

CS 225

Data Structures

February 27 – BTrees

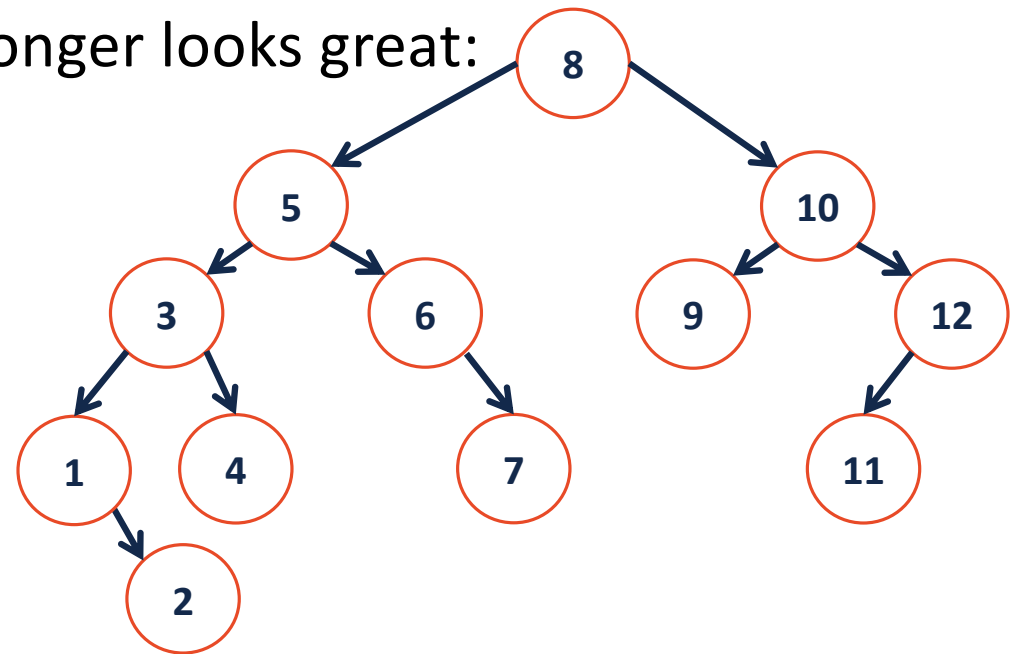
G Carl Evans

B-Tree Motivation

In Big-O we have assumed uniform time for all operations, but this isn't always true.

However, seeking data from the cloud may take 40ms+.

...an $O(\lg(n))$ AVL tree no longer looks great:





BTree Design Motivations

Knowing that we have large seek times for data, we want to:

BTree (of order m)

-3	8	23	25	31	42	43	55
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m=9

Goal: Minimize the number of reads!

Build a tree that uses _____ / node
[1 network packet]
[1 disk block]

BTree Insertion

A **BTree** of order **m** is an m-way tree:

- All keys within a node are ordered
- All leaves contain hold no more than **m-1** keys.



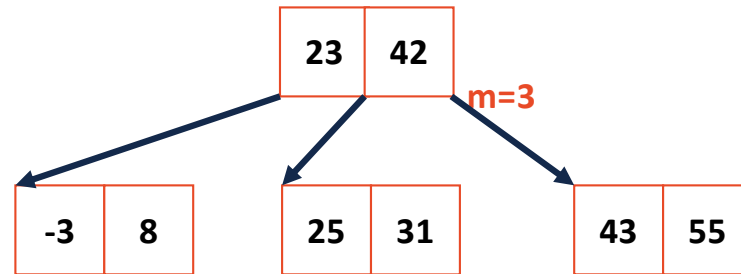


BTree Insertion

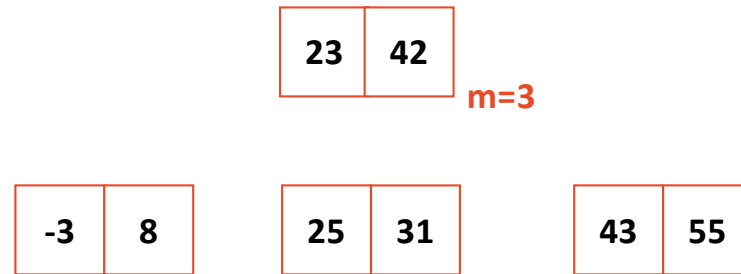
When a BTree node reaches **m** keys:



