String Algorithms and Data Structures

Tries

CS 199-225
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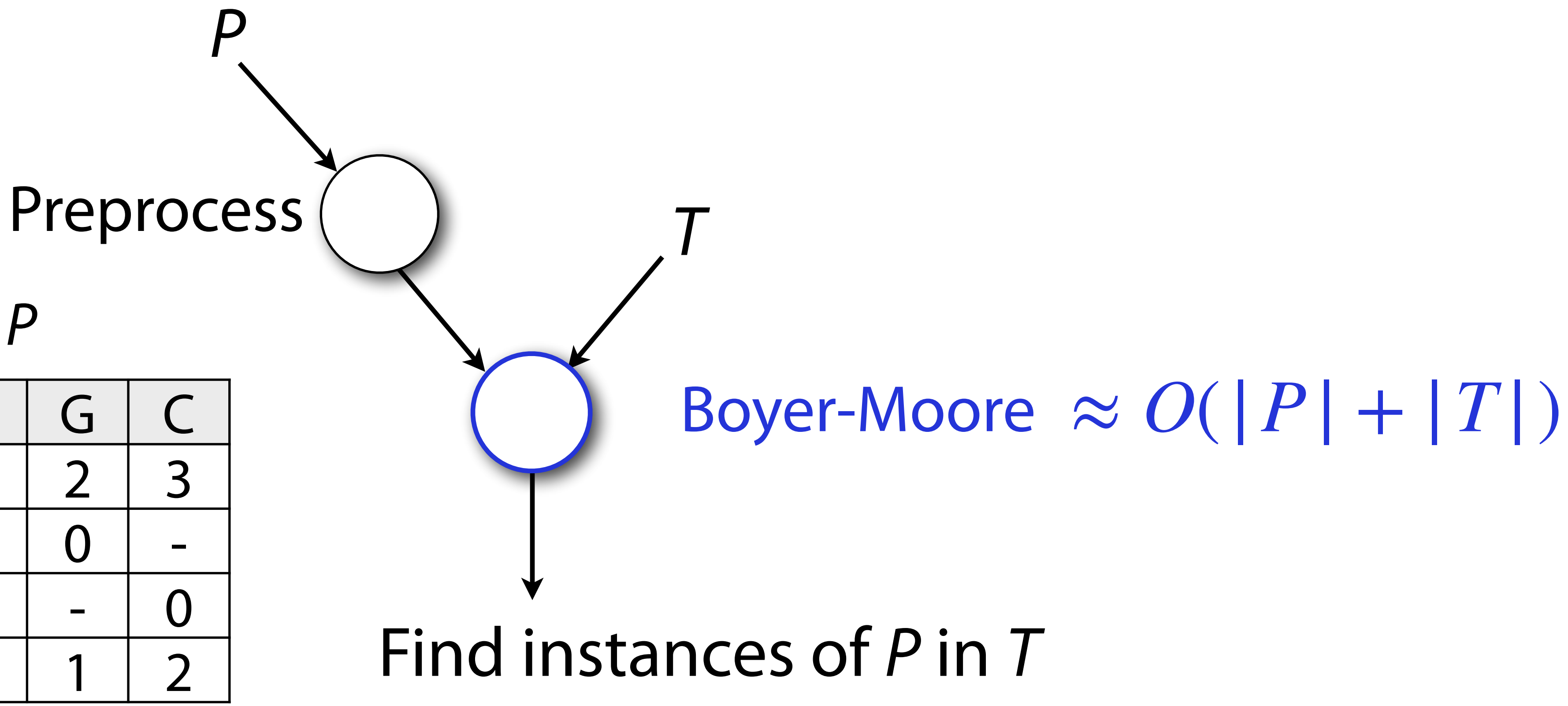
February 27, 2023

UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

Department of Computer Science

Exact pattern matching w/ Boyer-Moore

As seen in HW: sub-linear time *in practice*



Boyer-Moore $\approx O(|P|+|T|)$

Find instances of *P* in *T*

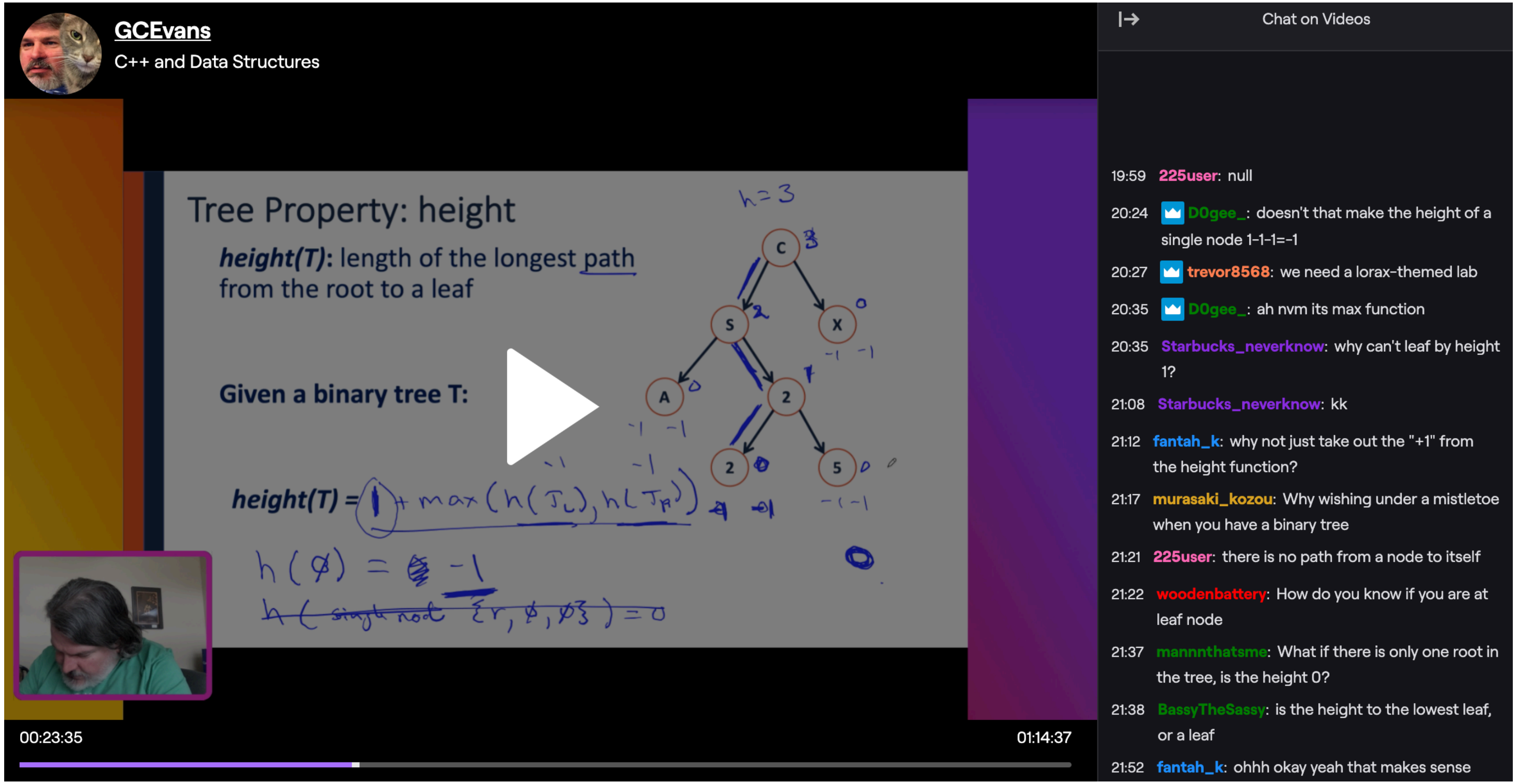
P

	T	C	G	C
A	0	1	2	3
C	0	-	0	-
G	0	1	-	0
T	-	0	1	2

Σ

Make lookup tables

Preprocessing: Live chat streams



Tree Property: height

height(T): length of the longest path from the root to a leaf

Given a binary tree T:

height(T) = 1 + max(h(T_L), h(T_R))

19:59 **225user**: null

20:24 **D0gee_**: doesn't that make the height of a single node 1-1-1=-1

20:27 **trevor8568**: we need a lorax-themed lab

20:35 **D0gee_**: ah nvm its max function

20:35 **Starbucks_neverknow**: why can't leaf by height 1?

21:08 **Starbucks_neverknow**: kk

21:12 **fantah_k**: why not just take out the "+1" from the height function?

