This assignment is best completed in groups. Choose the following roles for yourselves. If you only have three people, combine Reporter and Process Analyst.

Team Roles	**Team Member**
Manager Reads the questions out loud, keeps track of time, and makes sure everyone contributes appropriately.	
Reporter talks to the instructor and other teams.	
Quality Control records all answers and questions and provides team reflection to team and instructor.	
Process Analyst Considers how the team could work and learn more effectively.	

Objectives

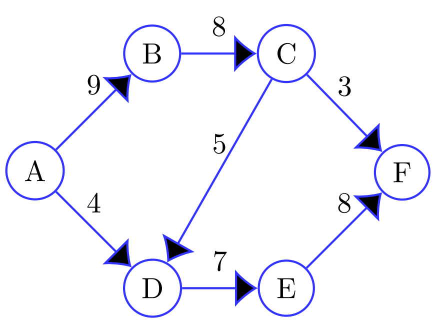
We are going to learn an algorithm for determining the network flow capacity of a graph. In the following graphs, the edge weights represent capacity. There are four sections: spend 10–15 minutes on each one.

Graph $G1$

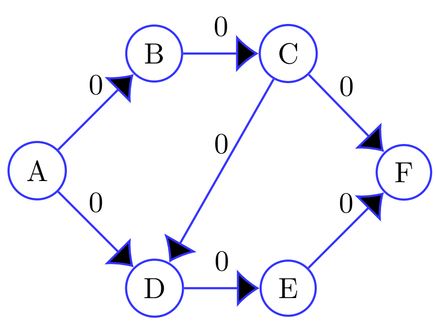
We are given as input graph $G1$.

We create two new graphs: a *flow graph* F and a *residual graph* R.

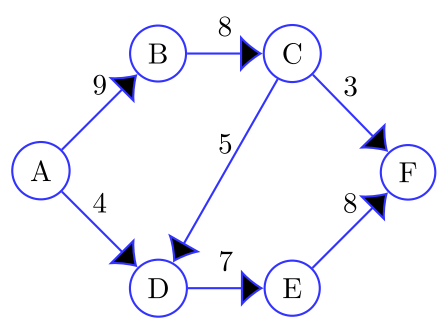
Graph G



Flow Graph F



Residual Graph R

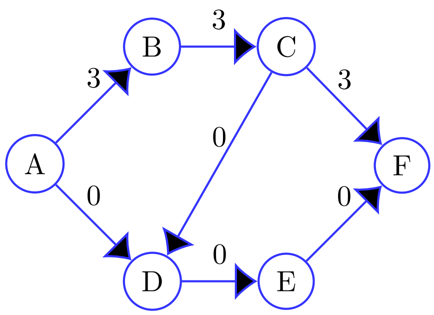


Problem 1.

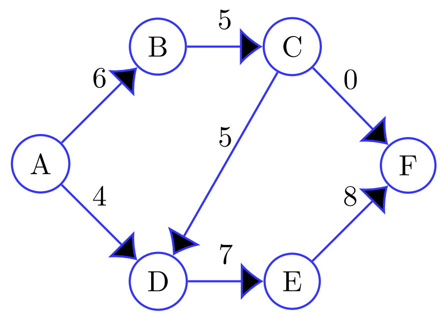
The algorithm works by selecting paths from the residual graph R. The first path selected is $A \to B \to C \to F$ in graph R. This path's flow capacity is 3. What do you think determines the flow capacity?

The algorithm uses the path to modify graphs F and R. Here is the result.

Graph F



Graph R



Problem 2.

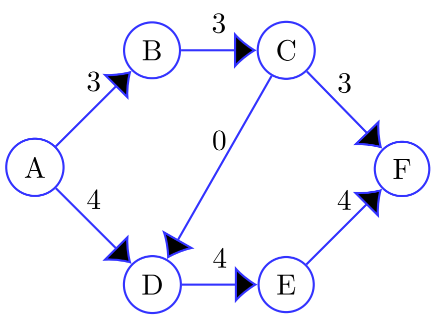
Examine the new versions of F and R above. What is being done with the path selected from R to modify these graphs?

Problem 3.

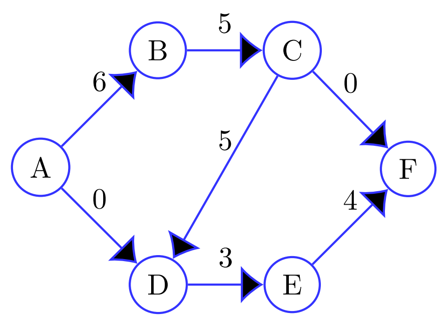
The next path selected was $A \rightarrow D \rightarrow E \rightarrow F$ in graph R. What is the flow capacity of that path?

The resulting working graphs are these:

Graph F



Graph R



Problem 4.

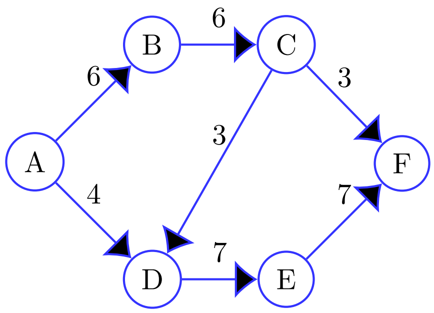
We select path $A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F$. What is the flow capacity of that path?

Problem 5.

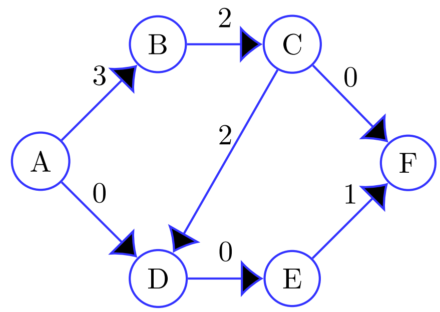
The paths selected always start from node A and end with node F. What is different about these nodes compared to the others?

Here are the final working graphs F and R.

Graph F



Graph R



Problem 6.

At this point, the algorithm is finished. How can we know the algorithm is done by examining graph R?

Problem 7.

For nodes B, C, D, and E, what is the relationship between the in-flows and the out-flows? Why does that relationship have to exist?

Problem 8.

Using the final flow graph F above, determine the maximum flow of graph $G1$.

Problem 9.

In graph F, the outflow of A is equal to the inflow of F. Should that always be the case?

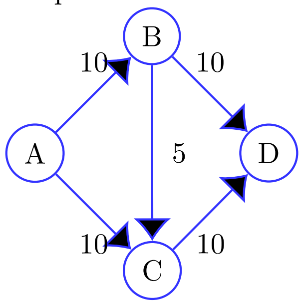
Problem 10.

Node A is called a *source node* and node F is called a *sink node*. Would this technique work if there were multiple source and sink nodes? Why or why not?

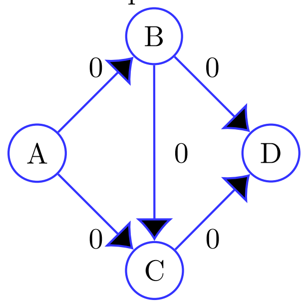
Graph $G2$

Now we are going to look at a case that messes up the algorithm.

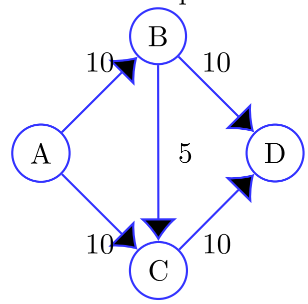
Graph $G2$



Flow Graph



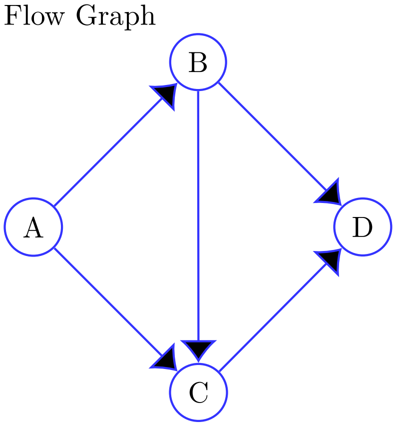
Residual Graph



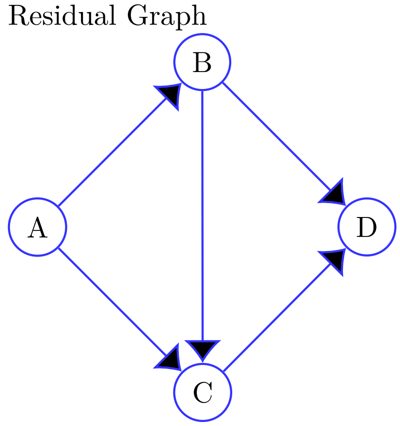
Problem 11.
The algorithm picks path $A \to B \to C \to D$. What is the capacity of that path?

Problem 12.
Update the flow and residual graphs as a result of selecting this path.

Flow Graph



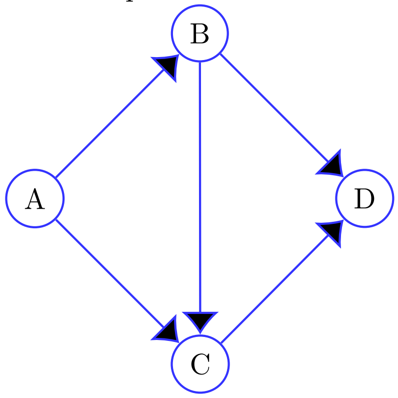
Residual Graph



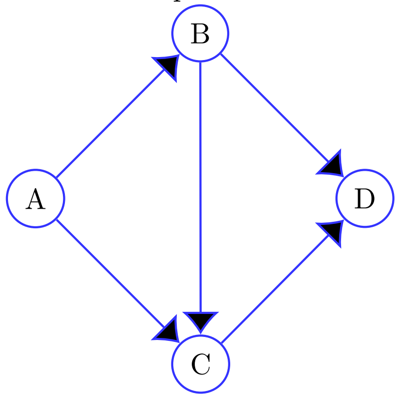
Problem 13.
Select path $A \to B \to D$ from the above residual graph. What is the capacity of that path?

Problem 14.
Update the flow and residual graphs as a result of selecting this path.

Flow Graph



Residual Graph



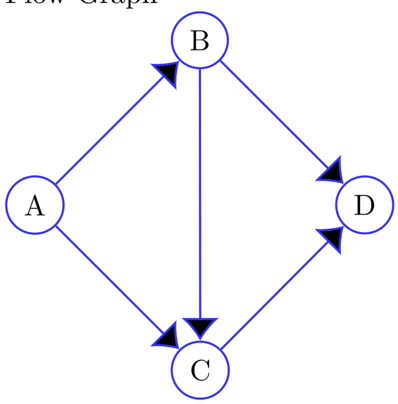
Problem 15.

Select path $A \to C \to D$. What is the capacity of that path?

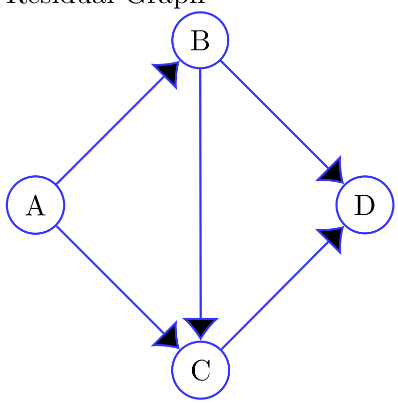
Problem 16.

Update the flow and residual graphs as a result of selecting this path.

Flow Graph



Residual Graph



At this point, the algorithm is finished.

Problem 17.

What is the maximum network flow of $G2$, according to the algorithm?

Problem 18.

Is this number correct? Why or why not? Examine $G2$ to verify your answer.

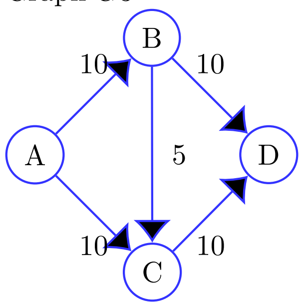
Problem 19.

Suppose a maximum flow is running through $G2$. What would be the flow on edge $B \to C$ in this situation? Would it change the total flow of $G2$ if we deleted $B \to C$?

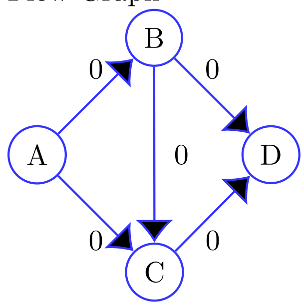
Graph $G3$

We are going to modify the algorithm. Starting again with the previous graph, we make a new kind of residual graph. The dotted edges are added, and are legal edges to be traversed in the residual graph.

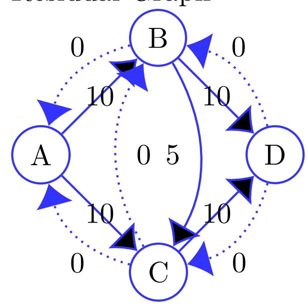
Graph $G3$



Flow Graph



Residual Graph

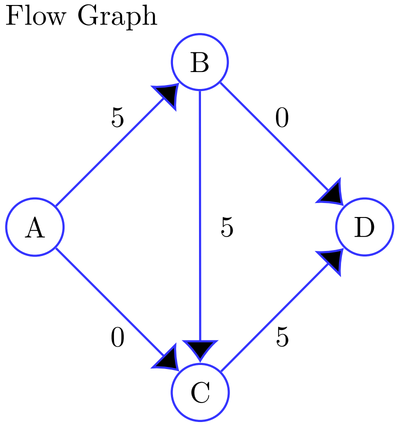


Problem 20.

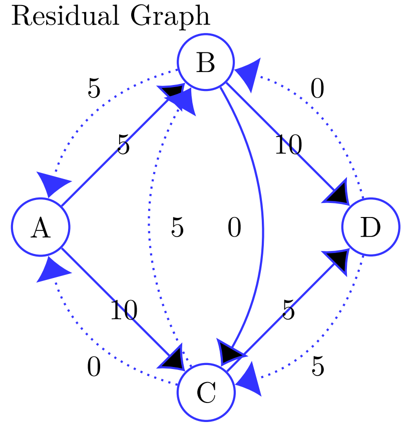
Select path $A \to B \to C \to D$. What is the capacity of that path?

Here are the updated flow and residual graphs:

Flow Graph

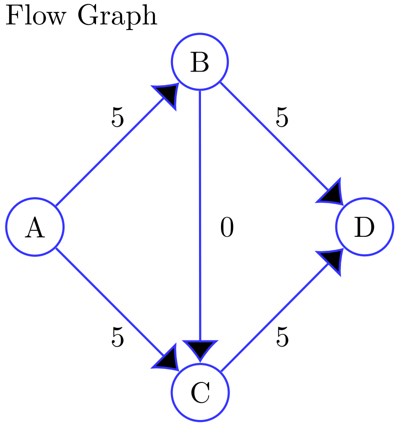


Residual Graph

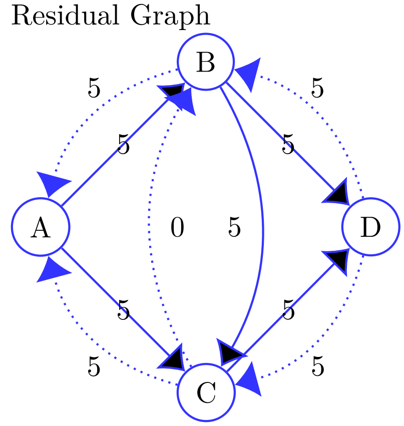


Now we select path $A \to C \to B \to D$.
Here are the updated flow and residual graphs:

Flow Graph



Residual Graph



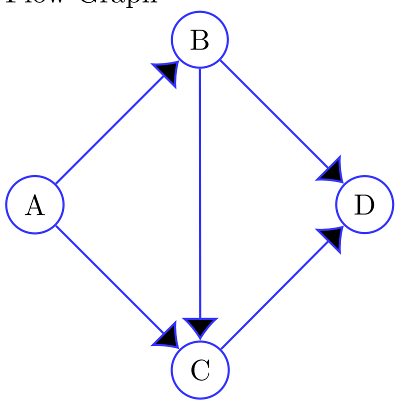
Problem 21.

Our algorithm adds capacity to the reverse edges when we update the residual graph. In your own words, can you explain what the reverse edges represent?

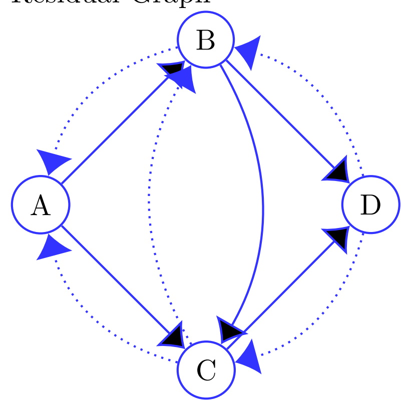
Problem 22.

Select path $A \to B \to C \to D$. (Yes, we are repeating this path.) What are the resulting flow and residual graphs?

Flow Graph



Residual Graph

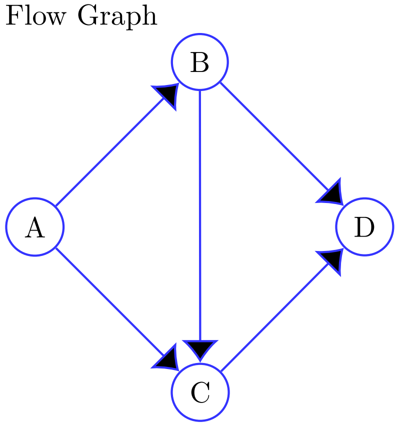


Problem 23.

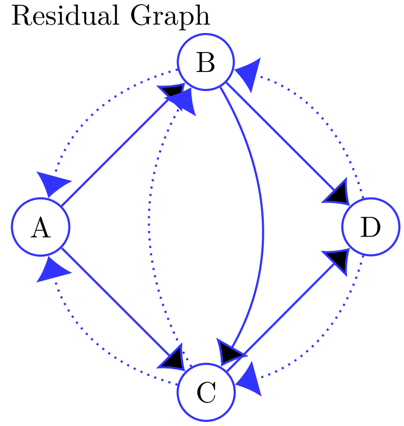
Now we select path $A \to C \to B \to D$.

What are the updated flow and residual graphs?

Flow Graph



Residual Graph



Problem 24.

At this point, the algorithm should be done. Is the final network flow accurate now?

Reflection

Congratulations! You have discovered the Ford-Fulkerson method of determining the maximum flow of a network!

Problem 25.
What was the strongest aspect of your performance as a team?

Problem 26.
How could your team perform even more effectively on the next activity?

Problem 27.
What insights did you have working on this activity?