1 \end{bmatrix}, U = \begin{bmatrix} 4 & 2 & 2 \\ 0 & 1.5 & 3.5 \\ 0 & 0 & 1 \end{bmatrix}, P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

What is the determinant of A?

(a) $\det(A) = 1$
(b) $\det(A) = -1$
(c) $\det(A) = 6$
(d) $\det(A) = -6$ (*)

23. Given the code below, compute the BEST asymptotic bound for the number of floating point operations needed to compute `x` based on the value n.

```
x(1) = b(1)/L(1,1);
for i = 2:n
   s = b(i);
   for j = 1:i-1
      s = s - L(i,j)*x(j);
   end
      x(i) = s/L(i,i);
end
```

(a) $O(n)$

(b) $O(n^2)$ (*)

(c) $O(n^3)$

(d) $O(n^4)$

24. Gaussian elimination without pivoting on the matrix,

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 4 & 4 & 2 \\ 4 & 6 & 5 \end{bmatrix}$$

would produce which of the following?

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & -4 & 0 \\ 2 & -6 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 2 \\ 0 & -4 & -6 \\ 0 & 0 & 0 \end{bmatrix}$ (*)

(c) $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 2 \\ -6 & -4 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

25. If you perform LU factoring with partial pivoting on the matrix,

$$A = \begin{bmatrix} 2 & 0 & 2 \\ -4 & 6 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

so that $P * A = L * U$ and you are given that,

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ What are L and U?

(a) $L = \begin{bmatrix} -4 & 6 & 0 \\ 0 & 3 & 2 \\ 0 & 0 & 4 \end{bmatrix}, U = \begin{bmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $L = \begin{bmatrix} 1 & -0.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, U = \begin{bmatrix} -4 & 0 & 0 \\ 6 & 3 & 0 \\ 0 & 2 & 4 \end{bmatrix}$

(c) $L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, U = \begin{bmatrix} -4 & 6 & 0 \\ 0 & 3 & 2 \\ 0 & 0 & 4 \end{bmatrix}$

(d) $L = \begin{bmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, U = \begin{bmatrix} -4 & 6 & 0 \\ 0 & 3 & 2 \\ 0 & 0 & 4 \end{bmatrix}$ (*)