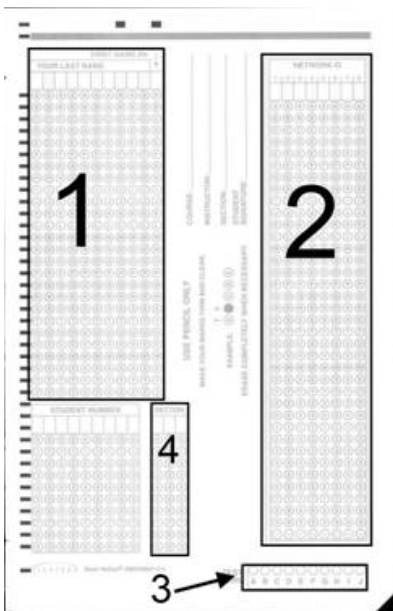


Exam #1 A Thursday, March 1st

READ and complete the following:

- Bubble your Scantron only with a No. 2 pencil.
- On your Scantron (shown in the figure below), bubble :
 1. Your Name
 2. Your NetID
 3. Form letter "A"



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- No electronic devices or books are allowed while taking this exam. However, you may use a "cheat sheet" - a single sheet of size 8.5" x 11" or smaller.
 - Please fill in the most correct answer on the provided Scantron sheet.
 - We will not answer any questions during the exam.
 - Each question has only ONE correct answer.
 - You must stop writing when time is called by the proctors.
 - Hand in both these exam pages and the Scantron.
 - DO NOT turn this page UNTIL the proctor instructs you to.

1. (True/False) Double precision positive floating point numbers in IEEE-754 are equally spaced.
 - (a) True
 - (b) False

2. (True/False) If a matrix is singular then it cannot have an LU factorization.
 - (a) True
 - (b) False

3. Which of the following is **NOT** true about **machine epsilon** (ϵ_m)?
 - (a) $(3 + \epsilon_m) - 2 = 1$ in floating point arithmetic .
 - (b) $\frac{\epsilon_m}{2}$ is the maximum relative error in representing nonzero real numbers.
 - (c) ϵ_m is the smallest representable positive normalized machine number.
 - (d) ϵ_m is the smallest floating point number for which $(1 + \epsilon_m) > 1$.

4. What is the output of the following Matlab script?

```
i = 3.0;
while i ~= 2.0
    i = i - 0.1;
end
i
```

 - (a) 2.0
 - (b) 0.0
 - (c) machine epsilon (ϵ_m)
 - (d) non-terminating loop

5. Consider computing the LU factorization (in IEEE double-precision) of the following matrix

$$A = \begin{bmatrix} k & 1 \\ -1 & 1 \end{bmatrix}$$

When is pivoting necessary for accuracy?

- (a) When $|k|$ is larger than $\frac{1}{\epsilon_m}$.
 - (b) When $|k|$ is smaller than ϵ_m .
 - (c) When $|k|$ is near 1.
 - (d) The LU factorization will be relatively accurate for all machine numbers k .
6. Statement A: The digits lost in cancellation are the most significant digits.
Statement B: The digits lost in rounding are the least significant digits.
- (a) Both A and B are true
 - (b) A is true, B is false
 - (c) A is false, B is true
 - (d) Both A and B are false

7. What is the Jacobian of the function

$$\mathbf{f}(x_1, x_2, x_3) = \begin{bmatrix} 3x_1^2x_2 + x_3^2 \\ 2x_1^2 + x_1x_3 \\ 3x_2^2 + x_3^3 \end{bmatrix}$$

at the point $(1, 2, 3)$?

(a) $\begin{pmatrix} 3 & 4 & 9 \\ 3 & 0 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} 12 & 3 & 6 \\ 7 & 0 & 1 \\ 0 & 12 & 27 \end{pmatrix}$

(c) $\begin{pmatrix} 12 & 7 & 0 \\ 3 & 0 & 12 \\ 6 & 1 & 27 \end{pmatrix}$

(d) $\begin{pmatrix} 15 \\ 5 \end{pmatrix}$

8. Suppose at the n^{th} step of finding the roots of the function using the secant method,

$$f(x) = x - x^2 - 1,$$

we get $x_n = 1$ and $x_{n-1} = -1$. What is the next value by this method?

(a) $x_{n+1} = 0$

(b) $x_{n+1} = 1$

(c) $x_{n+1} = 2$

(d) $x_{n+1} = 3$

9. Which of the following is NOT TRUE about the secant method?

(a) Its converging rate is normally slower than of the Newton's method.

(b) It might not converge if $f'(x) \approx 0$ is encountered.

(c) It needs more function evaluations than Newton's method.

(d) Iterates are not confined to the initial bracket.

10. What is the BEST asymptotic bound for the number of floating point operations in multiplying two $n \times n$ matrices based on the value n ?

- (a) $\mathcal{O}(n)$
- (b) $\mathcal{O}(n^2)$
- (c) $\mathcal{O}(n^3)$
- (d) $\mathcal{O}(n!)$

11. What is true about the following equation?

$$Ax = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

- (a) There is a unique solution.
- (b) There are infinitely many solutions.
- (c) There is no solution.
- (d) There are exactly two solutions.

12. Use the bisection method to find a root of the function $f(x) = x^2 - 1$ starting from interval $[0.9, 1.3]$. What is the size(diameter) of the interval when a root is found? Note that the root will be the midpoint value of the interval.

- (a) 0.05
- (b) 0.2
- (c) 0.4
- (d) 1.0

13. Using Newton's method to find a root of the function $f(x) = x^2 - 1$, which of the following starting values will converge to the root $x = 1$?
- (a) 2.0
 - (b) 0.0
 - (c) -2.0
 - (d) all of above

14. What is the convergence rate of following sequence?

$$10^{-1}, 10^{-3}, 10^{-6}, 10^{-10}, 10^{-15}$$

- (a) superlinear
 - (b) quadratic
 - (c) linear
 - (d) sublinear
15. What is the next term in the Taylor expansion $e^x = 1 + x + x^2/2 + \dots$?
- (a) x^3
 - (b) $x^3/6$
 - (c) $x^4/24$
 - (d) x^2

16. What is the Taylor expansion of function $f(x, y, z) = x \sin(y) + 3z^2x$ about $(x, y, z) = (0, 0, 0)$ for terms $x^{k_1}y^{k_2}z^{k_3}$ where $|k| = 0, 1, 2, 3, 4, 5$ and $|k| = k_1 + k_2 + k_3$?

(a) $xy + 3z^2x$

(b) $xy - xy^3/6 + 6zx$

(c) $xy - xy^3/6 + 3z^2x$

(d) $x + y + 6z$

17. Mesopotamian culture had iterative formulas for computing algebraic quantities such as roots. In particular, they approximated \sqrt{N} by repeatedly applying the formula obtained by Newton's method. The formula for $n = 0, 1, 2, 3, \dots$ is :

(a) $x_{n+1} = \frac{1}{2}(x_n + N)$

(b) $x_{n+1} = (x_n + \frac{N}{2x_n})$

(c) $x_{n+1} = \frac{1}{2}(x_n + \frac{N}{x_n})$

(d) None of the above

18. Given the $n \times 1$ vectors u and v , what is the cost in floating point operations of computing uv^T ?

(a) $\mathcal{O}(1)$

(b) $\mathcal{O}(n)$

(c) $\mathcal{O}(n^2)$

(d) $\mathcal{O}(n^3)$

19. What is the relative condition number (C) of $f(x) = \sqrt{x}$ for $x > 0$?

(a) $C = \frac{1}{4}$

(b) $C = \frac{1}{2}$

(c) $C = 1$

(d) $C = 2$

20. What is the inverse of the following 2 by 2 matrix?

$$A = \begin{bmatrix} 2 & 4 \\ -3 & 1 \end{bmatrix}$$

(a) $\begin{bmatrix} \frac{1}{14} & \frac{1}{7} \\ \frac{1}{2} & \frac{3}{14} \end{bmatrix}$

(b) $\begin{bmatrix} \frac{1}{14} & \frac{-2}{7} \\ \frac{3}{14} & \frac{1}{7} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{3}{7} & \frac{1}{14} \\ \frac{2}{7} & \frac{5}{14} \end{bmatrix}$

(d) $\begin{bmatrix} \frac{3}{14} & \frac{1}{7} \\ \frac{1}{14} & \frac{-3}{14} \end{bmatrix}$

21. What is the interval value of $[1, 2]/[5, 10]$ where $/$ is interval divide?

(a) $[0.1, 0.4]$

(b) $[0.2, 0.4]$

(c) $[0.1, 0.2]$

(d) $[0.0, 0.4]$

22. In the LU-decomposition of a matrix A we get

$$LU = PA,$$

where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0.25 & 1 & 0 \\ 0.5 & 0 & 1 \end{bmatrix}, U = \begin{bmatrix} 4 & 2 & 2 \\ 0 & 1.5 & 3.5 \\ 0 & 0 & 1 \end{bmatrix}, P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

What is the determinant of A ?

(a) $\det(A) = 1$

(b) $\det(A) = -1$

(c) $\det(A) = 6$

(d) $\det(A) = -6$

23. Given the code below, compute the BEST asymptotic bound for the number of floating point operations needed to compute \mathbf{x} based on the value n .

```
x(1) = b(1)/L(1,1);
for i = 2:n
    s = b(i);
    for j = 1:i-1
        s = s - L(i,j)*x(j);
    end
    x(i) = s/L(i,i);
end
```

- (a) $O(n)$
- (b) $O(n^2)$
- (c) $O(n^3)$
- (d) $O(n^4)$

24. Gaussian elimination without pivoting on the matrix,

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 4 & 4 & 2 \\ 4 & 6 & 5 \end{bmatrix}$$

would produce which of the following?

- (a) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & -4 & 0 \\ 2 & -6 & 0 \end{bmatrix}$
- (b) $\begin{bmatrix} 1 & 2 & 2 \\ 0 & -4 & -6 \\ 0 & 0 & 0 \end{bmatrix}$
- (c) $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$
- (d) $\begin{bmatrix} 1 & 2 & 2 \\ -6 & -4 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

25. If you perform LU factoring with partial pivoting on the matrix,

$$A = \begin{bmatrix} 2 & 0 & 2 \\ -4 & 6 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

so that $P * A = L * U$ and you are given that,

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ What are } L \text{ and } U?$$

$$\text{(a) } L = \begin{bmatrix} -4 & 6 & 0 \\ 0 & 3 & 2 \\ 0 & 0 & 4 \end{bmatrix}, U = \begin{bmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{(b) } L = \begin{bmatrix} 1 & -0.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, U = \begin{bmatrix} -4 & 0 & 0 \\ 6 & 3 & 0 \\ 0 & 2 & 4 \end{bmatrix}$$

$$\text{(c) } L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, U = \begin{bmatrix} -4 & 6 & 0 \\ 0 & 3 & 2 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\text{(d) } L = \begin{bmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, U = \begin{bmatrix} -4 & 6 & 0 \\ 0 & 3 & 2 \\ 0 & 0 & 4 \end{bmatrix}$$