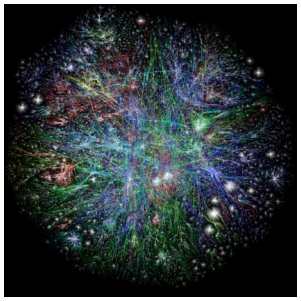
CS 225

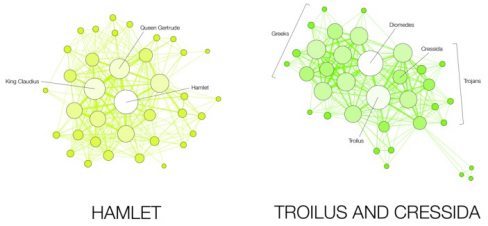
Data Structures

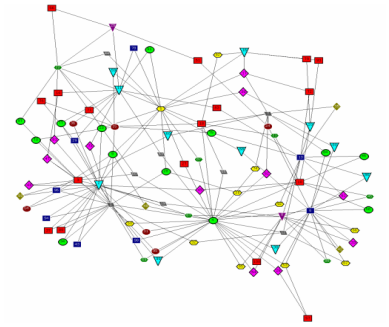
March 31 – Graph Traversals

G Carl Evans

Graphs

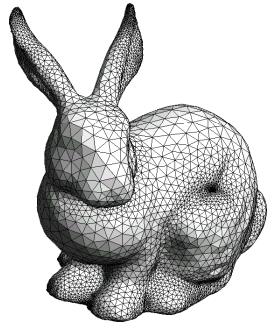


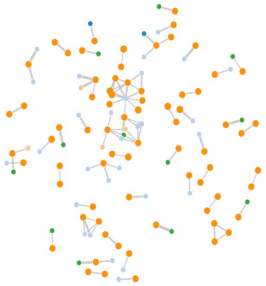


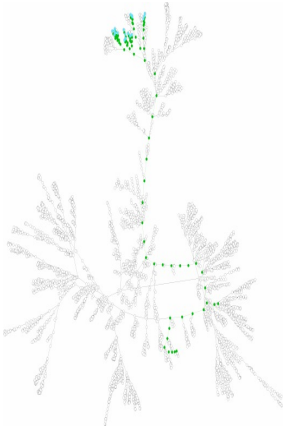


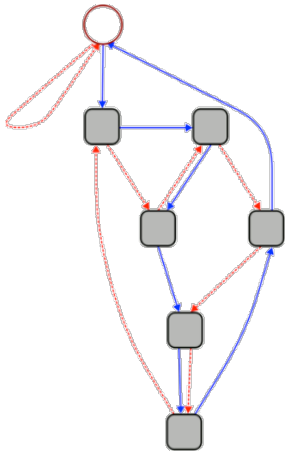
To study all of these structures:

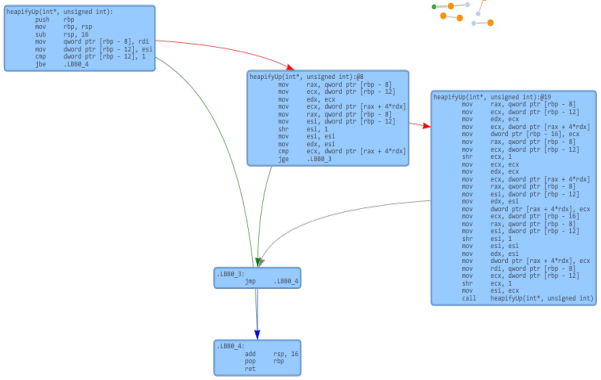
1. A common vocabulary
2. Graph implementations
3. Graph traversals
4. Graph algorithms











```
 1  BFS(G):
 2    Input: Graph, G
 3    Output: A labeling of the edges on
 4        G as discovery and cross edges
 5
 6    foreach (Vertex v : G.vertices()):
 7      setLabel(v, UNEXPLORED)
 8    foreach (Edge e : G.edges()):
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23          foreach (Vertex w : G.adjacent(v)):
24            q.enqueue(w,v)
25      else
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```

BFS Analysis

Q: Does our implementation handle disjoint graphs? If so, what code handles this?

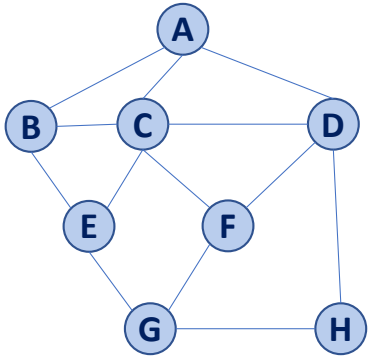
- ***How do we use this to count components?***

Q: Does our implementation detect a cycle?

- ***How do we update our code to detect a cycle?***

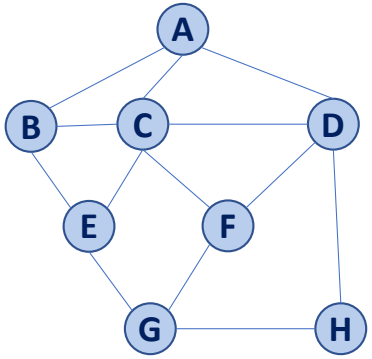
Q: What is the running time?

Traversal: BFS



v	d	P	Adjacent Edges
A			
B			
C			
D			
E			
F			
G			
H			

Traversal: BFS



v	d	P	Adjacent Edges
A	0	-	C B D
B			A C E
C			B A D E F
D			A C F H
E			B C G
F			C D G
G			E F H
H			D G

A

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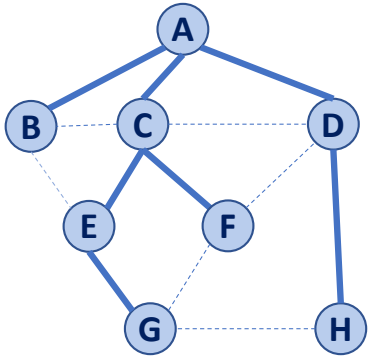
Running time of BFS



While-loop at **:18**?

For-loop at **:23**?

v	d	P	Adjacent Edges
A	0	-	C B D
B	1	A	A C E
C	1	A	B A D E F
D	1	A	A C F H
E	2	C	B C G
F	2	C	C D G
G	3	E	E F H
H	2	D	D G

G H F E D B C A

BFS Observations

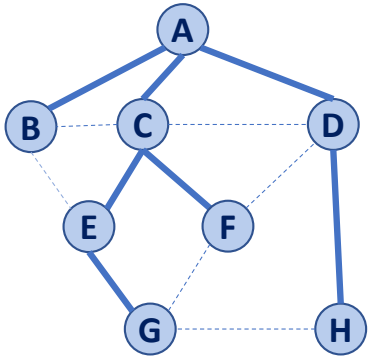
Q: What is a shortest path from **A** to **H**?

Q: What is a shortest path from **E** to **H**?

Q: How does a cross edge relate to **d**?

Q: What structure is made from discovery edges?

v	d	P	Adjacent Edges
A	0	-	**C B D**
B	1	**A**	**A C E**
C	1	**A**	**B A D E F**
D	1	**A**	**A C F H**
E	2	**C**	**B C G**
F	2	**C**	**C D G**
G	3	**E**	**E F H**
H	2	**D**	**D G**



BFS Observations

Obs. 1: BFS can be used to count components.

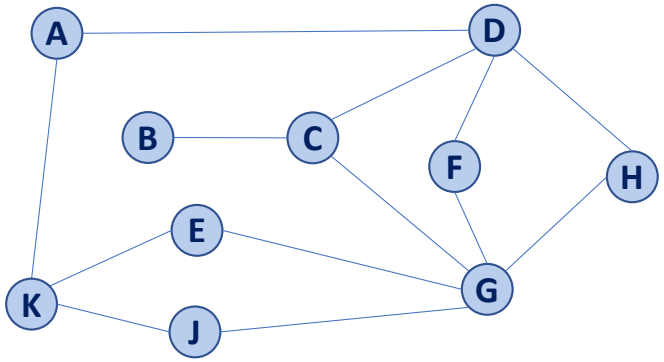
Obs. 2: BFS can be used to detect cycles.

Obs. 3: In BFS, **d** provides the shortest distance to every vertex.

Obs. 4: In BFS, the endpoints of a cross edge never differ in distance, **d**, by more than 1:

$$|d(u) - d(v)| = 1$$

Traversal: DFS



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```

```
 1  DFS(G):
 2    Input: Graph, G
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 4        G as discovery and back edges
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 6    foreach (Vertex v : G.vertices()):
 7      setLabel(v, UNEXPLORED)
 8    foreach (Edge e : G.edges()):
 9      setLabel(e, UNEXPLORED)
10    foreach (Vertex v : G.vertices()):
11      if getLabel(v) == UNEXPLORED:
12         DFS(G, v)
```

```
14  DFS(G, v):
15    Stack q
16    q.enqueue(v)
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18    while !q.empty()
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21          setLabel(v, VISITED)
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23          foreach (Vertex w : G.adjacent(v)):
24            q.enqueue(w,v)
25      else
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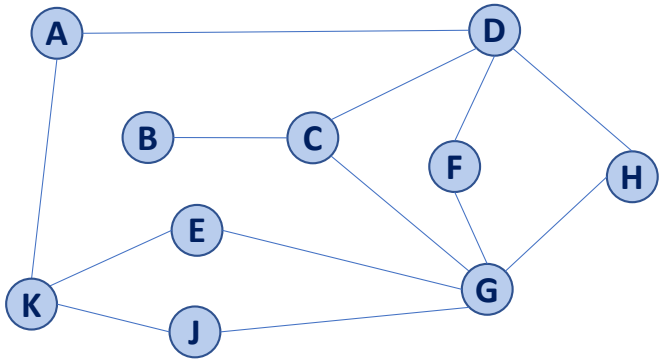
Traversal: DFS



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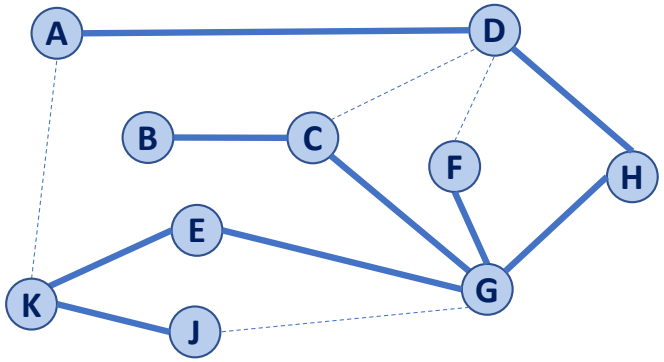

Discovery Edge

Back Edge

Running time of DFS

Labeling:

- Vertex:

- Edge:

Queries:

- Vertex:

- Edge: