Data Structures and Algorithms
Bloom and Counting Sketches

CS 225
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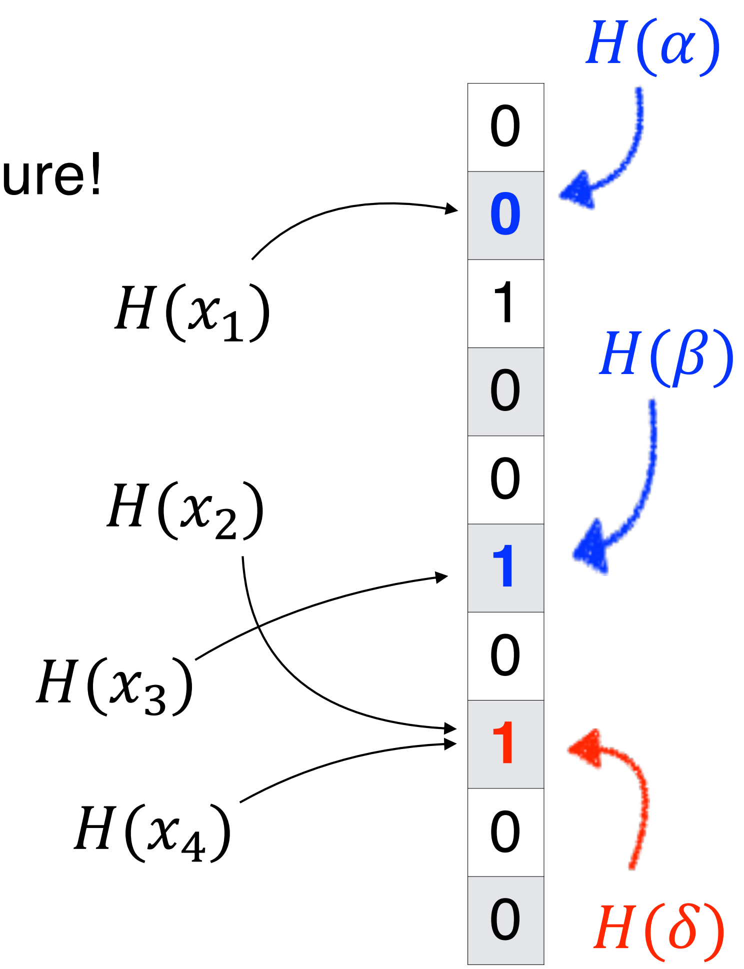
Bloom Filter: Search

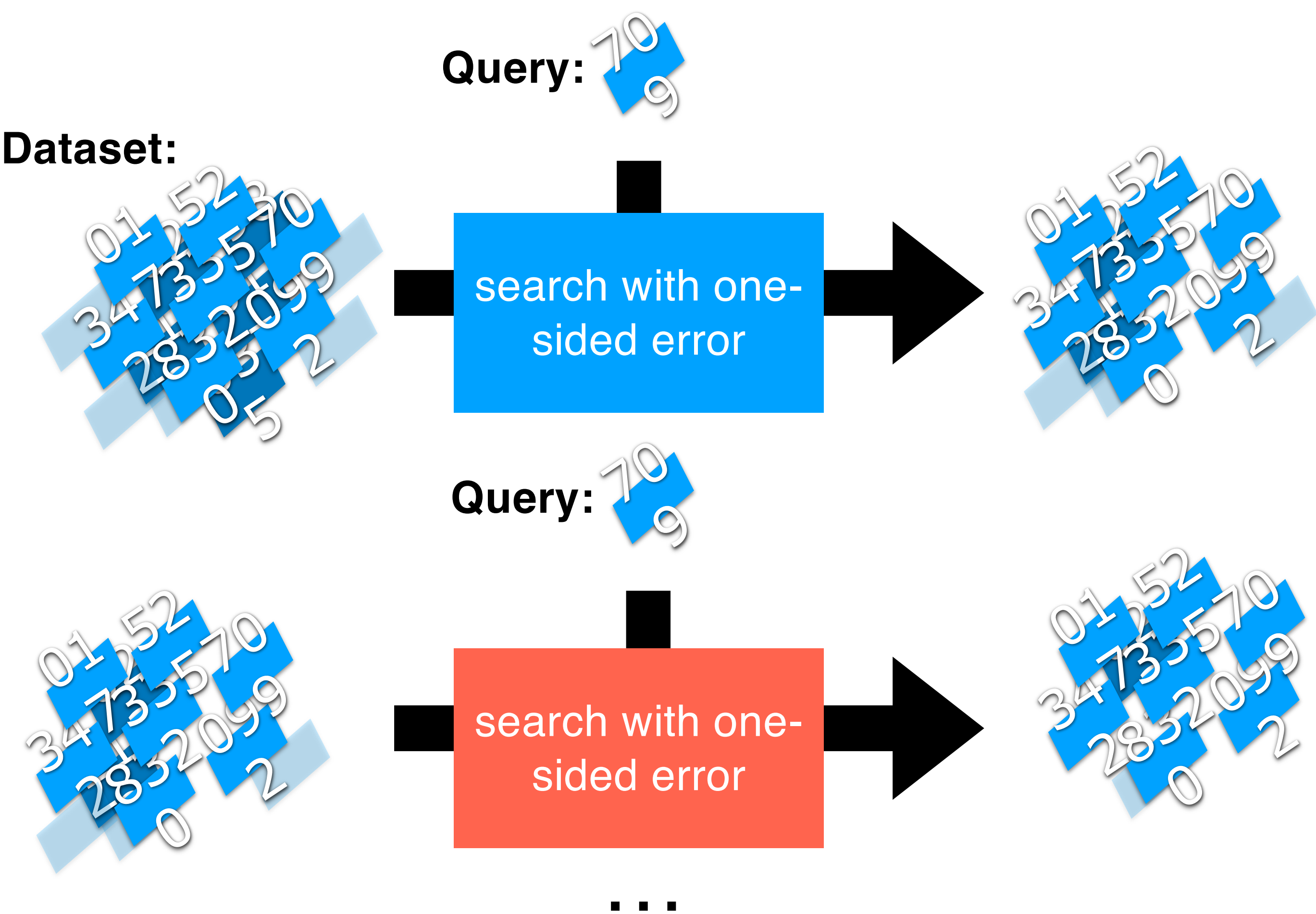
The bloom filter is a *probabilistic* data structure!

If the value in the BF is 0:

If the value in the BF is 1:



Probabilistic Accuracy: One-sided error

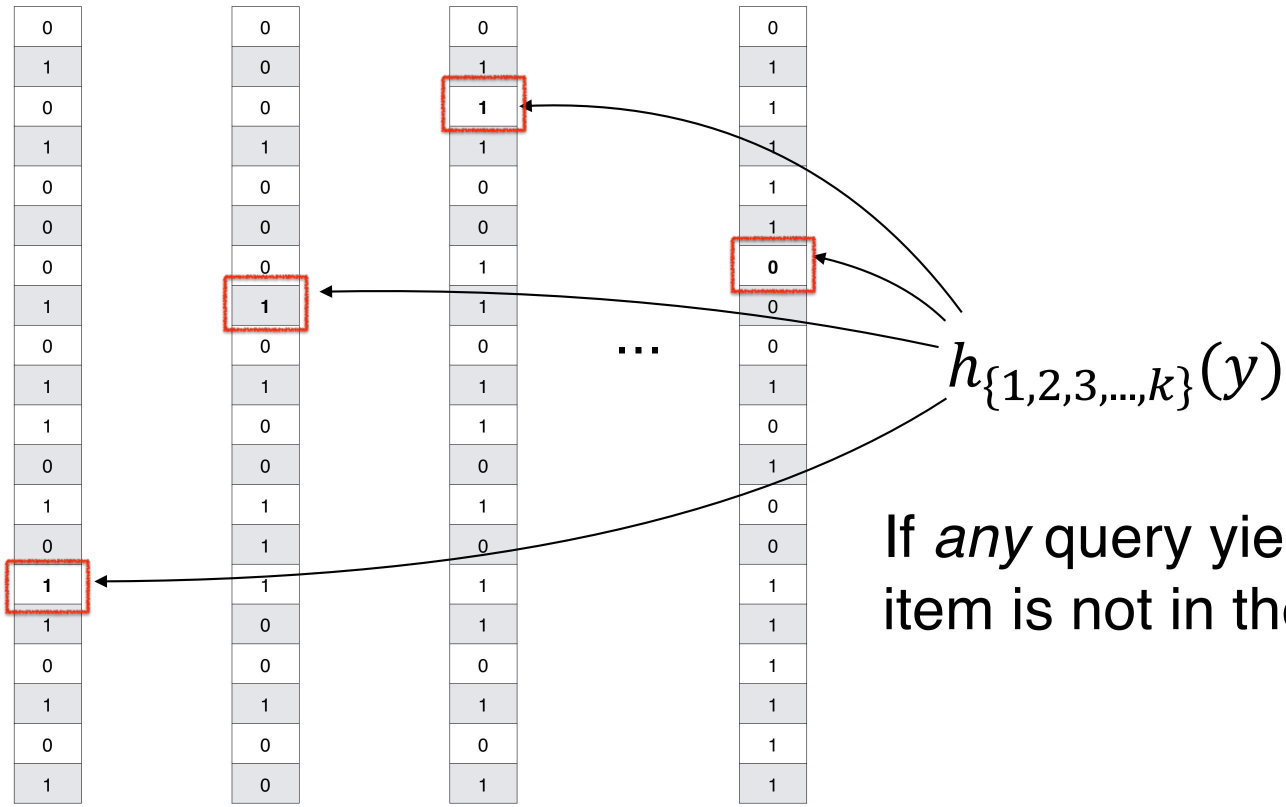


Bloom Filter: Repeated Trials

0	0	0		0
1	0	1		1
0	0	1		1
1	1	1		1
0	0	0		1
0	0	0		1
0	0	1		0
1	1	1		0
0	0	0	…	0
1	1	1		1
1	0	1		0
0	0	0		1
1	1	1		0
0	1	0		0
1	1	1		1
1	0	1		1
0	0	0		1
1	1	1		1
0	0	0		1
1	0	1		1

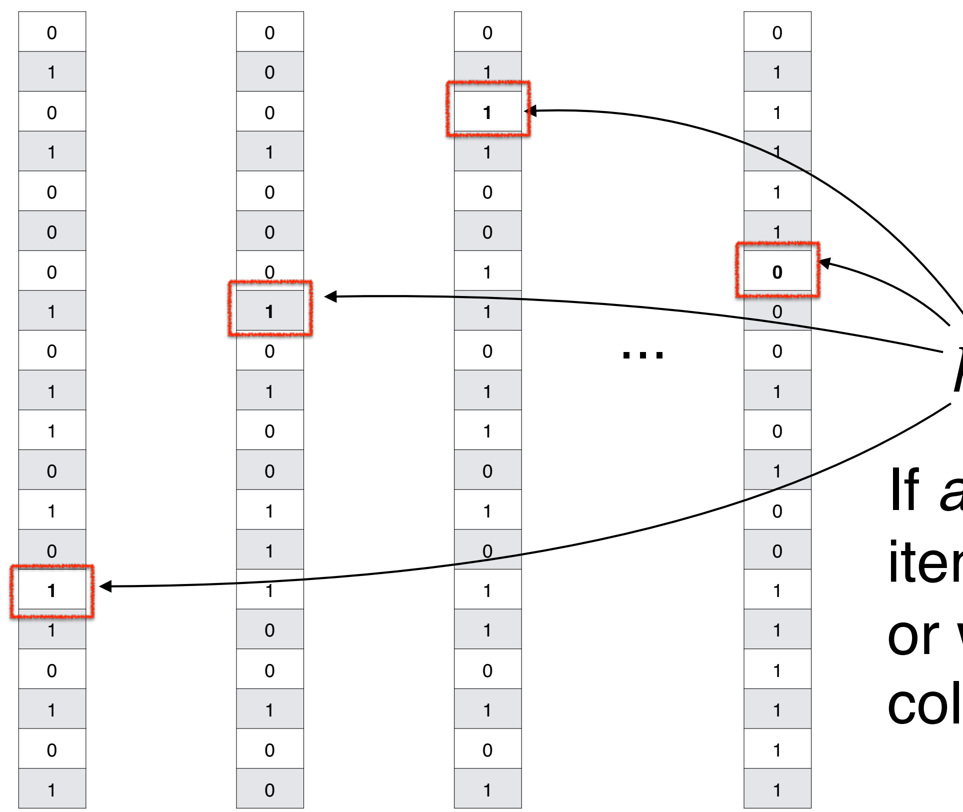
$$h_{\{1,2,3,\ldots,k\}}(y)$$

Bloom Filter: Repeated Trials



If *any* query yields 0, item is not in the set

Bloom Filter: Repeated Trials



If *all* queries yield 1, item *may* be in the set; or we might have collided k times

Bloom Filter: Repeated Trials

But doesn’t this hurt our storage costs by storing k separate filters?

h_1	h_2	h_3	$\cdots$	h_k
0	0	0		0
1	0	1		1
0	0	1		1
1	1	1		1
0	0	0		1
0	0	0		1
0	0	1		0
1	1	1		0
0	0	0		0
1	1	1		1
1	0	1		0
0	0	0		1
1	1	1		0
0	1	0		0
1	1	1		1
1	0	1		1
0	0	0		1
1	1	1		1
0	0	0		1
1	0	1		1

Bloom Filter: Repeated Trials

Rather than use a new filter for each hash, one filter can use k hashes

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

$S = \{ 6, 8, 4 \}$

$h_1(x) = x \% 10$ **$h_2(x) = 2x \% 10$** **$h_3(x) = (5+3x) \% 10$**

Bloom Filter: Repeated Trials

Rather than use a new filter for each hash, one filter can use k hashes

0	0
1	0
2	1
3	1
4	1
5	0
6	1
7	1
8	1
9	1

$h_1(x) = x \% 10$ **$h_2(x) = 2x \% 10$** **$h_3(x) = (5+3x) \% 10$**

`_find(1)`

`_find(16)`

Bloom Filter

A probabilistic data structure storing a set of values

Built from a bit vector of length m and k hash functions

Insert / Find runs in: ________________

Delete is not possible (yet)!

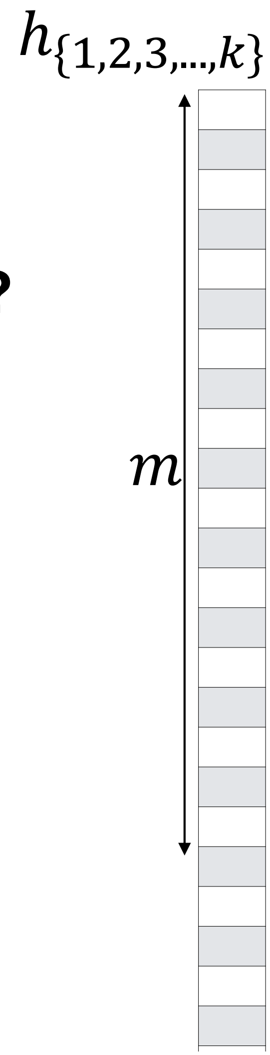
$$H = \{h_1, h_2, \ldots, h_k\}$$

0
0
1
0
0
1
0
1
0
0

Bloom Filter: Error Rate

Given bit vector of size m and k SUHA hash function

What is our expected FPR after n objects are inserted?

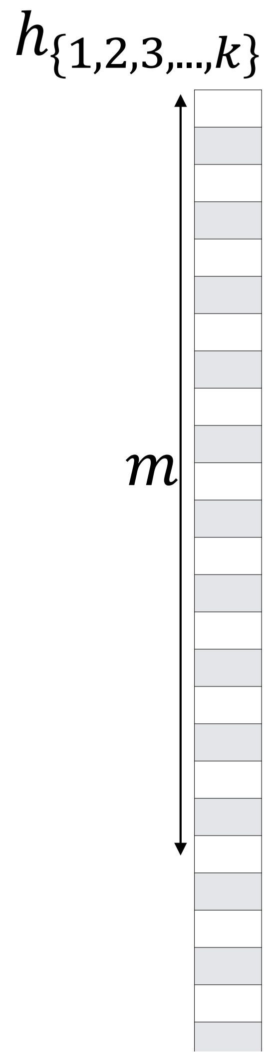


Bloom Filter: Error Rate

Given bit vector of size m and 1 SUHA hash function

What's the probability a specific bucket is 1 after one object is inserted?

Same probability given k SUHA hash function?

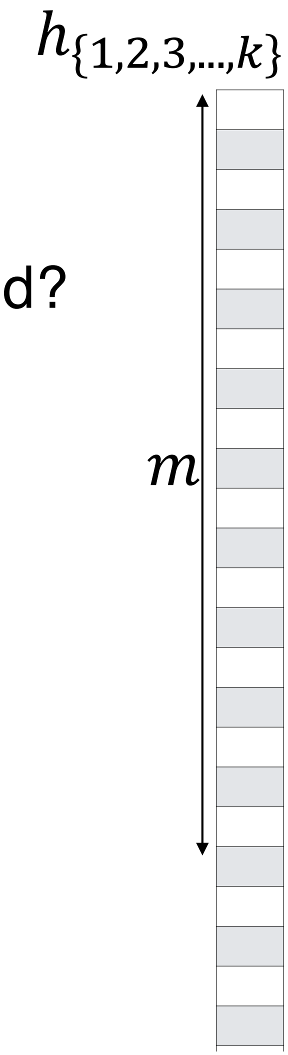


Bloom Filter: Error Rate

Given bit vector of size m and k SUHA hash function

Probability a specific bucket is 0 after one object is inserted?

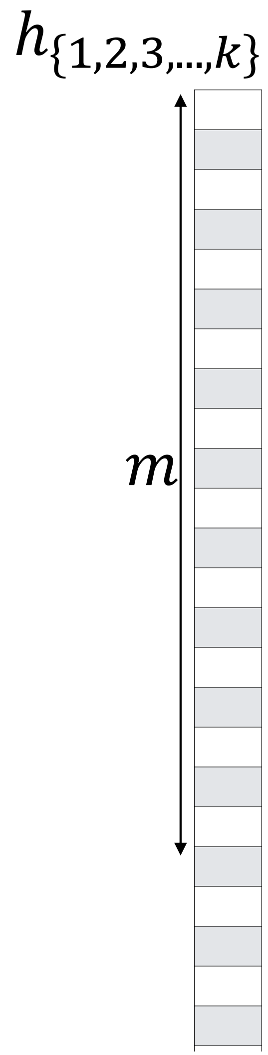
After n objects are inserted?



Bloom Filter: Error Rate

Given bit vector of size m and k SUHA hash function

What's the probability a specific bucket is 1 after n objects are inserted?



Bloom Filter: Error Rate

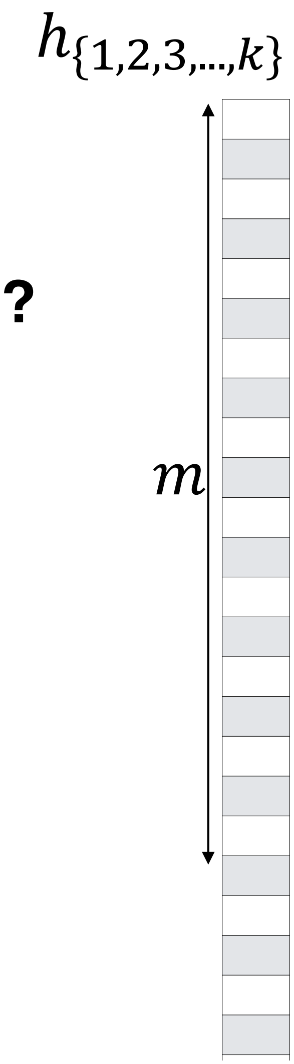
Given bit vector of size m and k SUHA hash function

What is our expected FPR after n objects are inserted?

The probability my bit is 1 after n objects inserted

$$\left(1-\left(1-\frac{1}{m}\right)^{nk}\right)^{k}$$

The number of [assumed independent] trials



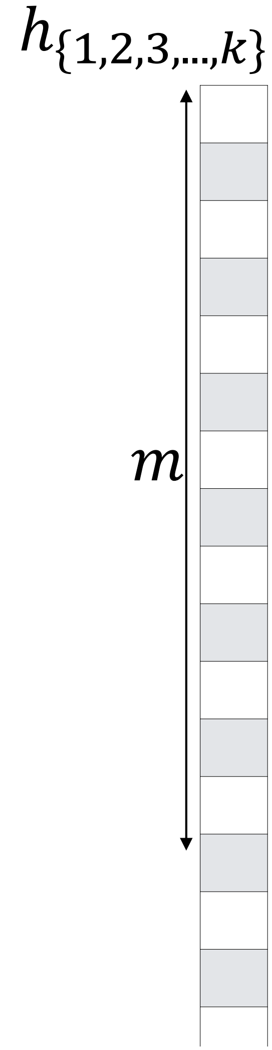
Bloom Filter: Error Rate

Vector of size m, k SUHA hash function, and n objects

To minimize the FPR, do we prefer…

(A) large k **(B) small k**

$$\left(1-\left(1-\frac{1}{m}\right)^{nk}\right)^{k}$$



Bloom Filter: Optimal Error Rate

So how can we find the minimum error rate?

Bloom Filter: Optimal Error Rate

Claim: The optimal hash function is when $k * = ln2 \cdot \frac{m}{n}$

$$\left(1 - \left(1 - \frac{1}{m}\right)^{nk}\right)^{k} \approx \left(1 - e^{\frac{-nk}{m}}\right)^{k}$$

$$\frac{d}{dk}\left(1 - e^{\frac{-nk}{m}}\right)^{k} \approx \frac{d}{dk}\left(kln(1 - e^{\frac{-nk}{m}})\right)$$

Derivative is zero when $k^{*} = \ln 2 \cdot \frac{m}{n}$

Bloom Filter: Error Rate

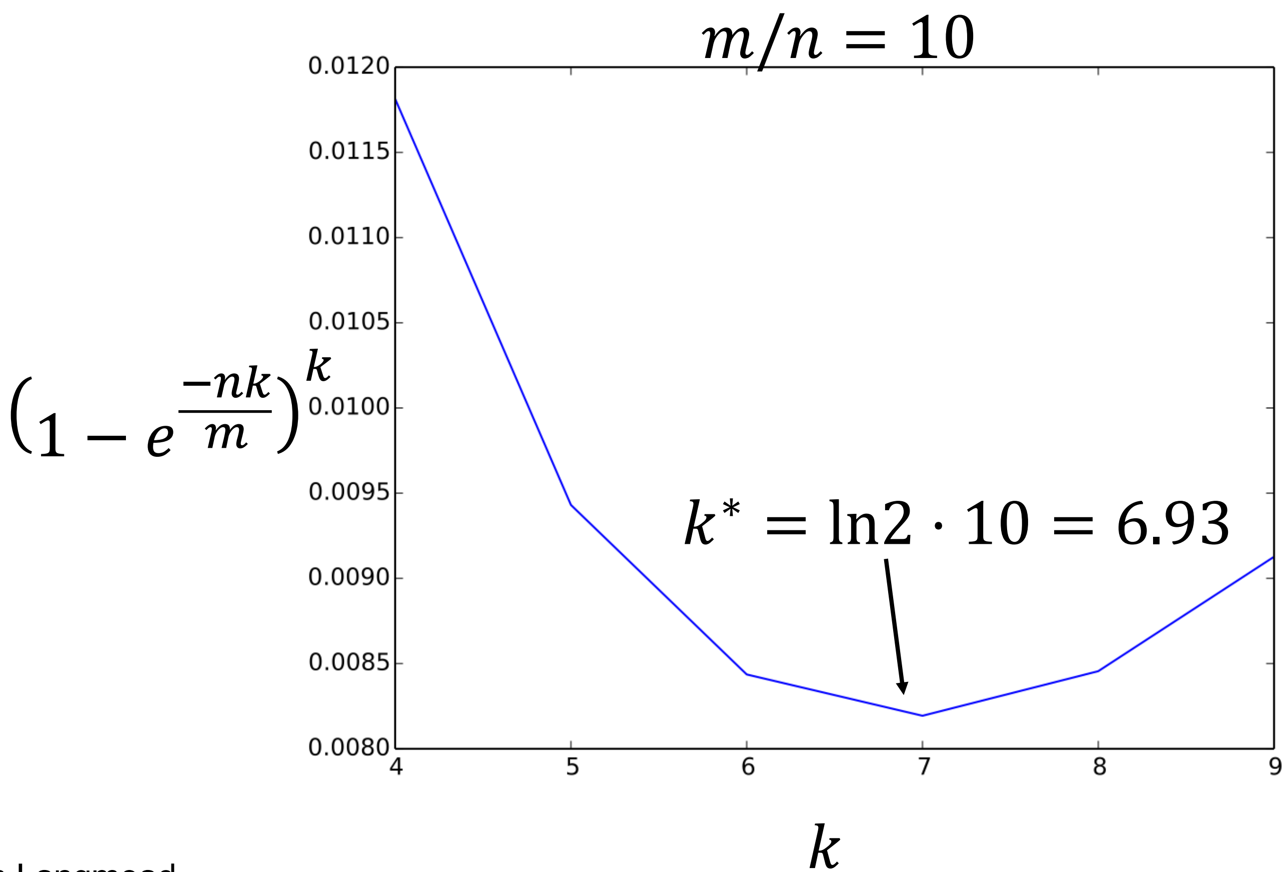


Figure by Ben Langmead

Bloom Filter: Optimal Parameters

$$\boxed{k^* = \ln 2 \cdot \frac{m}{n}}$$

Given any two values, we can optimize the third

$n = 100$ items $k = 3$ hashes $m =$

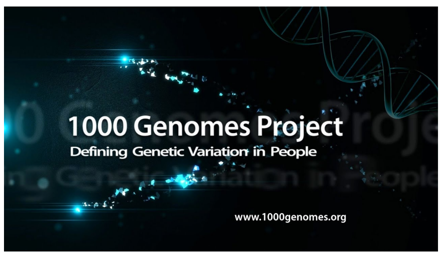
$m = 100$ bits $n = 20$ items $k =$

$m = 100$ bits $k = 2$ items $n =$

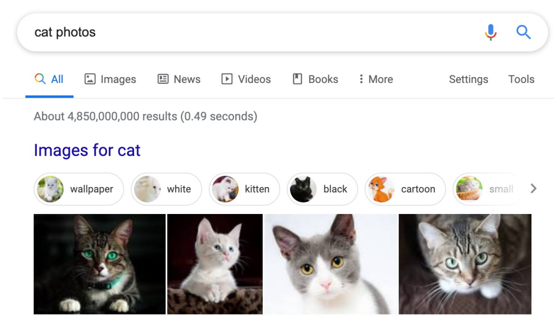
Bloom Filter: Optimal Parameters

$$\boxed{m = \frac{nk}{\ln 2} \approx 1.44 \cdot nk}$$

Optimal hash function is still O(n)!

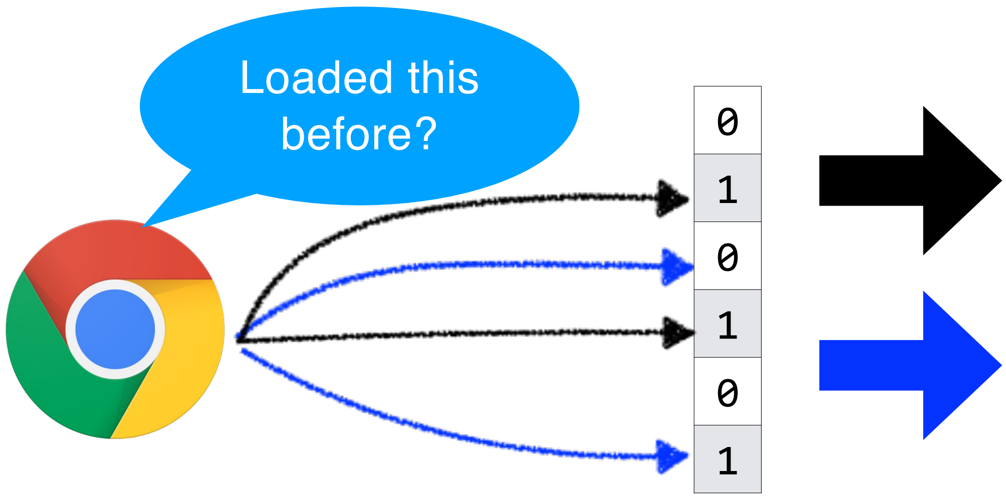


n = 250,000 files vs ~10^{15} nucleotides vs 260 TB



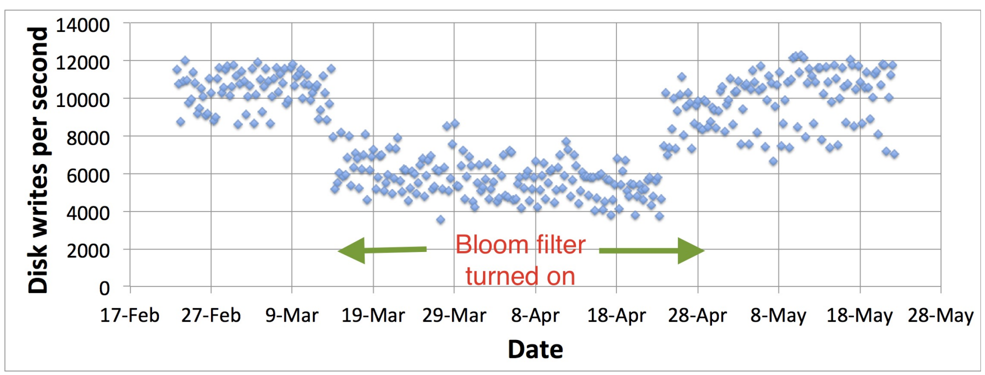
n = 60 billion — 130 trillion

Bloom Filter: Website Caching



Cache this page!

Add to filter (but don't cache!)

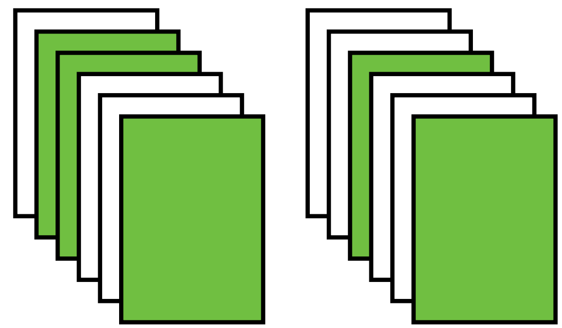


Maggs, Bruce M., and Ramesh K. Sitaraman. **Algorithmic nuggets in content delivery.** *ACM SIGCOMM Computer Communication Review* 45.3 (2015): 52-66.

Sequence Bloom Trees

Imagine we have a large collection of text…

And our goal is to search these files for a query of interest…



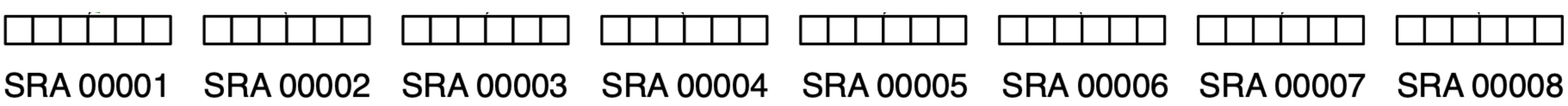
Bloom Filters: Unioning

Bloom filters can be trivially merged using bit-wise union.

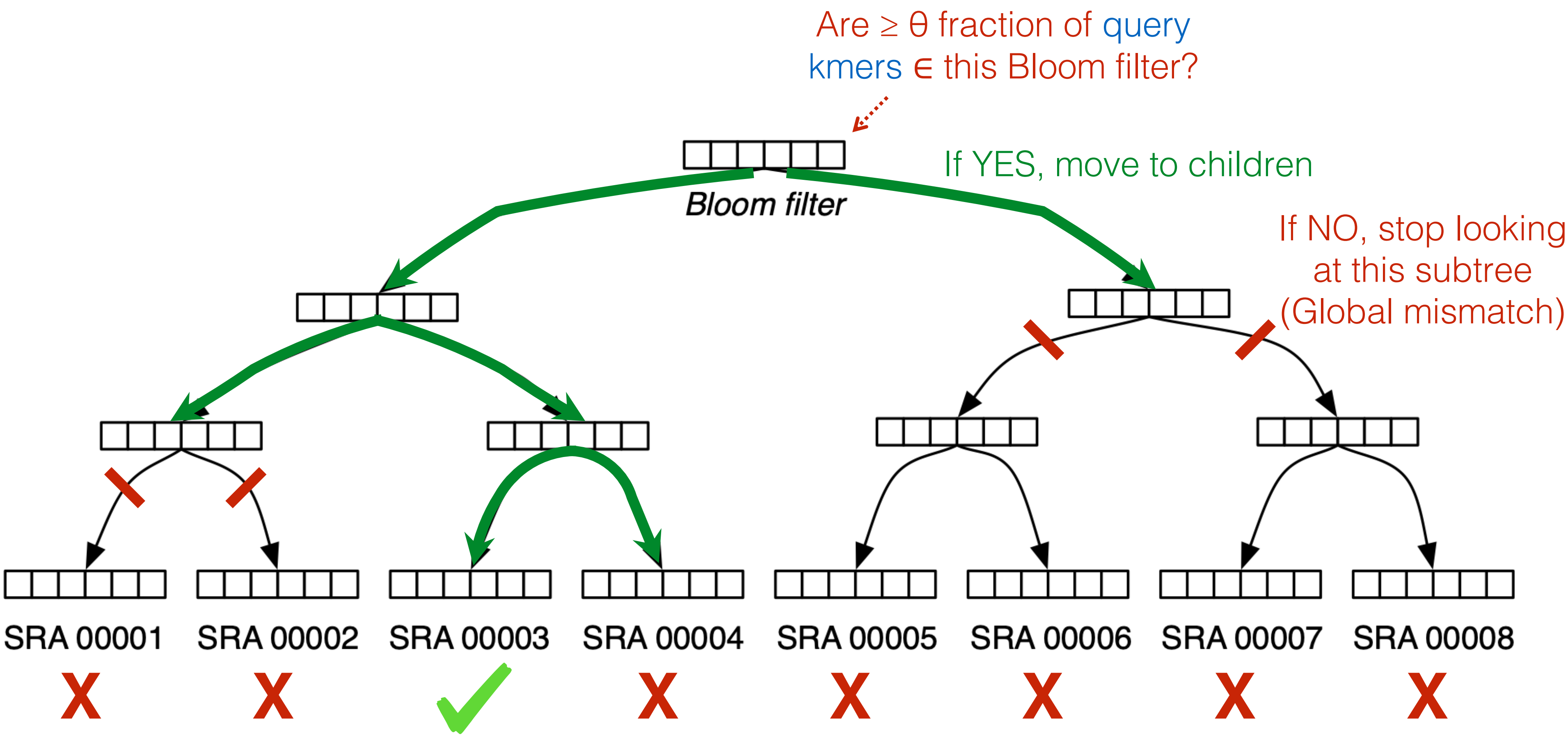
	A		B		Result
0	1	0	0	0	
1	0	1	1	1	
2	1	2	1	2	
3	1	3	0	3	
4	0	4	0	4	
5	0	5	0	5	
6	1	6	1	6	
7	0	7	1	7	
8	0	8	1	8	
9	1	9	1	9	

$A \cup B = \text{Result}$

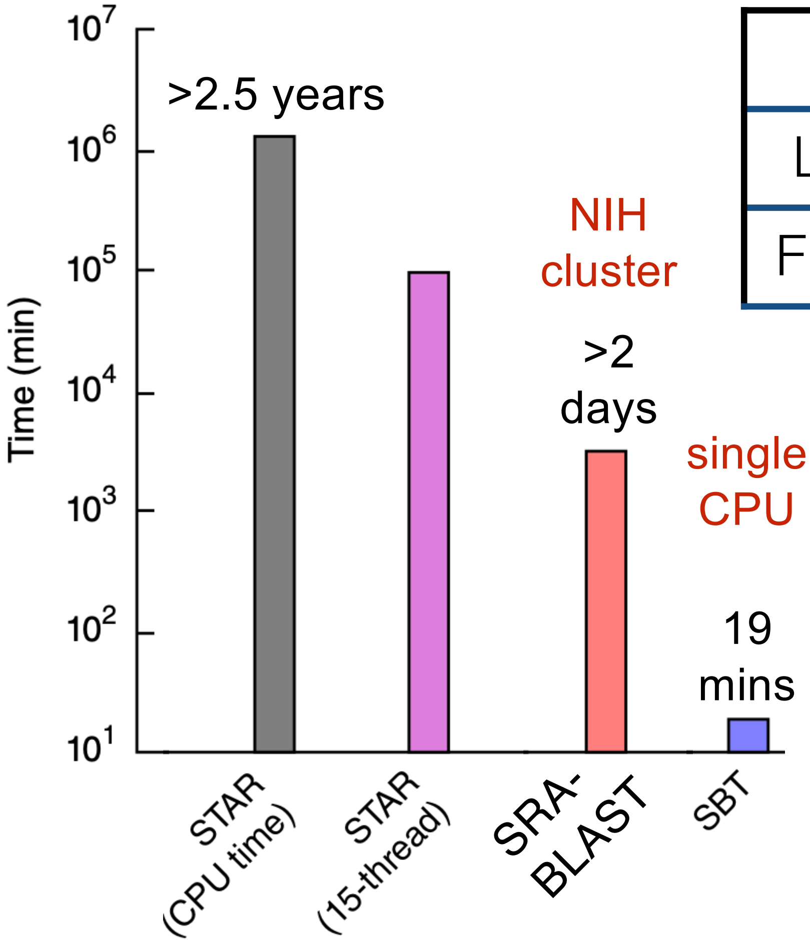
Sequence Bloom Trees



Sequence Bloom Trees



Sequence Bloom Trees



	SRA	FASTA.gz	SBT
Leaves	4966 GB	2692 GB	63 GB
Full Tree	-	-	200 GB

Solomon, Brad, and Carl Kingsford. "Fast search of thousands of short-read sequencing experiments." *Nature biotechnology* 34.3 (2016): 300-302.

Solomon, Brad, and Carl Kingsford. "Improved search of large transcriptomic sequencing databases using split sequence bloom trees." *International Conference on Research in Computational Molecular Biology*. Springer, Cham, 2017.

Sun, Chen, et al. "Allsome sequence bloom trees." *International Conference on Research in Computational Molecular Biology*. Springer, Cham, 2017.

Harris, Robert S., and Paul Medvedev. "Improved representation of sequence bloom trees." *Bioinformatics* 36.3 (2020): 721-727.

Bloom Filters: Tip of the Iceberg

Cohen, Saar, and Yossi Matias. "Spectral bloom filters." *Proceedings of the 2003 ACM SIGMOD international co*

Fan, Bin, et al. "Cuckoo filter: Practically better than bloom." *Proceedings of the 10th ACM International on Conference on emerging Networking Experiments and Technologies*. 2014.

Nayak, Sabuzima, and Ripon Patgiri. "countBF: A General-purpose High Accuracy and Space Efficient Counting Bloom Filter." *2021 17th International Conference on Network and Service Management (CNSM)*. IEEE, 2021.

Mitzenmacher, Michael. "Compressed bloom filters." *IEEE/ACM transactions on networking* 10.5 (2002): 604-612

Crainiceanu, Adina, and Daniel Lemire. "Bloofi: Multidimensional bloom filters." *Information Systems* 54 (2015): 3

Chazelle, Bernard, et al. "The bloomier filter: an efficient data structure for static support lookup tables." *Proceedings of the fifteenth annual ACM-SIAM symposium on Discrete algorithms*. 2004.

There are many more than shown here…

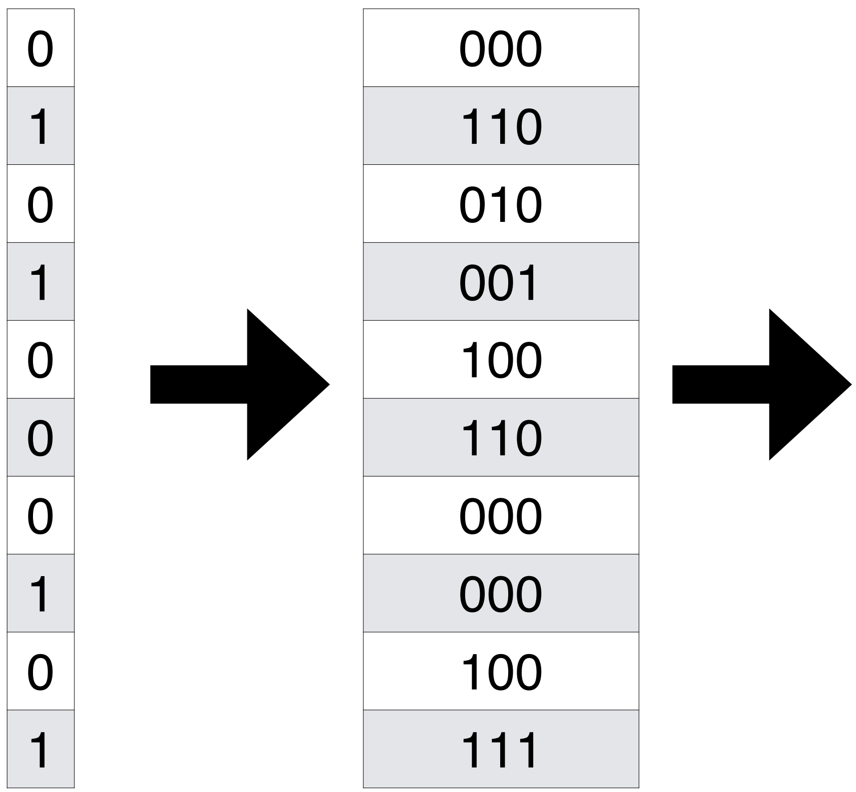
Counting Sketches

A **sketch** is a compact (reduced) representation of a dataset that acts as a replacement for calculations.

Sometimes we need more information than 'presence/absence'…

3201
946
5581
8945
6145
8126
3887
8925
1246
8324
4549
9100
3887
8499
8970

Count Min Sketch



Count Min Sketch

0
1
0
1
0
0
0
1
0
1

000
110
010
001
100
110
000
000
100
111

0	6	2	1	4	6	0	0	4	7
5	0	5	2	0	5	0	3	5	5
3	3	3	3	3	3	3	3	3	3
4	2	4	2	4	2	4	2	4	2
30	0	0	0	0	0	0	0	0	0

Count Min Sketch

A **count-min sketch** has k rows, each with its own hash function.

Each bucket is b-bits (allowing 2^b count values)

$h_1(x)$	0	0	0	0	0	0	0	0	0	0
$h_2(x)$	0	0	0	0	0	0	0	0	0	0
$h_3(x)$	0	0	0	0	0	0	0	0	0	0
$h_4(x)$	0	0	0	0	0	0	0	0	0	0
$h_5(x)$	0	0	0	0	0	0	0	0	0	0

Count Min Sketch Insertion

S = { 1, 3, 8, 16}

$h_1(k)$ = k % 7 $h_2(k)$ = k+3(k%2) % 7 $h_3(k)$ = |k - 4| % 7

h_1	0	0	0	0	0	0	0
h_2	0	0	0	0	0	0	0
h_3	0	0	0	0	0	0	0

Count Min Sketch Find

S = { 1, 3, 8, 16}

$h_1(k) = k \% 7$ $h_2(k) = k+3(k\%2) \% 7$ $h_3(k) = |k - 4| \% 7$

`_find(16)`

`_find(1)`

h_1	0	2	1	1	0	0	0
h_2	0	1	1	0	1	0	1
h_3	0	1	0	1	1	1	0

Count Min Sketch Find

What is our estimated count of x?

How many **known** collisions?

$h_1(x)$	10	5	$\boxed{13}$	17	8	1	6
$h_2(x)$	3	7	20	12	2	$\boxed{9}$	7
$h_3(x)$	12	6	7	5	$\boxed{9}$	18	3
$h_4(x)$	4	19	$\boxed{26}$	1	4	5	1
$h_5(x)$	6	6	$\boxed{8}$	11	6	7	16

Improving Count Min Sketch

Given what we know about collisions here, can we improve our insert strategy?

$h_1(x)$	10	5	$\boxed{13}$	17	8	1	6
$h_2(x)$	3	7	20	12	2	$\boxed{9}$	7
$h_3(x)$	12	6	7	5	$\boxed{9}$	18	3
$h_4(x)$	4	19	$\boxed{26}$	1	4	5	1
$h_5(x)$	6	6	$\boxed{8}$	11	6	7	16

Count Min Sketch Improved Insertion

S = { … , 1, 3, 8, 16, …}

$h_1(k) = k \% 7$ $h_2(k) = k+3(k\%2) \% 7$ $h_3(k) = |k - 4| \% 7$

h_1	7	9	10	1	5	7	6
h_2	4	8	7	5	4	10	7
h_3	15	12	2	5	8	2	1

Count Min Sketch Insertion

Minimal Increase: When inserting, only update the minimum count.

Default

h_1	7	**11**	**11**	**2**	5	7	6
h_2	4	**9**	**8**	5	**5**	10	**8**
h_3	15	**13**	2	**6**	**9**	**3**	1

Minimal Increase

h_1	7	9	10	**2**	5	7	6
h_2	4	**9**	7	5	**5**	10	7
h_3	15	12	2	5	**9**	**3**	1

Count-Min Sketch

A probabilistic data structure storing a set of values

Has **four** key properties:

- k, number of hash functions
- n, expected number of insertions
- m, filter size in ***registers***
- b**, number of bits per register**

$h_{\{1,2,3,\ldots,k\}}$

0	3	1	0
0	0	2	3
2	0	1	0
3	1	0	1
0	0	2	2

Minimal increase reduces overcounting by identifying collisions.

Count value returned here [underestimates / overestimates / matches] the true count of the query.

Trivia Point 1: Deletion is Possible!

$h_1(x)$	0	1	2	2	3	1	1
$h_2(x)$	3	2	0	0	2	2	1
$h_3(x)$	0	1	1	2	4	2	0
$h_4(x)$	4	1	3	0	2	0	0
$h_5(x)$	1	1	5	1	0	1	1

Trivia Point 2: Counting Bloom Filters

$h_1(x)$	10	5	13	17	8	1	6
$h_2(x)$	3	7	20	12	2	9	7
$h_3(x)$	12	6	7	5	9	18	3
$h_4(x)$	4	19	26	1	4	5	1
$h_5(x)$	6	6	8	11	6	7	16

Trivia Point 2: Counting Bloom Filters

S = { 16, 8, 4, 13, 29, 11, 22 }

$h_1(k) = k \% 7$ $h_2(k) = 2k+1 \% 7$

0	
1	
2	
3	
4	
5	
6	

Trivia Point 2: Counting Bloom Filters

S = { 16, 8, 4, 13, 29, 11, 22 }

$h_1(k) = k \% 7$ **$h_2(k) = 2k+1 \% 7$**

0	0
1	3
2	3
3	3
4	2
5	1
6	2

Next Time: Cardinality

Cardinality: how many *distinct* values in a data stream?

946
5581
8945
6145
8126
3887
8925
1246
8324
4549
9100
5598
8499
8970
3921
8575
4859
4960