



CS 225

Data Structures

March 25 – Graph Traversals

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```

1 BFS(G) :
2   Input: Graph, G
3   Output: A labeling of the edges on
4           G as discovery and cross edges
5
6   foreach (Vertex v : G.vertices()): ) O(n)
7     setLabel(v, UNEXPLORED)
8   foreach (Edge e : G.edges()): ) O(m)
9     setLabel(e, UNEXPLORED)
10  foreach (Vertex v : G.vertices()): | O(n)
11    if getLabel(v) == UNEXPLORED:
12      BFS(G, v)

```

not
O(n)

```

14 BFS(G, v) :
15   Queue q
16   q.enqueue(v)
17
18   while !q.empty()
19     [v,p] = q.dequeue()
20     if( getLabel(v) == UNEXPLORED)
21       setLabel(v, VISITED)
22       setLabel(p, DISCOVERY)
23       foreach (Vertex w : G.adjacent(v)):
24         q.enqueue(w,v)
25     else
26       setLabel(p, CROSS)
27

```

O(m)?

$\sum_{v \in V} \text{deg}(v)$
 $= 2m$
 $O(m)$
 total

BFS Analysis

Q: Does our implementation handle disjoint graphs?
If so, what code handles this?

- ***How do we use this to count components?***

each call to $\text{BFS}(G, v)$ is a component

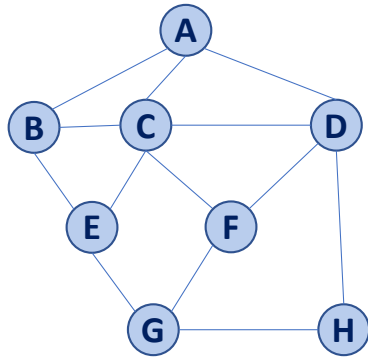
Q: Does our implementation detect a cycle?

- ***How do we update our code to detect a cycle?***

any cross edges

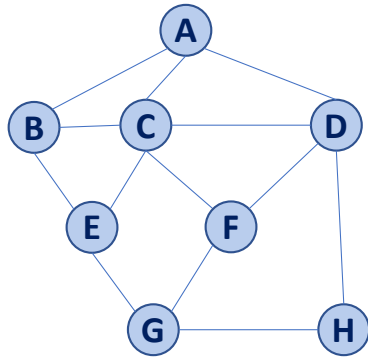
Q: What is the running time? $O(m+n)$

Traversal: BFS



v	d	P	Adjacent Edges
A			
B			
C			
D			
E			
F			
G			
H			

Traversal: BFS



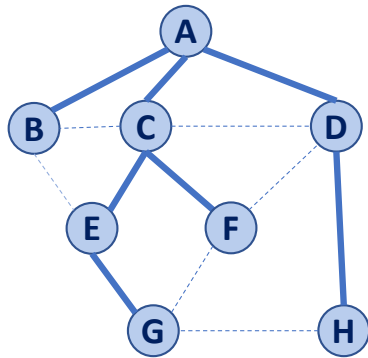
v	d	P	Adjacent Edges
A	0	-	C B D
B			A C E
C			B A D E F
D			A C F H
E			B C G
F			C D G
G			E F H
H			D G

A

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```

Running time of BFS



While-loop at **:18?**

For-loop at **:23?**

v	d	P	Adjacent Edges
A	0	-	C B D
B	1	A	A C E
C	1	A	B A D E F
D	1	A	A C F H
E	2	C	B C G
F	2	C	C D G
G	3	E	E F H
H	2	D	D G



BFS Observations

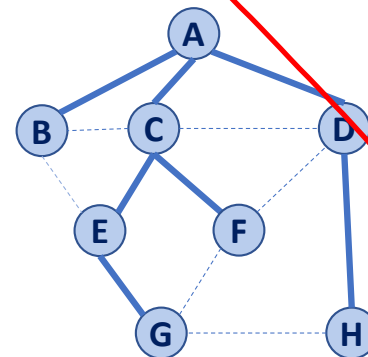
Q: What is a shortest path from **A** to **H**?

Q: What is a shortest path from **E** to **H**?

Q: How does a cross edge relate to **d**?

Q: What structure is made from discovery edges?

v	d	P	Adjacent Edges
A	0	-	C B D
B	1	A	A C E
C	1	A	B A D E F
D	1	A	A C F H
E	2	C	B C G
F	2	C	C D G
G	3	E	E F H
H	2	D	D G





BFS Observations

Obs. 1: BFS can be used to count components.

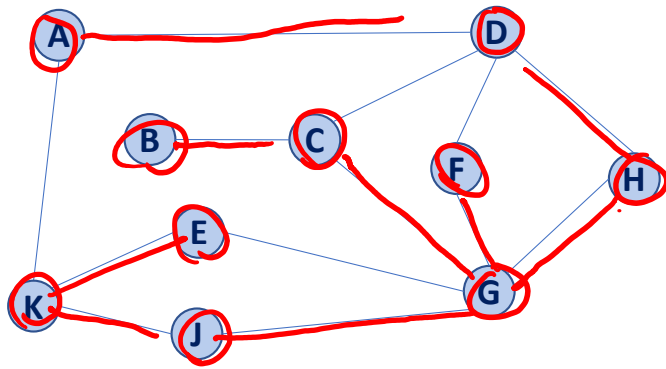
Obs. 2: BFS can be used to detect cycles.

Obs. 3: In BFS, d provides the shortest distance to every vertex.

Obs. 4: In BFS, the endpoints of a cross edge never differ in distance, d , by more than 1:

$$|d(u) - d(v)| = 1$$

Traversal: DFS



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```
1 DFS(G) :
2   Input: Graph, G
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6   foreach (Vertex v : G.vertices()):
7     setLabel(v, UNEXPLORED)
8   foreach (Edge e : G.edges()):
9     setLabel(e, UNEXPLORED)
10  foreach (Vertex v : G.vertices()):
11    if getLabel(v) == UNEXPLORED:
12      DFS(G, v)
```

```
14 DFS(G, v) :
15   Stack q
16   q.enqueue(v)
17
18   while !q.empty()
19     [v,p] = q.dequeue()
20     if( getLabel(v) == UNEXPLORED)
21       setLabel(v, VISITED)
22       setLabel(p, DISCOVERY)
23       foreach (Vertex w : G.adjacent(v)):
24         q.enqueue(w,v)
25     else
26       setLabel(p, BACK)
27
```

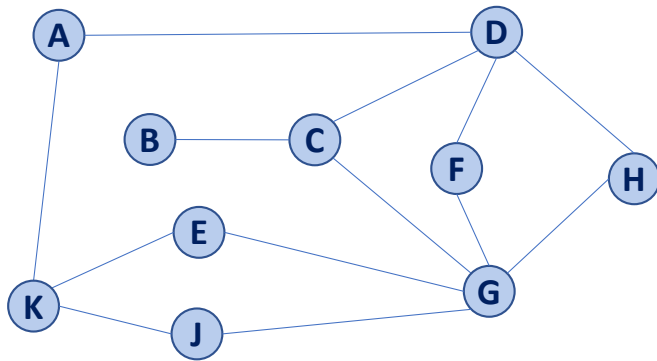
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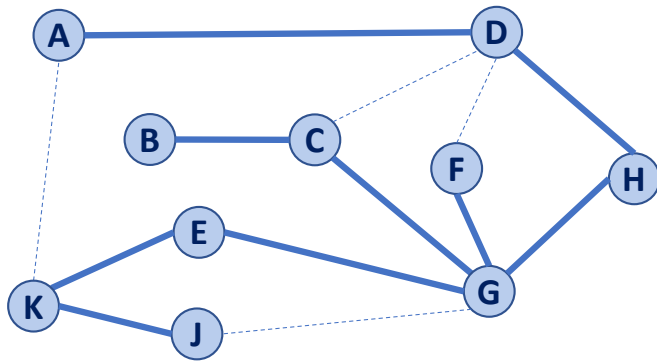
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9     setLabel(e, UNEXPLORED)
10  foreach (Vertex v : G.vertices()):
11    if getLabel(v) == UNEXPLORED:
12      DFS(G, v, null)
```

```
14 DFS(G, v, p) :
15 Stack q
16 q.enqueue(v)
17
18 while !q.empty()
19 [v,p] = q.dequeue()
20   if( getLabel(v) == UNEXPLORED)
21     setLabel(v, VISITED)
22     setLabel(p, DISCOVERY)
23     foreach (Vertex w : G.adjacent(v)):
24       q.enqueue(w,v) DFS(G, w, v)
25   else
26     setLabel(p, BACK)
27
```

Traversal: DFS



Traversal: DFS



————— Discovery Edge

..... Back Edge

Running time of DFS

Labeling:

- Vertex:
- Edge:

Queries:

- Vertex:
- Edge:

