

Bisection 1D

check signs  $f(a), f(b)$

$$\Delta t_{k+1} = \frac{\Delta t_k}{2}$$

$$\Delta t_k = \frac{\Delta t_0}{2^k}$$

Newton 1D

$$x_{k+1} = x_k + h$$

$$h = -f(x_k)/f'(x_k)$$

Newton 2D

$$\underline{x}_{k+1} = \underline{x}_k + \underline{s}$$

$$\underline{J}(x_k) \underline{s} = -\underline{f}(x_k)$$

Secant 1D

$$x_{k+1} = x_k + h$$

$$h = -f(x_k)/\tilde{f}'(x_k)$$

$$\tilde{f}' = \frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}}$$

"COST"

1 function eval  
per iteration

(no need for derivatives)

2 fc eval per iteration

need  $f(x)$  and  $f'(x)$

One solve ( $O(n^3)$ ) per  
iteration!

To obtain  $J(x)$  for dense  
matrix  $\Rightarrow n^2$  evaluations

1 fc eval per iteration  
needs two initial guesses  
no need to know  $f'(x)$

CONVERGENCE

$$\lim_{k \rightarrow \infty} \frac{\|e_{k+1}\|}{\|e_k\|^r} = C$$

$r = 1$ ,  $C = 0.5$   
linear

• quadratic (close  
to root)  
• depends on initial  
guess (not guaranteed  
to converge)

$r = 2$ ,  $C < 1$   
(close to root!)

• superlinear  $1 < r < 2$   
• local convergence  
(depends on initial  
guess)  
 $r = 1.618$