BTree Recursive Insert



BTree Recursive Insert





BTree Visualization/Tool

<https://www.cs.usfca.edu/~galles/visualization/BTree.html>



Btree Properties

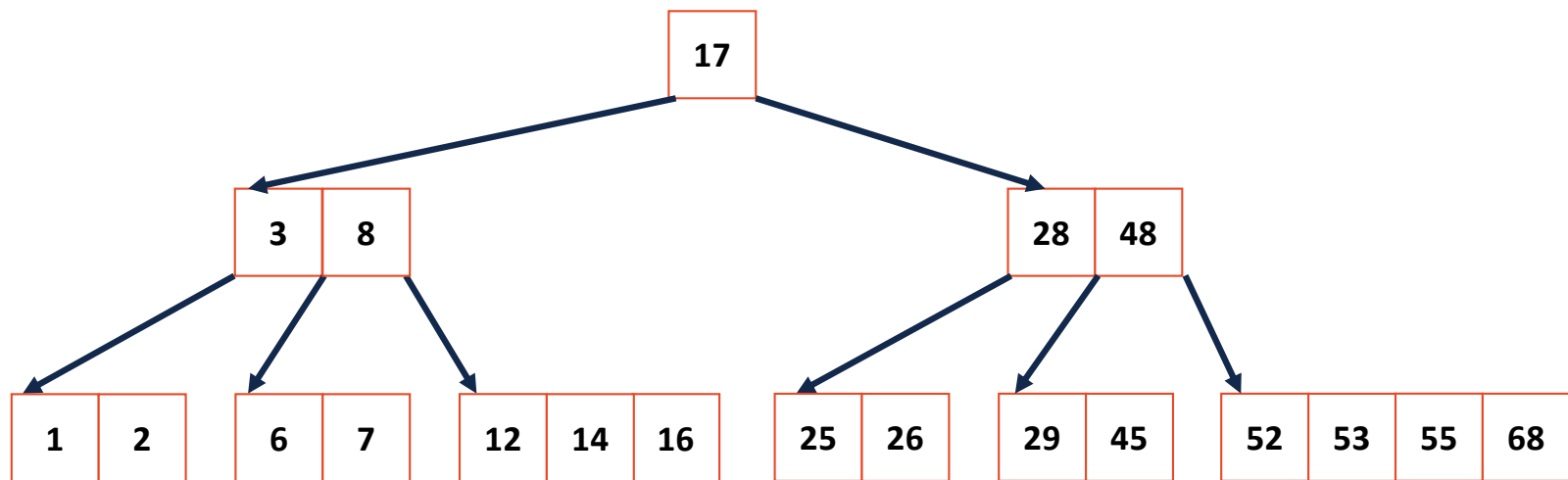
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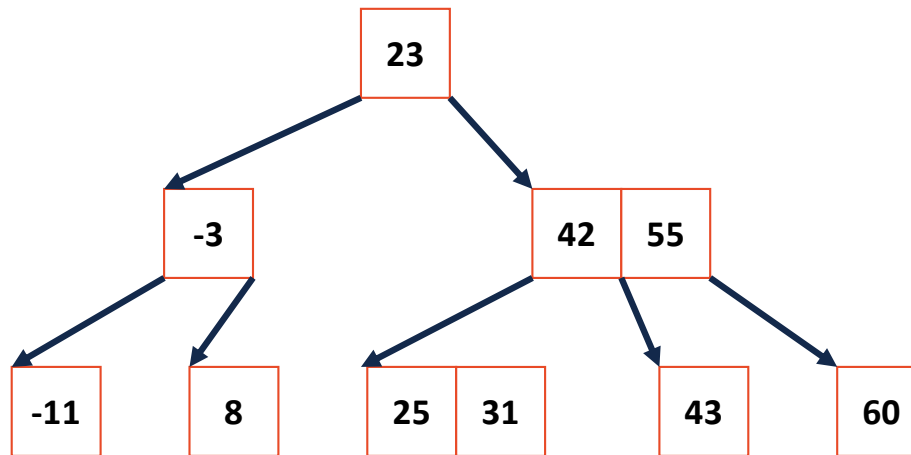
- All internal nodes have exactly **one more child than keys**
- Root nodes can be a leaf or have **[2, m]** children.
- All non-root, internal nodes have **[ceil(m/2), m]** children.

- All leaves are on the same level

BTree

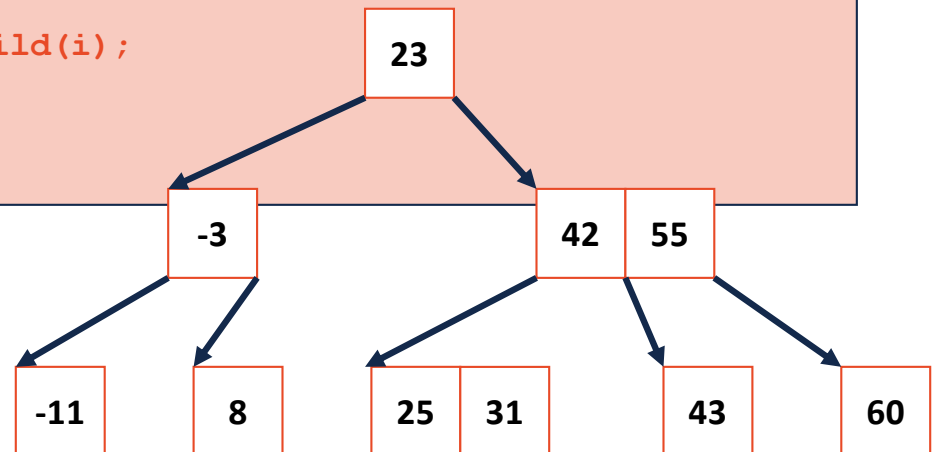


BTree Search



BTree Search

```
1 bool Btree::_exists(BTreeNode & node, const K & key) {
2
3     unsigned i;
4     for ( i = 0; i < node.keys_ct_ && key > node.keys_[i]; i++) { }
5
6     if ( i < node.keys_ct_ && key == node.keys_[i] ) {
7         return true;
8     }
9
10    if ( node.isLeaf() ) {
11        return false;
12    } else {
13        BTreeNode nextChild = node._fetchChild(i);
14        return _exists(nextChild, key);
15    }
16 }
```





BTree Analysis

The height of the BTree determines maximum number of _____ possible in search data.

...and the height of the structure is: _____.

Therefore: The number of seeks is no more than _____.

...suppose we want to prove this!



BTree Analysis

In our AVL Analysis, we saw finding an upper bound on the height (given n) is the same as finding a lower bound on the nodes (given h).

We want to find a relationship for BTrees between the number of keys (n) and the height (h).



BTree Analysis

Strategy:

We will first count the number of nodes, level by level.

Then, we will add the minimum number of keys per node (**n**).

The minimum number of nodes will tell us the largest possible height (**h**), allowing us to find an upper-bound on height.



BTree Analysis

The minimum number of **nodes** for a BTree of order m **at each level:**

root:

level 1:

level 2:

level 3:

...

level h :



BTree Analysis

The **total number of nodes** is the sum of all of the levels:



BTree Analysis

The **total number of keys:**



BTree Analysis

The **smallest total number of keys** is:

So an inequality about **n** , the total number of keys:

Solving for **h** , since **h** is the number of seek operations:



BTree Analysis

Given $m=101$, a tree of height $h=4$ has:

Minimum Keys:

Maximum Keys: