



CS 225

Data Structures

*March 1 – Btrees Analysis and
Functions as Data*

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Btree Properties

A **BTree** of order **m** is an m-way tree:

- All keys within a node are ordered
- All leaves contain no more than **m-1** keys.

- All internal nodes have exactly **one more child than keys**
- Root nodes can be a leaf or have **[2, m]** children.
- All non-root, internal nodes have **[ceil(m/2), m]** children.

- All leaves are on the same level



BTree Analysis

The height of the BTree determines maximum number of _____ possible in search data.

...and the height of the structure is: _____.

Therefore: The number of seeks is no more than _____.

...suppose we want to prove this!



BTree Analysis

In our AVL Analysis, we saw finding an upper bound on the height (given n) is the same as finding a lower bound on the nodes (given h).

We want to find a relationship for BTrees between the number of keys (n) and the height (h).



BTree Analysis

Strategy:

We will first count the number of nodes, level by level.

Then, we will add the minimum number of keys per node (**n**).

The minimum number of nodes will tell us the largest possible height (**h**), allowing us to find an upper-bound on height.



BTree Analysis

The minimum number of **nodes** for a BTree of order m **at each level:**

root:

level 1:

level 2:

level 3:

...

level h :



BTree Analysis

The **total number of nodes** is the sum of all of the levels:



BTree Analysis

The **total number of keys:**



BTree Analysis

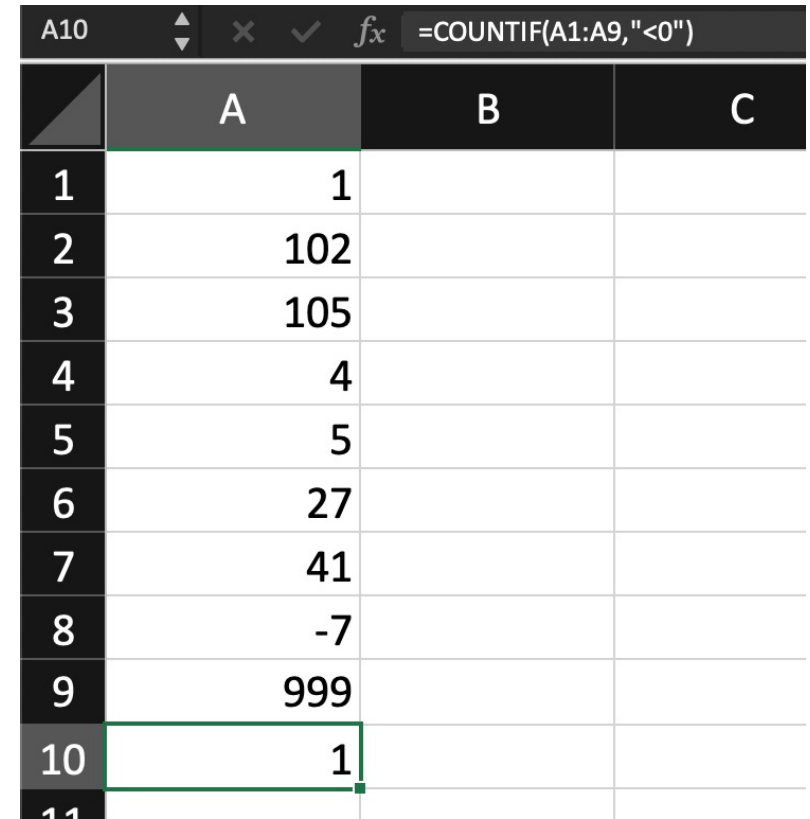
The **smallest total number of keys** is:

So an inequality about **n** , the total number of keys:

Solving for **h** , since **h** is the number of seek operations:

Functions As Data

Consider the function from Excel
`COUNTIF(range, criteria)`



The screenshot shows an Excel spreadsheet with a formula bar at the top displaying `=COUNTIF(A1:A9,"<0")`. The spreadsheet has three columns labeled A, B, and C, and rows numbered 1 through 11. Column A contains the following values: 1, 102, 105, 4, 5, 27, 41, -7, 999, 1, and 1. The cell containing the value 1 in row 10, column A is highlighted with a green border, indicating it is the result of the COUNTIF function applied to the range A1:A9 with the criteria "<0".

	A	B	C
1	1		
2	102		
3	105		
4	4		
5	5		
6	27		
7	41		
8	-7		
9	999		
10	1		
11	1		

COUNTIF in C++

Countif.hpp

```
10 template <typename Iter, typename Pred>
11 int Countif(Iter begin, Iter end, Pred pred) {
12     int count = 0;
13     auto cur = begin;
14
15     while(cur != end) {
16         if(pred(*cur))
17             ++count;
18         ++cur;
19     }
20
21     return count;
22 }
```

Ways to use Countif()

main.cpp

```
12 bool isNegative(int num) { return (num < 0); }
13
14 class IsNegative {
15 public:
16     bool operator() (int num) { return (num < 0); }
17 };
18
19 int main() {
20     std::vector<int> numbers = {1, 102, 105, 4, 5, 27, 41, -7, 999};
21
22     auto isnegl = [](int num) { return (num < 0); };
23     auto isnegfp = isNegative;
24     auto isnegfactor = IsNegative();
25
26     std::cout << "There are " << Countif(numbers.begin(), numbers.end(), _____)
27         << " negative numbers" << std::endl;
```



Lambdas in C++ (functions with no name)

```
[           ](           ){           }
```

Power of the lambda

main.cpp

```
29 int big;
30 std::cout << "How big is big? ";
31 std::cin >> big;
32
33 auto isbig = [big](int num) { return (num >= big); };
34
35 std::cout << "There are " << Countif(numbers.begin(), numbers.end(), isbig)
36 << " big numbers" << std::endl;
37 }
38
```