CS 225

Data Structures

February 16 – BST Rotations

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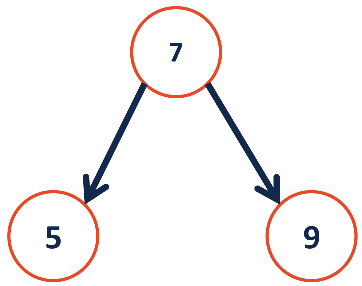
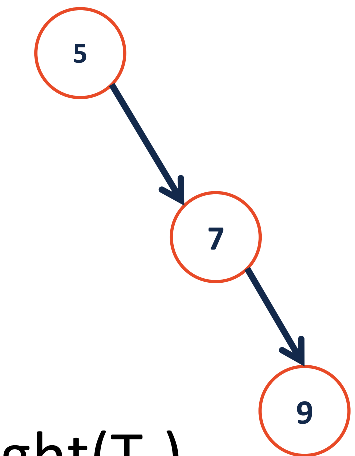
Height-Balanced Tree

What tree makes you happier?





Height balance: $b = \text{height}(T_R) - \text{height}(T_L)$

A tree is height balanced if:

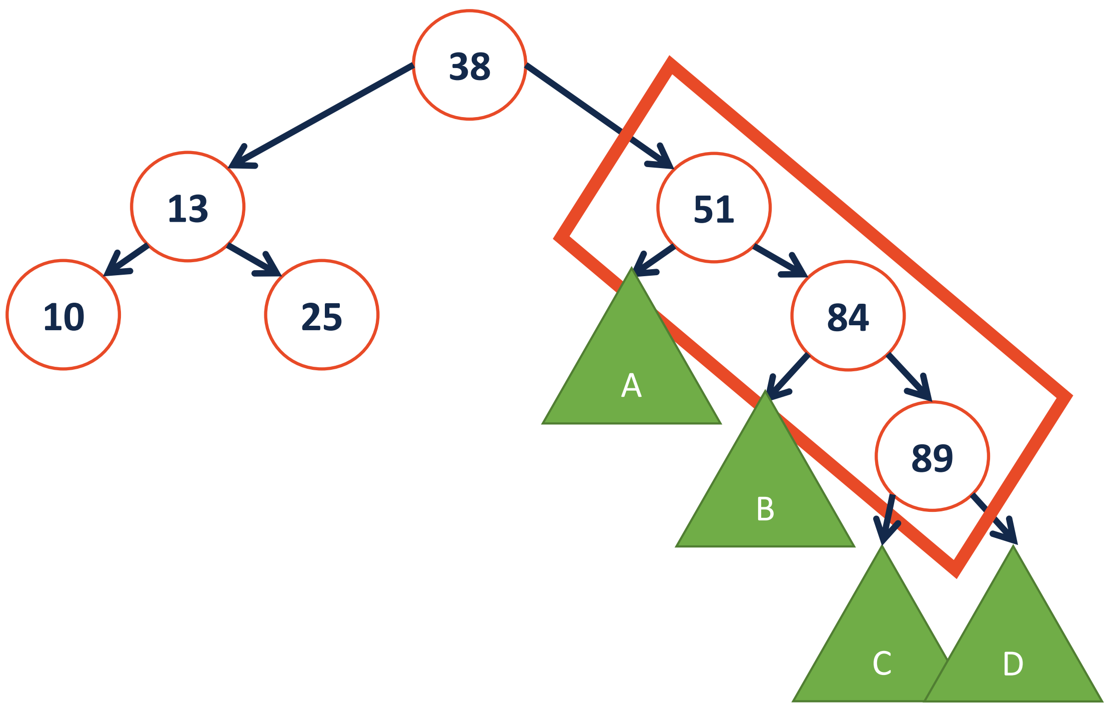
For all nodes in the tree $|b| < 2$.

BST Rotation

We will perform a rotation that maintains two properties

1. **Maintain the BST property**

2. **Change a “stick” into a “mountain”**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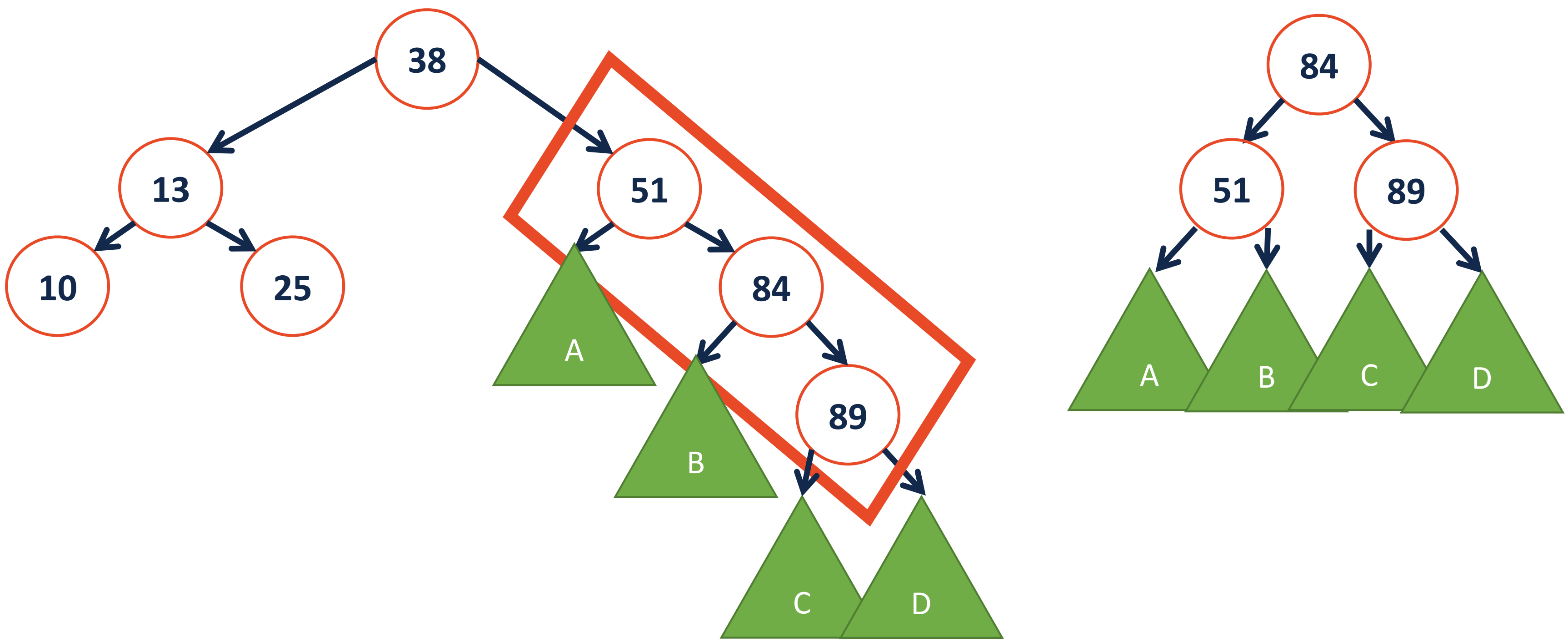
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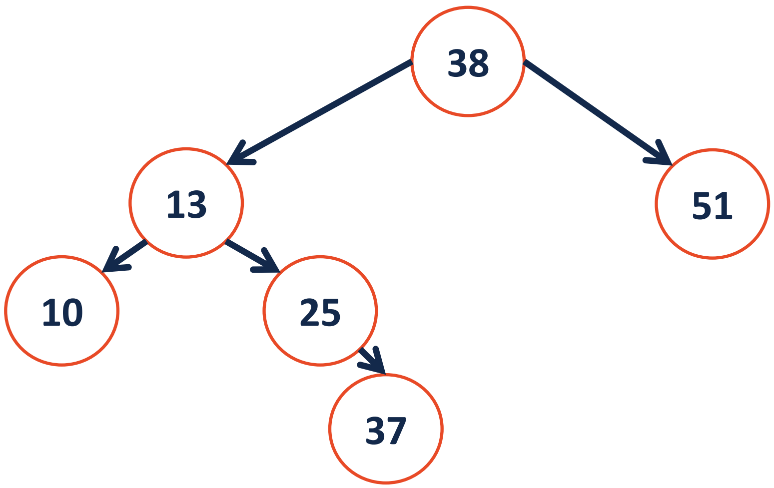
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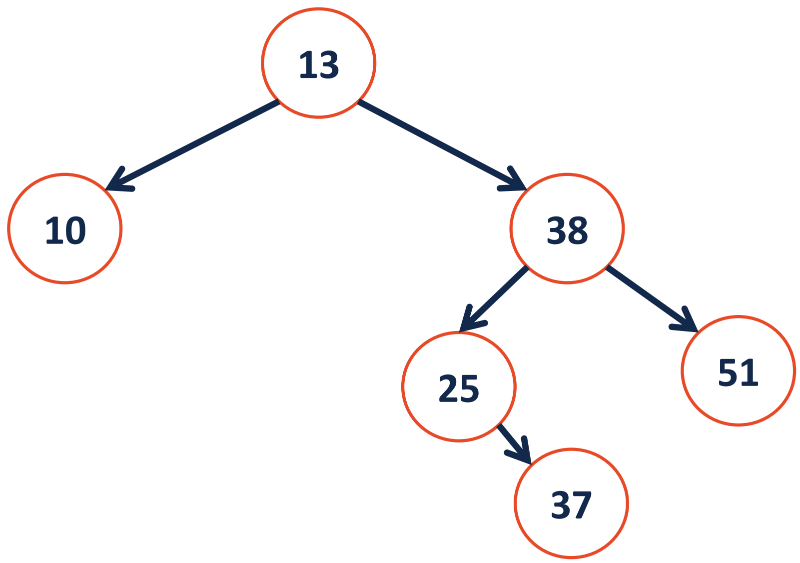
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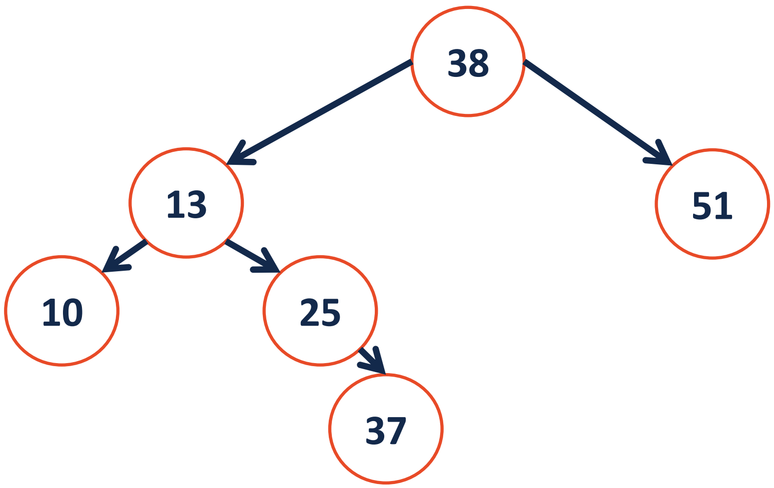
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BST Rotation Summary

- Four kinds of rotations (L, R, LR, RL)
- All rotations are local (subtrees are not impacted)
- All rotations are constant time: O(1)
- BST property maintained

GOAL:

We call these trees:

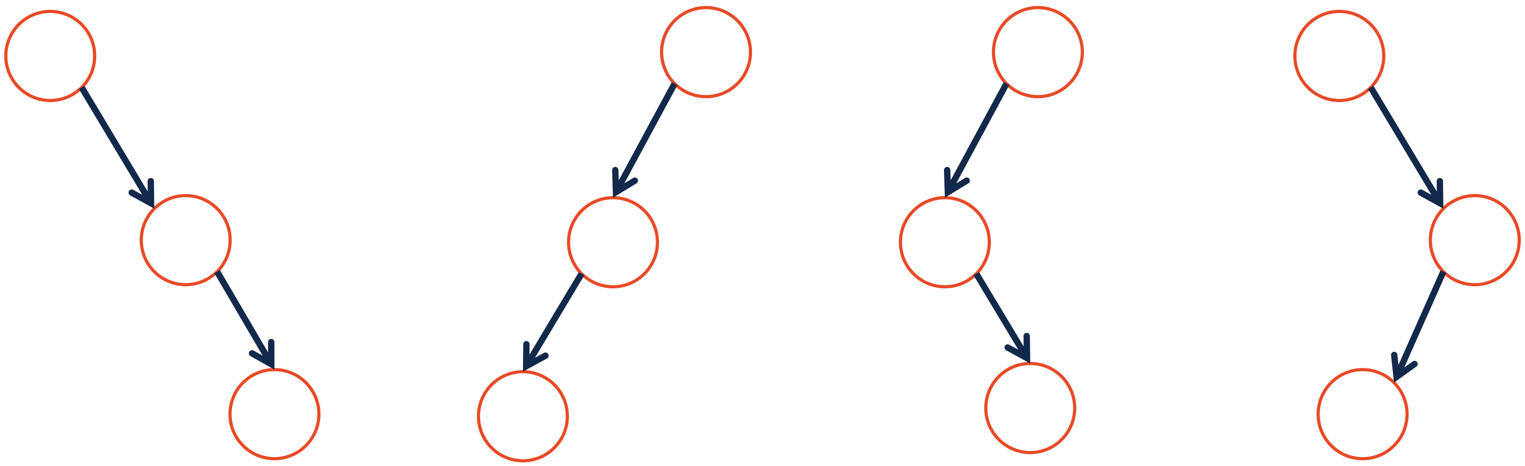
AVL Trees

Three issues for consideration:

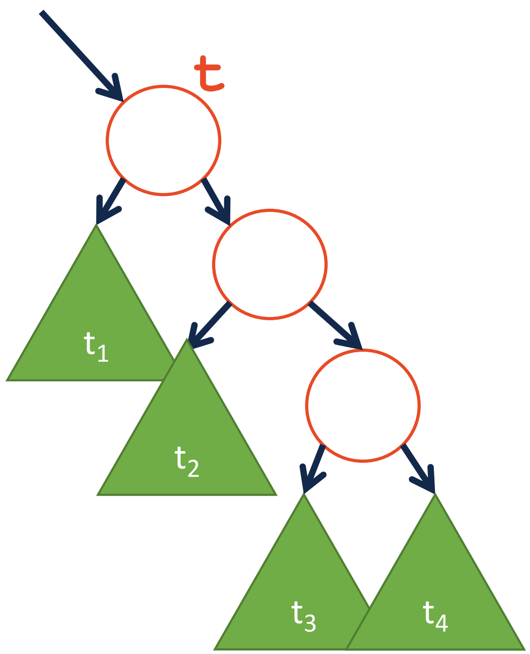
- Rotations
- Maintaining Height
- Detecting Imbalance

AVL Tree Rotations

Four templates for rotations:



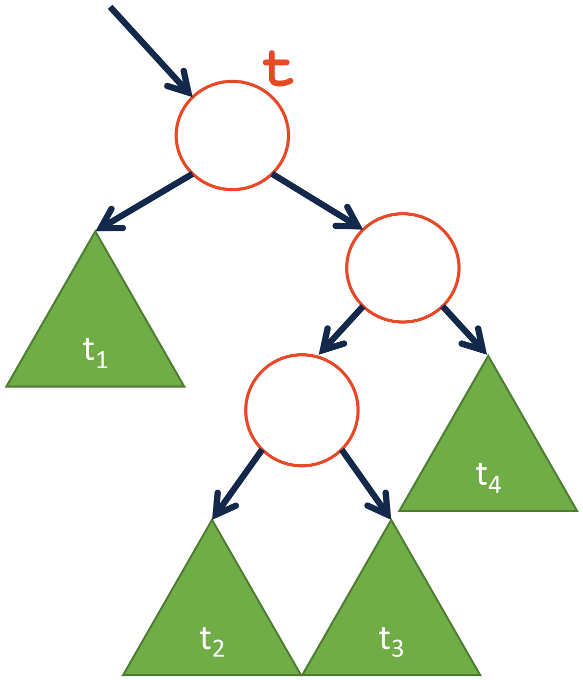
Finding the Rotation on Insert



Theorem:
If an insertion occurred in subtrees $\mathbf{t_3}$ or $\mathbf{t_4}$ and a subtree was detected at **t**, then a __________ rotation about **t** restores the balance of the tree.

We gauge this by noting the balance factor of **t->right** is ______.

Finding the Rotation on Insert

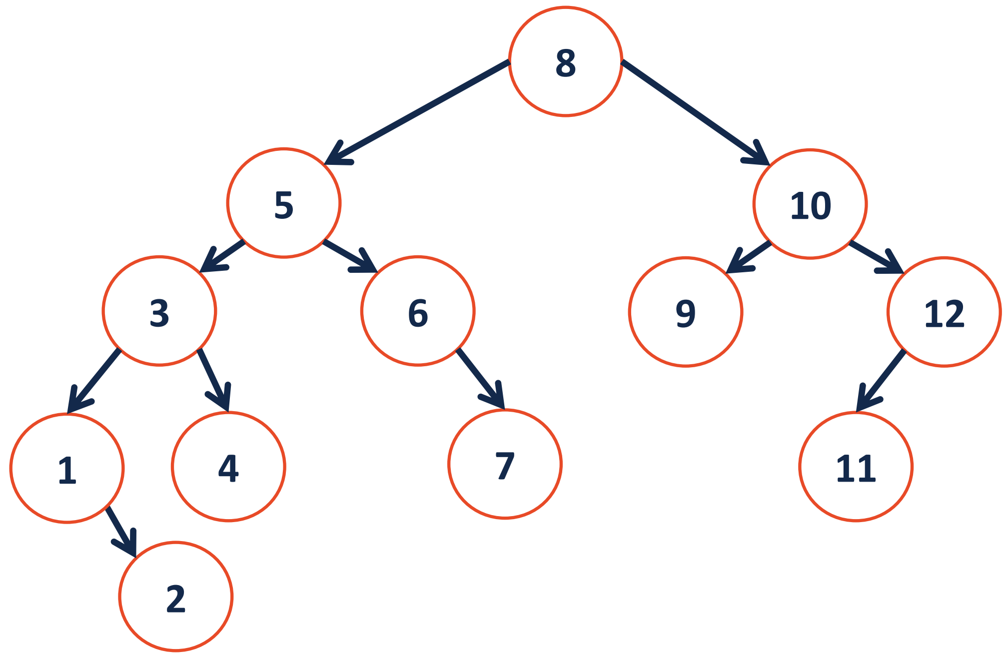


Theorem:
If an insertion occurred in subtrees **t_2** or **t_3** and a subtree was detected at **t**, then a ___________ rotation about **t** restores the balance of the tree.

We gauge this by noting the balance factor of **t->right** is ______.

Insertion into an AVL Tree

`_insert(6.5)`




```
struct TreeNode {
  T key;
  unsigned height;
  TreeNode *left;
  TreeNode *right;
};
```

Insertion into an AVL Tree

`_insert(6.5)`

Insert (pseudo code):
1: Insert at proper place
2: Check for imbalance
3: Rotate, if necessary
4: Update height

```
struct TreeNode {
  T key;
  unsigned height;
  TreeNode *left;
  TreeNode *right;
};
```