21:17 **murasaki_kozou**: Why wishing under a mistletoe when you have a binary tree

21:21 **225user**: there is no path from a node to itself

21:22 **woodenbattery**: How do you know if you are at leaf node

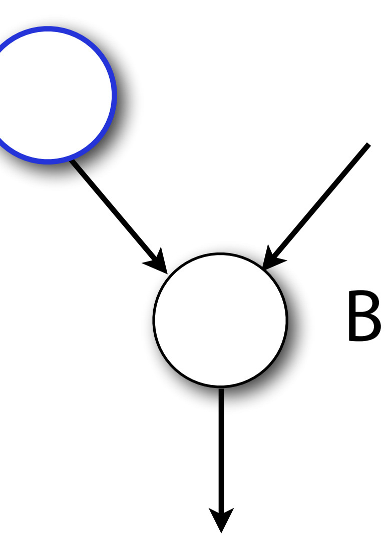
21:37 **mannnthatsme**: What if there is only one root in the tree, is the height 0?

21:38 **BassyTheSassy**: is the height to the lowest leaf, or a leaf

21:52 **fantah_k**: ohhh okay yeah that makes sense

Patterns: banned phrases   **Text:** Chat messages

Preprocessing: Live chat streams



!@#$!%

Preprocess

Thanks for the help!

I don't understand that…

You are a !@#$!% teacher

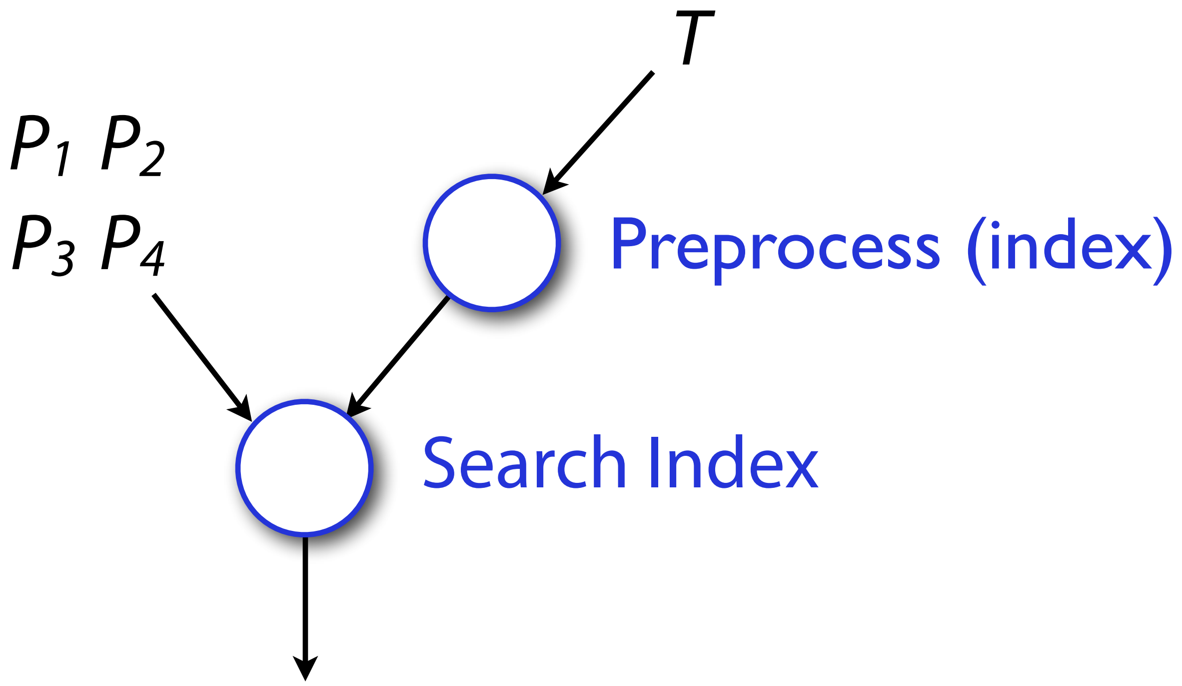
Boyer-Moore

Find instances of *P* in *T*

Amortize cost of preprocessing *P* over many *T*

Exact pattern matching *w/ indexing*

Conventionally $T >> P$:



T

P_1 P_2

P_3 P_4

Preprocess (index)

Search Index

Find instances of P in T

Amortize cost of preprocessing T over many P

Preprocessing: Libraries

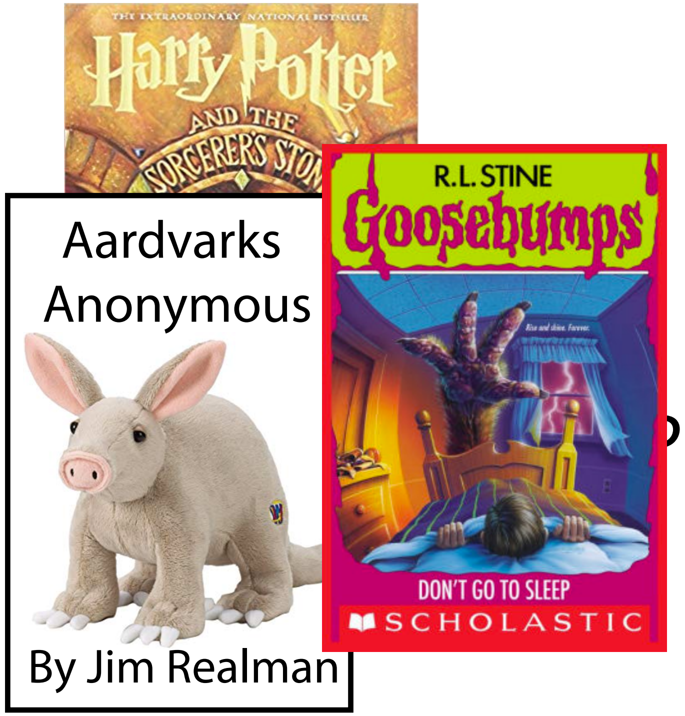
Patterns: Book of interest

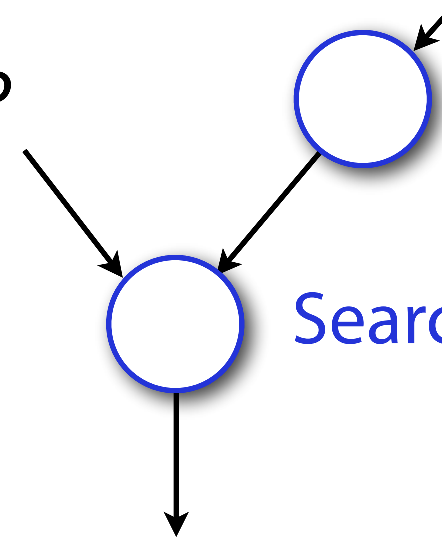
Text: All books in library

Preprocessing: Libraries

Preprocess the library by *indexing* all the books

Preprocessing: Libraries





List of all library books

Preprocess (index)

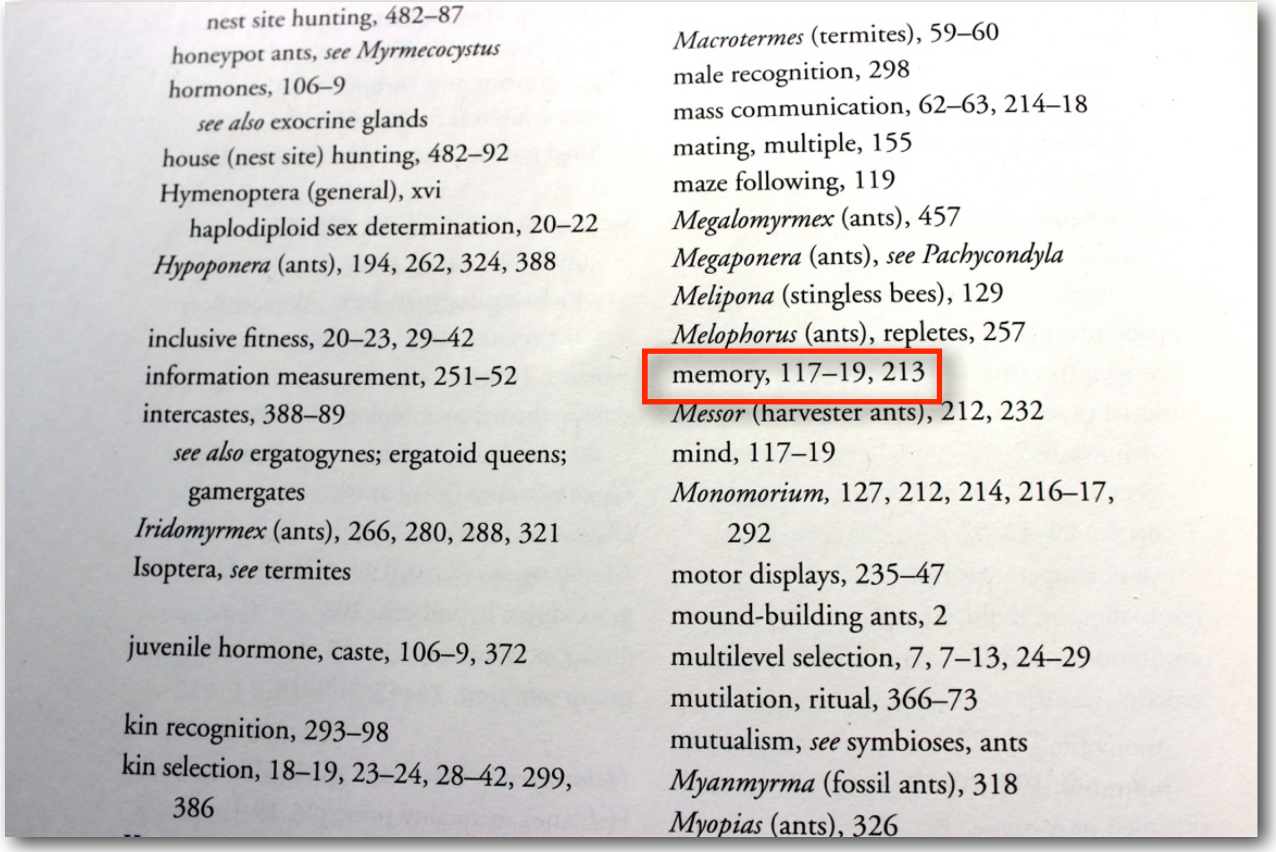
Search Index

Find instances of *P* in *T*

Given full library, built an index once* that is re-used

Preprocessing: Glossaries

What method of preprocessing is this?



- nest site hunting, 482–87
- honeypot ants, *see Myrmecocystus*
- hormones, 106–9
  - *see also* exocrine glands
- house (nest site) hunting, 482–92
- Hymenoptera (general), xvi
  - haplodiploid sex determination, 20–22
- *Hypoponera* (ants), 194, 262, 324, 388

- inclusive fitness, 20–23, 29–42
- information measurement, 251–52
- intercastes, 388–89
  - *see also* ergatogynes; ergatoid queens; gamergates
- *Iridomyrmex* (ants), 266, 280, 288, 321
- Isoptera, *see* termites

- juvenile hormone, caste, 106–9, 372

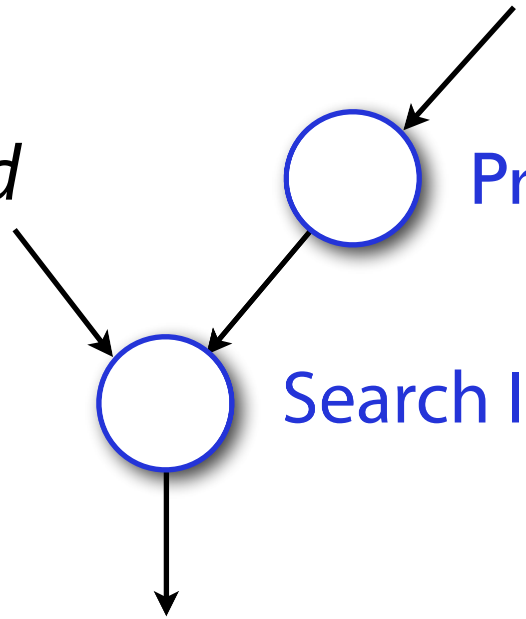
- kin recognition, 293–98
- kin selection, 18–19, 23–24, 28–42, 299, 386

- *Macrotermes* (termites), 59–60
- male recognition, 298
- mass communication, 62–63, 214–18
- mating, multiple, 155
- maze following, 119
- *Megalomyrmex* (ants), 457
- *Megaponera* (ants), *see Pachycondyla*
- *Melipona* (stingless bees), 129
- *Melophorus* (ants), repletes, 257
- memory, 117–19, 213
- *Messor* (harvester ants), 212, 232
- mind, 117–19
- *Monomorium*, 127, 212, 214, 216–17, 292
- motor displays, 235–47
- mound-building ants, 2
- multilevel selection, 7, 7–13, 24–29
- mutilation, ritual, 366–73
- mutualism, *see* symbioses, ants
- *Myanmyrma* (fossil ants), 318
- *Myopias* (ants), 326

Patterns: Key terms

Text: All text in the book

Preprocessing: Glossaries



Memory

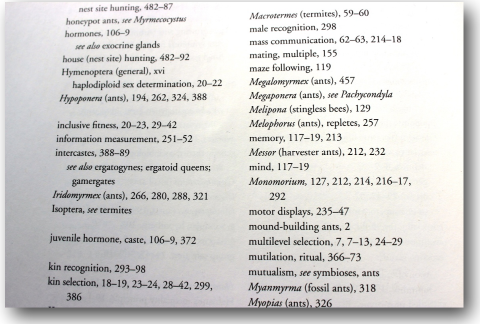
kin recognition

mind

Biology Textbook

Preprocess (index)

Search Index



nest site hunting, 482–87
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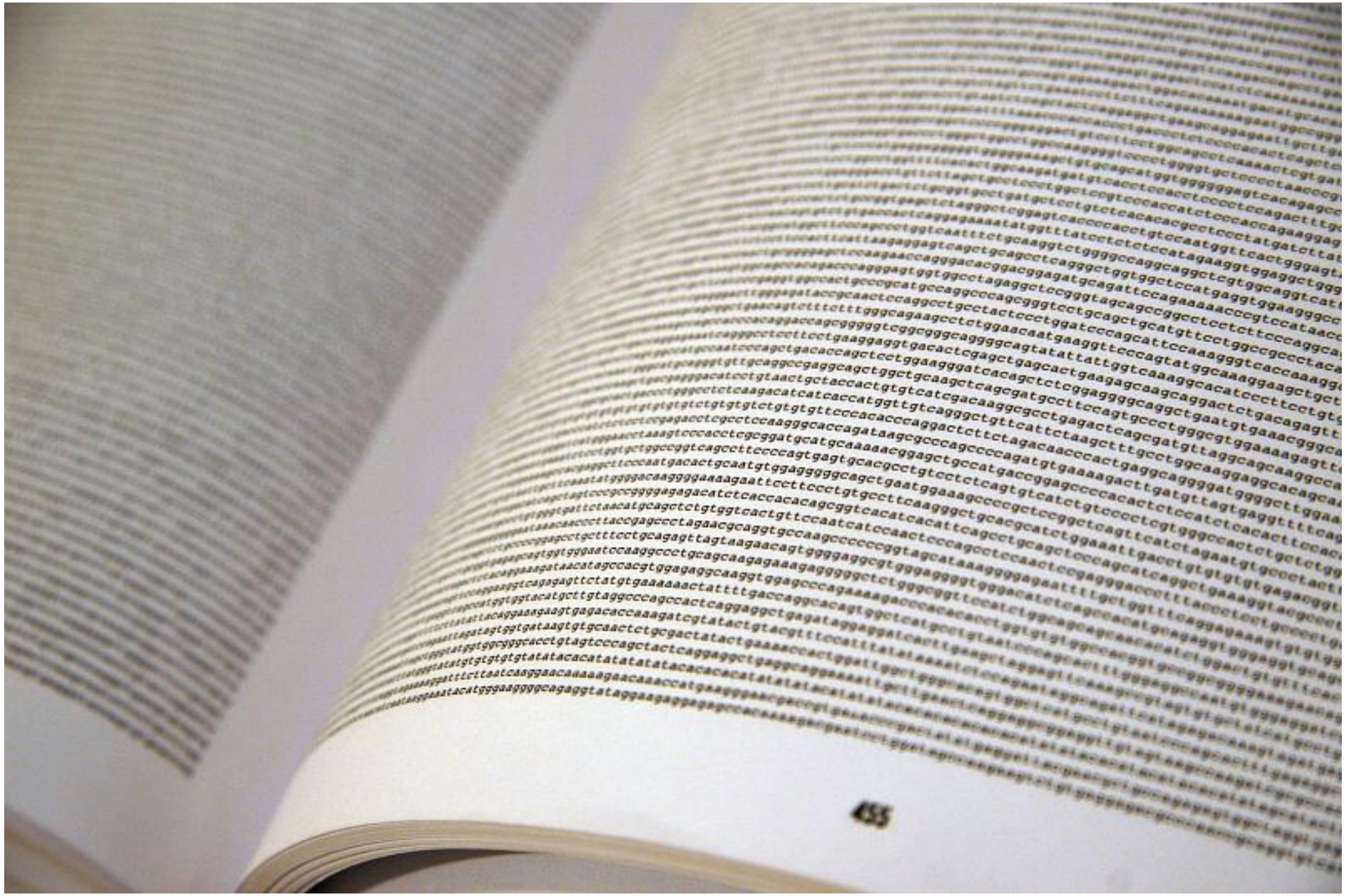
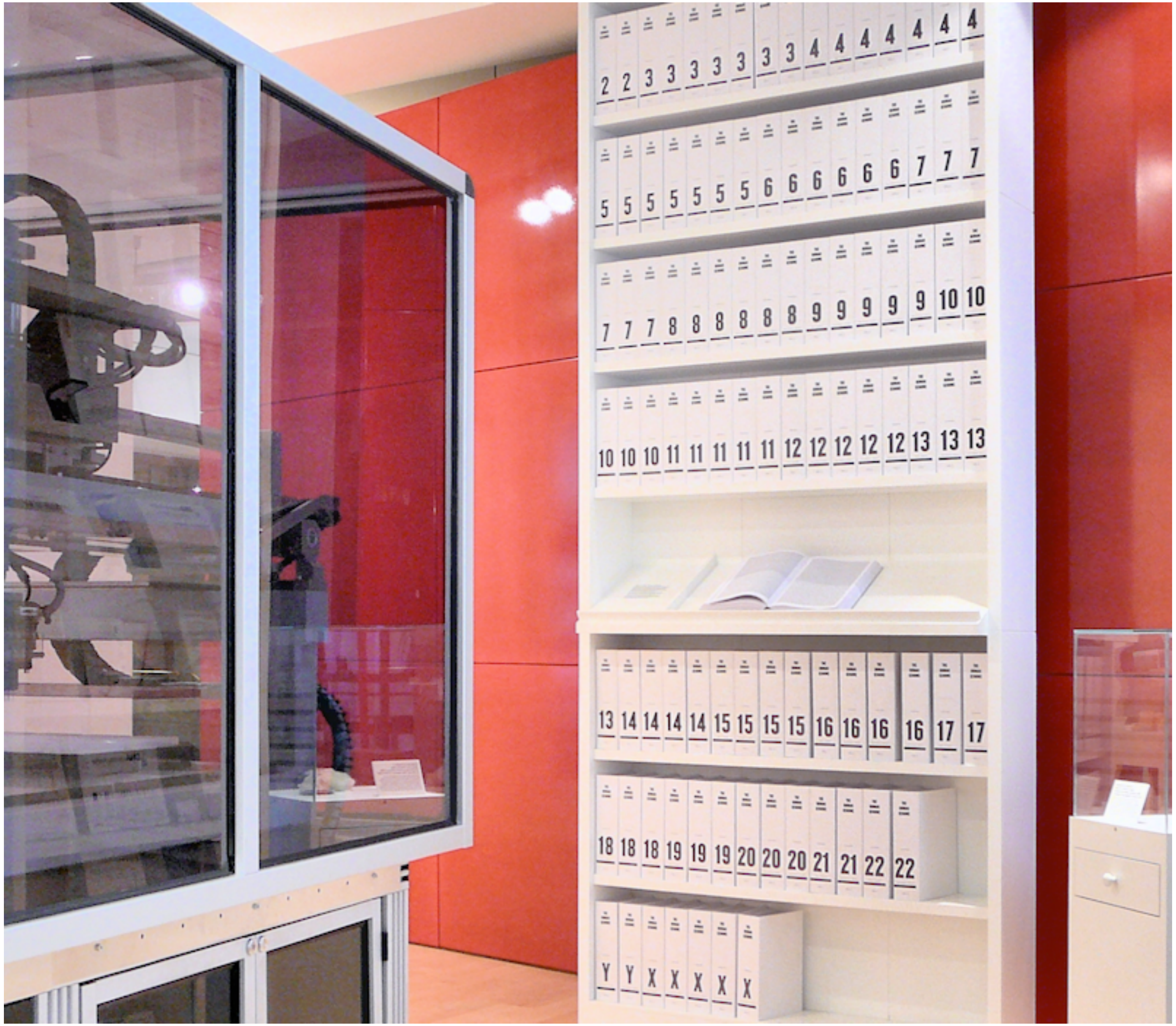
mutualism, *see* symbioses, ants
Myanmyrma (fossil ants), 318
Myopias (ants), 326

Find instances of *P* in *T*

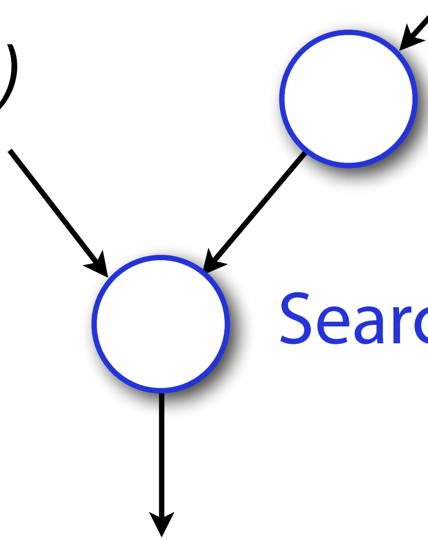
Glossary built on total contents *T*, useful for multiple *P*



455


2 2 3 3 3 3 3 3 3 4 4 4 4 4 4 4
5 5 5 5 5 5 5 6 6 6 6 6 7 7 7
7 7 7 8 8 8 8 8 8 9 9 9 9 9 10 10
10 10 10 11 11 11 11 11 12 12 12 12 12 13 13 13
13 14 14 14 14 15 15 15 15 16 16 16 16 17 17
18 18 18 19 19 19 20 20 20 21 21 22 22
Y Y X X X X X

Exact pattern matching *w/ indexing*



The human genome (~3 billion)

1 Billion reads (~100)

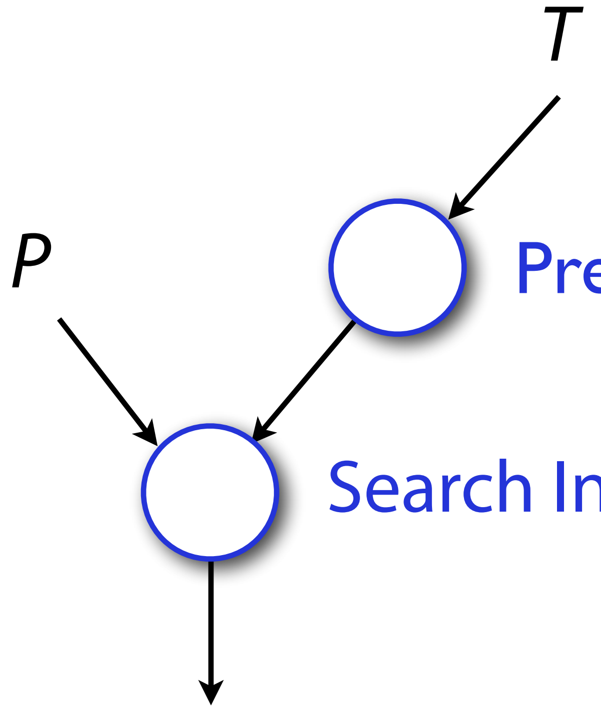
Preprocess (index) $\approx O(|T|)$

Search Index $\approx O(|P|)$

Find instances of *P* in *T*

Amortize cost of preprocessing *T* over many *P*

Exact pattern matching *w/ indexing*



T

P

Preprocess (index) $\approx O(|T|)$

Search Index $\approx O(|P|)$

Find instances of *P* in *T*

What information from *T* do we need to search for *P*?

Preprocessing for exact pattern matching

T: **C G T G C**

P:

Search(*P*, *T*):

P:

Search(*P*, *T*):

P:

Search(*P*, *T*):

Preprocessing for exact pattern matching

T: **C G T G C**

C	0
G	1
T	2
G	3
C	4
C G	0
G T	1
T G	2
G C	3
C G T	0
G T G	1
T G C	2
A substring *S*	The position of *S* in *T*

Preprocessing for exact pattern matching

T: **C G T G C**

C
G
T
G
C

|T|

C G
G T
T G
G C

|T-1|

C G T
G T G
T G C

|T-2|

Key	Value
C	0
G	1
T	2
G	3
C	4
CG	0
GT	1
TG	2
…	…

?

Preprocessing for exact pattern matching

T: **C G T G C**

C
G
T
G
C

|T|

C G
G T
T G
G C

|T-1|

C G T
G T G
T G C

|T-2|

Key	Value
C	0
G	1
T	2
G	3
C	4
CG	0
GT	1
TG	2
…	…

$$\frac{|T|(|T|+1)}{2}$$

Preprocessing for exact pattern matching

Because our keys are strings, this is sometimes possible!

Key	Value
C	0
G	1
T	2
G	3
C	4
CG	0
GT	1
TG	2
…	…

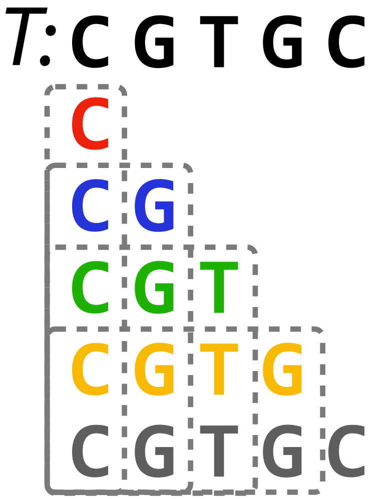
$$\frac{|T|(|T|+1)}{2}$$

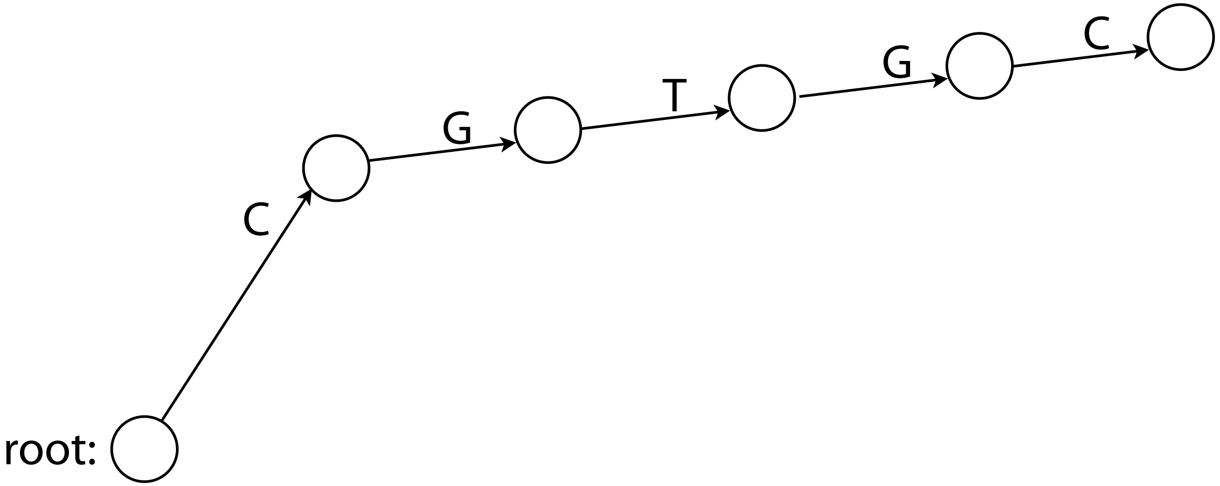
We want to search in $O(|P|)$ without $O(|T|^2)$ space!

Preprocessing for exact pattern matching

Strings consist of individual characters!

… and these characters can overlap:





Preprocessing for exact pattern matching

Strings consist of individual characters!

… and these characters can overlap:

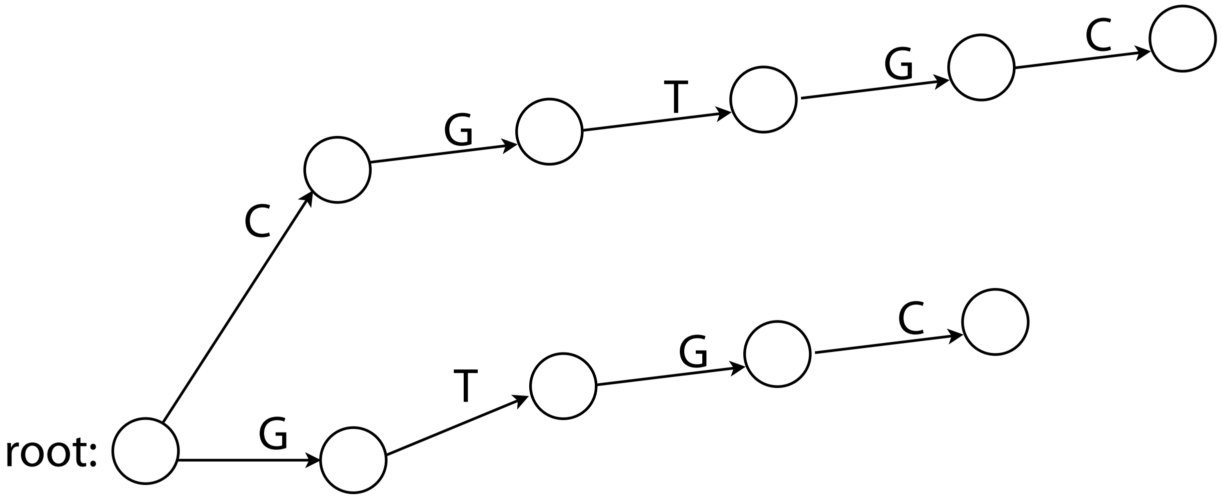
T: **C G T G C**

G

G T

G T G

G T G C



Preprocessing for exact pattern matching

Strings consist of individual characters!

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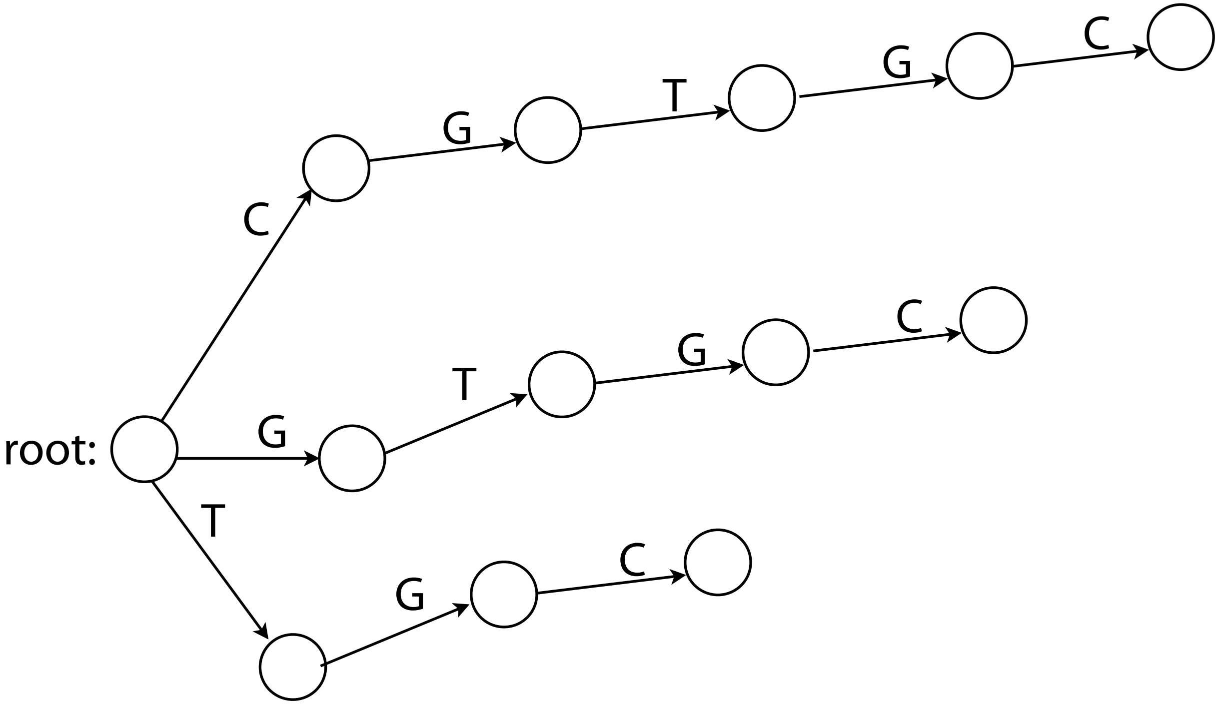
T: **C G T G C**

T

T G

T G C



Preprocessing for exact pattern matching

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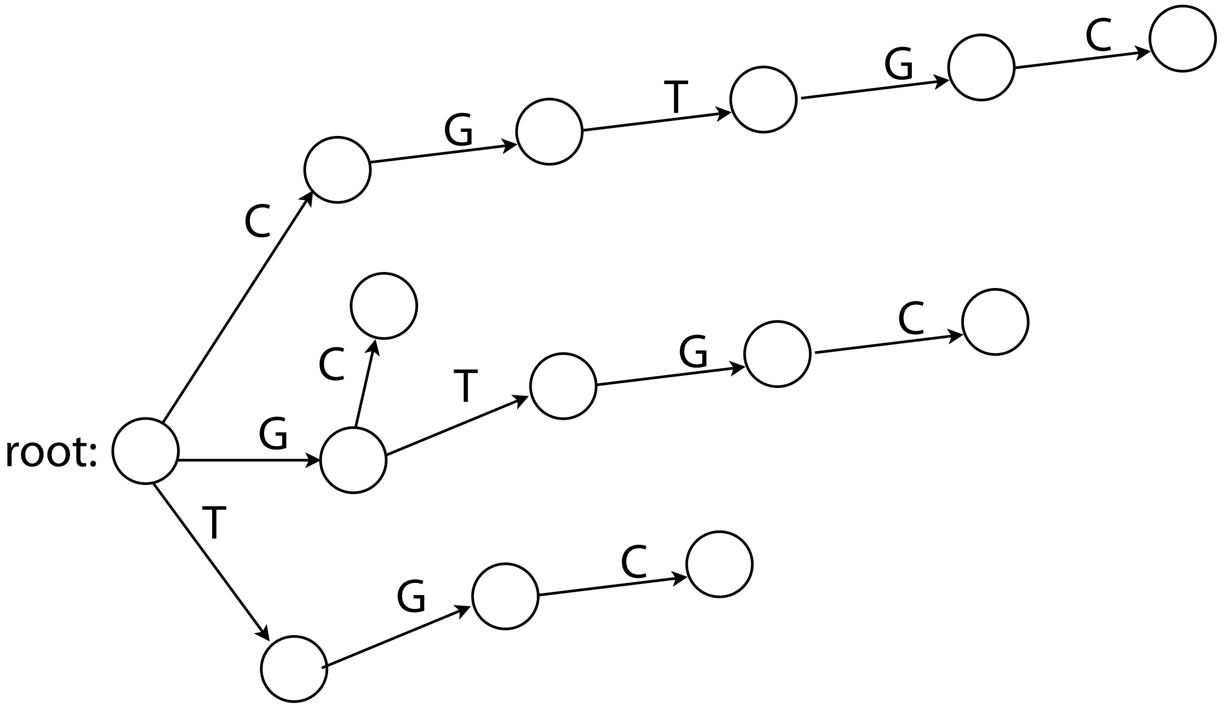
… and these characters can overlap:

T: **C G T G C**

G

G C



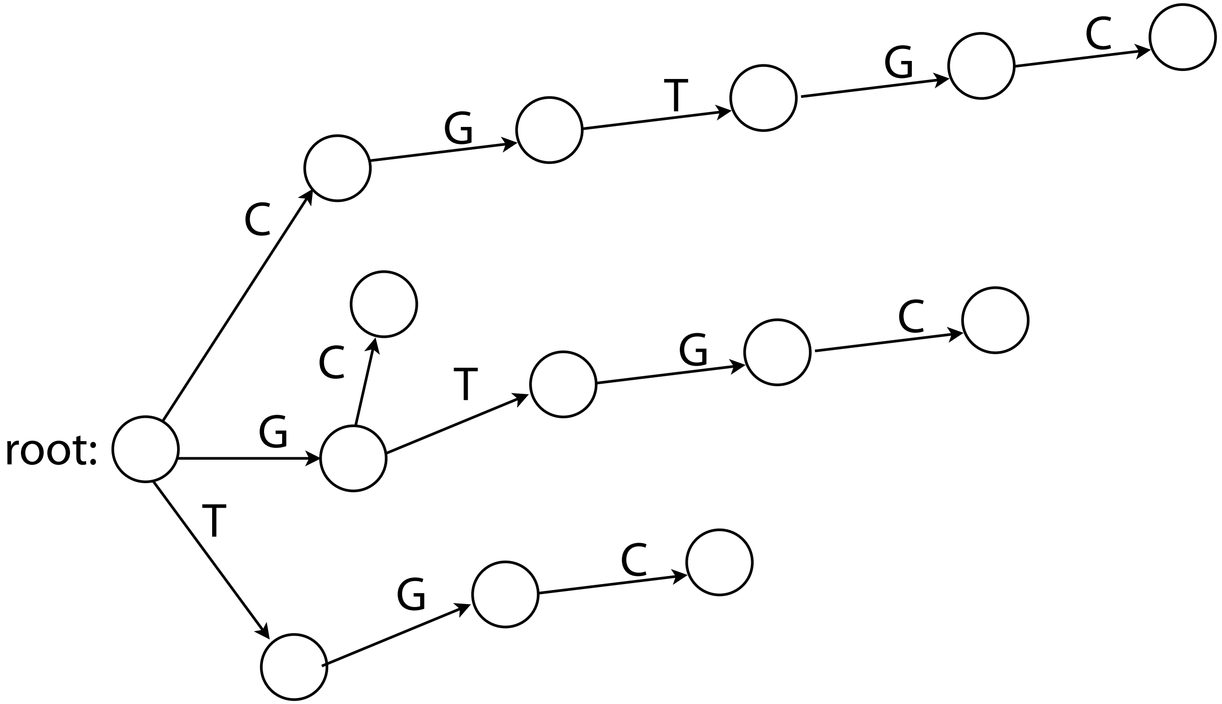
Preprocessing for exact pattern matching

Strings consist of individual characters!

… and these characters can overlap:

T: **C G T G C**

C



String indexing with Tries

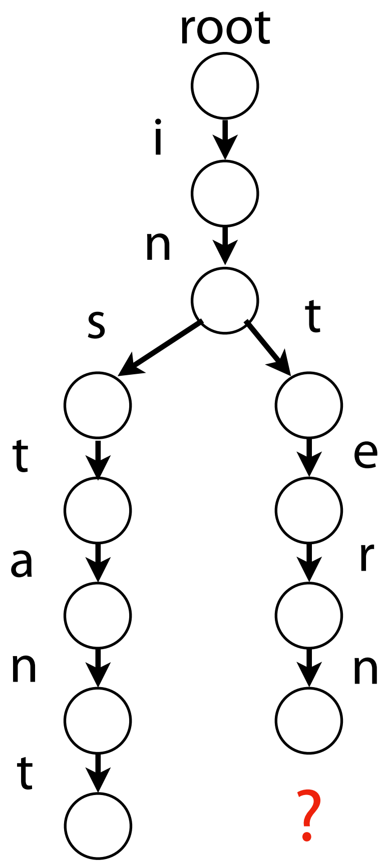
Trie: A rooted tree storing a collection of (key, value) pairs

Keys:	Values:
`instant`	1
`internal`	2
`internet`	3

Each edge is labeled with a character $c \in \Sigma$

For given node, at most one child edge has label c, for any $c \in \Sigma$

Each key is "spelled out" along some path starting at root



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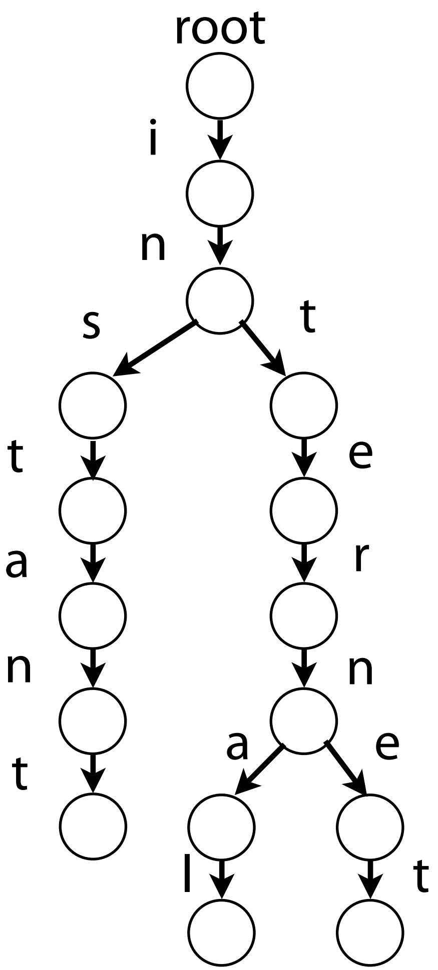
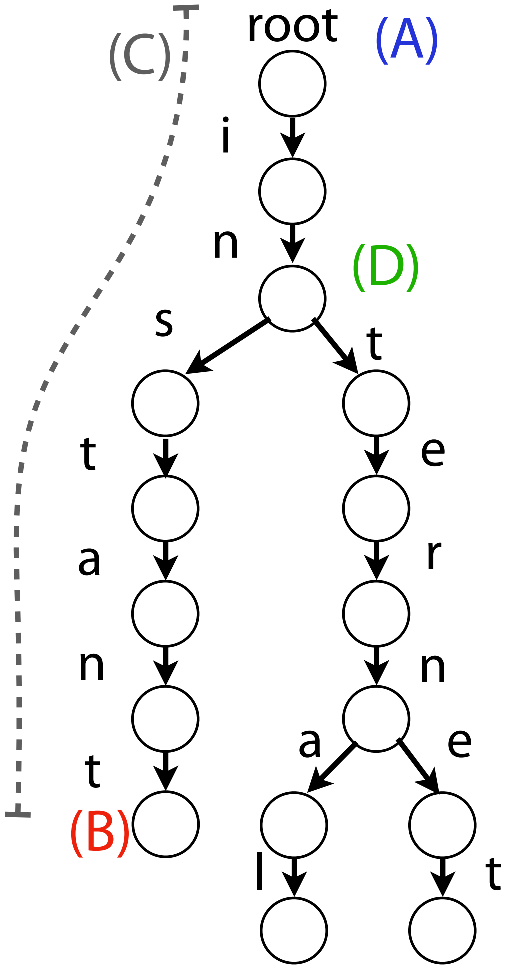


String indexing with Tries

Trie: A rooted tree storing a collection of (key, value) pairs

Keys:	Values:
`instant`	1
`internal`	2
`internet`	3

Where should I store the value 1?

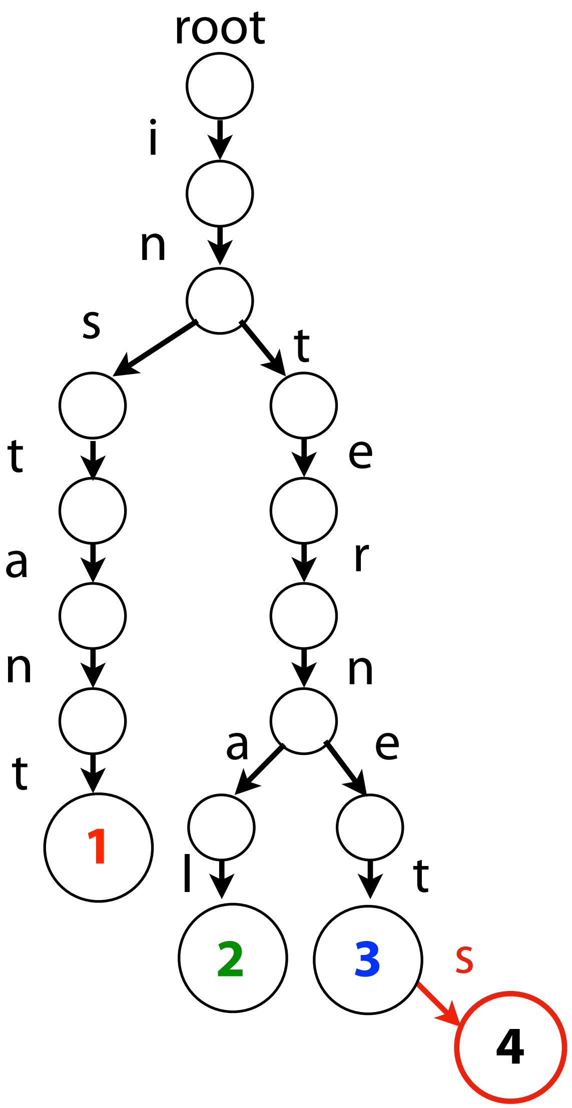


String indexing with Tries

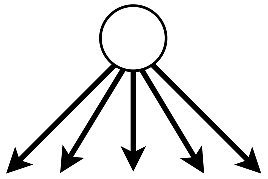
Trie: A rooted tree storing a collection of (key, value) pairs

Keys:	Values:
`instant`	1
`internal`	2
`internet`	3
`internets`	4

Each key's value is stored at the last node in the path



The Node Implementation



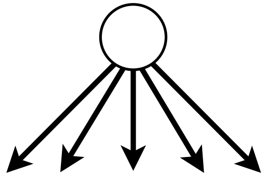
Each node in my trie has $\leq |\Sigma|$ edges!

Each edge is a (potentially NULL) pointer.

How can we encode this?

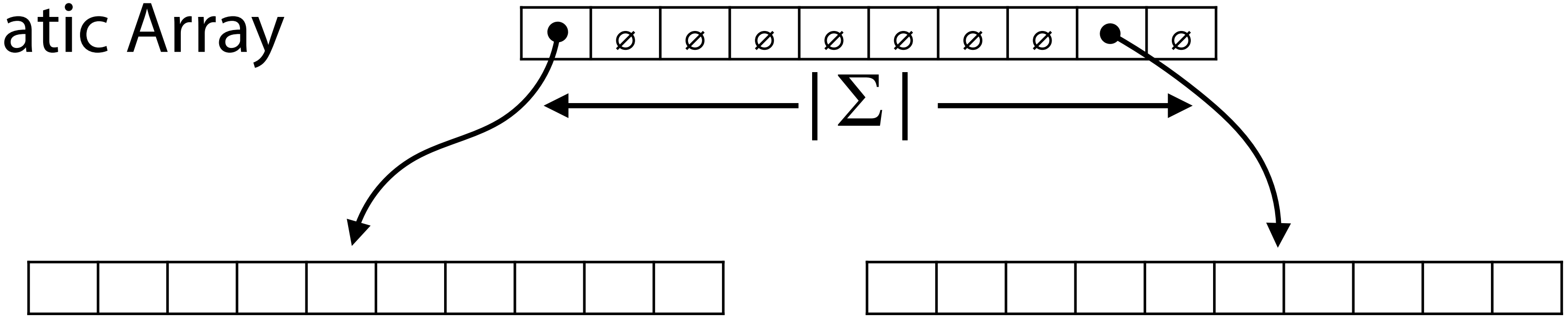
The Node Implementation

Each node in my trie has $\leq |\Sigma|$ edges!

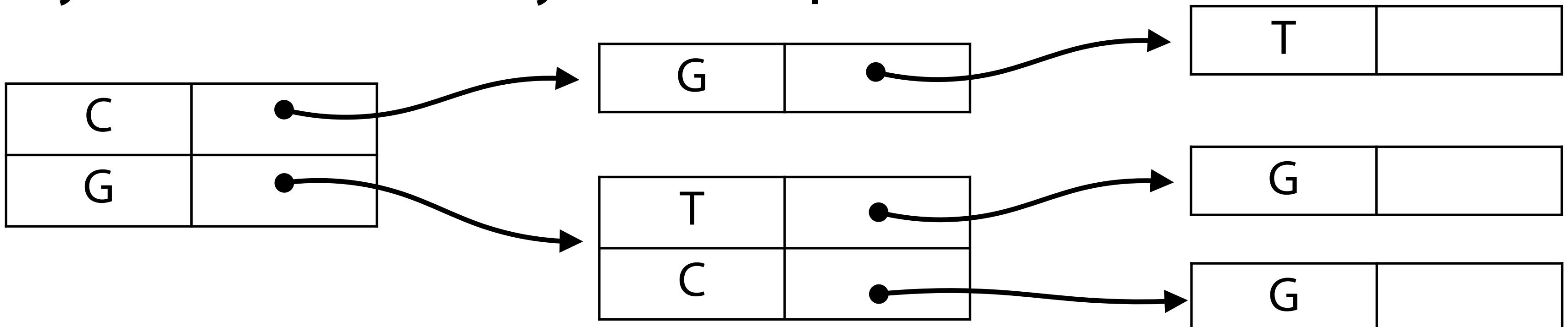


Each edge is a (potentially NULL) pointer.

1) Static Array



2) Dynamically-sized Dictionary (std::map)



Trie Node Implementation

NaryTree.h

```
class NaryTree
{
  public:
    struct Node {
      std::vector<int> index;
      std::map<char, Node*> children;

      Node(std::string s, int i)
      {
        if(s.length() > 0 ){
          children[s[0]] = new Node(s.substr(1), i);
        } else {
          index.push_back(i);
        }
      }
    };
  protected:
    Node* root;
…
}
```

Trie Node Implementation

NaryTree.h

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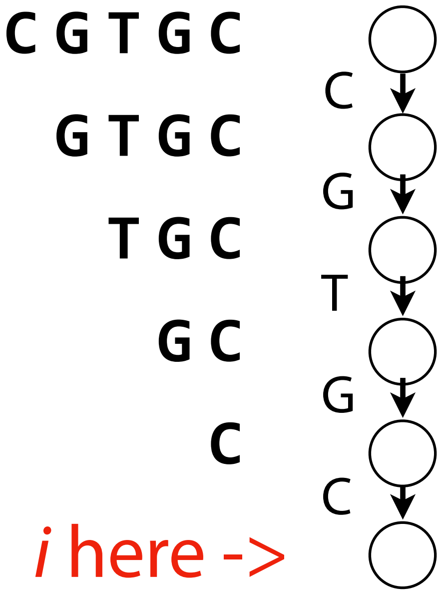

What if we have more than one string?

Trie Node Implementation

main.cpp

```
NaryTree myT;
myTree.print();

myTree.insert("AB",0);
myTree.print();

myTree.insert("ABA",1);
myTree.print();

myTree.insert(“ABB",2);
myTree.print();

myTree.insert("BAB",3);
myTree.print();

myTree.insert(“BBB",4);
myTree.print();
```

x

Trie Node Implementation

main.cpp

```
NaryTree myT;
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myTree.print();

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myTree.insert(“ABB",2);
myTree.print();

myTree.insert("BAB",3);
myTree.print();

myTree.insert(“BBB",4);
myTree.print();
```

```
o
+--A--o      0
      +--B--o
```

Trie Node Implementation

main.cpp

```
NaryTree myT;
myTree.print();

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myTree.print();

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myTree.print();

myTree.insert(“BBB",4);
myTree.print();




```

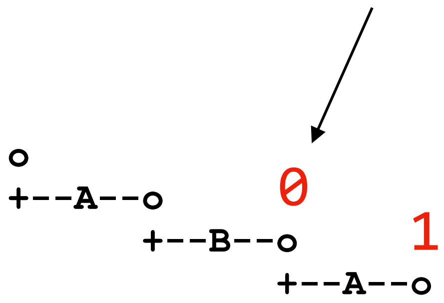

Former leaf node, still holds value

```
o
+--A--o      0
      +--B--o      1
            +--A--o
```

```
struct Node {
        std::vector<int> index;
        std::map<char, Node*> children;
}
```

Trie Node Implementation

main.cpp

```cpp
NaryTree myT;
myTree.print();

myTree.insert("AB",0);
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myTree.print();

myTree.insert("BAB",3);
myTree.print();

myTree.insert(“BBB",4);
myTree.print();
```

```
o
+--A--o      0
      +--B--o      1
            +--A--o
            +--B--o  2
```

Trie Node Implementation

main.cpp

```
NaryTree myT;
myTree.print();

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myTree.print();

myTree.insert(“BBB",4);
myTree.print();
```

```
o
+--A--o     0
|     +--B--o     1
|           +--A--o
|           +--B--o 2
+--B--o
      +--A--o
            +--B--o 3
```

Trie Node Implementation

main.cpp

```
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myTree.print();

myTree.insert("AB",0);
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myTree.insert("ABA",1);
myTree.print();

myTree.insert(“ABB",2);
myTree.print();

myTree.insert("BAB",3);
myTree.print();

myTree.insert(“BBB",4);
myTree.print();
```

```
o
+--A--o      0
|     +--B--o      1
|           +--A--o
|           +--B--o  2
+--B--o
      +--A--o
      |     +--B--o  3
      +--B--o
            +--B--o  4
```

Trie Node Implementation

NaryTree.h

```
void NaryTree::insert(const std::string& s, int i)
{
    insert(root, s, int i);
}

void NaryTree::insert(Node*& node, const std::string & s, int i)
{
    // If we're at a NULL pointer, we make a new Node
    if (node == NULL) {
        node = new Node(s, i);
    } else {
        if(s.length() > 0 ){
            if(node->children.count(s[0]) > 0){
               insert(node->children[s[0]],s.substr(1), i);
            }else{
                node->children[s[0]] = new Node(s.substr(1), i);
            }
        } else{
            node->index.push_back(i);
        }

    }
}
```

Assignment 5: a_narytree

Learning Objective:

Store all substrings in a trie using NaryTree implementation

Implement exact pattern matching using this trie

Consider: How many insertions are we doing for each string?
Is there a better or faster way to do this?

Trie Node Implementation

main.cpp

```cpp
NaryTree myT;

myTree.insert("AB",0);

myTree.insert(“AB",2);

myTree.print();
```

```
o          ??
+--A--o
      +--B--o
```

Trie Node Implementation

main.cpp

```
NaryTree myT;

myTree.insert("AB",0);

myTree.insert(“AB",2);

myTree.print();
```

```
o
+--A--o     0,2
      +--B--o
```

```
struct Node {
    std::vector<int> index;
    std::map<char, Node*> children;
}
```

```
if(s.length() > 0 ){
    if(node->children.count(s[0]) > 0){
        insert(node->children[s[0]],s.substr(1), i);
    }else{
        node->children[s[0]] = new Node(s.substr(1), i);
    }
} else{
    node->index.push_back(i);
}
```

Searching a Trie

Given *P*, search the trie for keys and return values

Pattern: `i n f e r`

`i n f e r`

`i n f e r`

`i n f e r`

Lets break that down using *recursion*:

Starting at root:

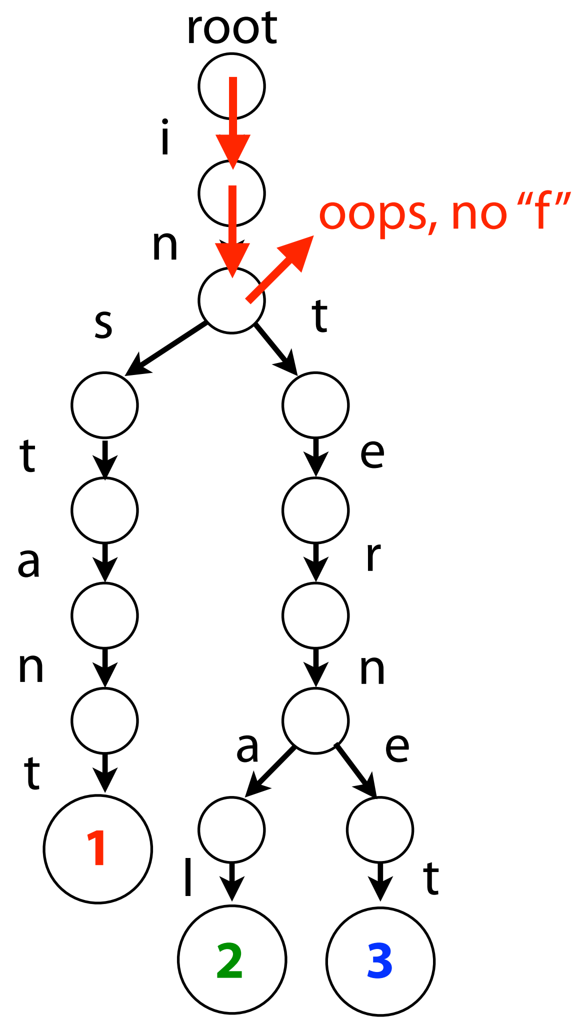
(1) Try to match front character

(2) If match, move to appropriate child

(2.5) Set pattern equal to remainder

(2.5) Go back to (1)

(3) If mismatch, *P* is not a key!



Searching a Trie

Given *P*, search the trie for keys and return values

Pattern: interesting

interesting

interesting

Lets break that down using *recursion*:

Starting at root:

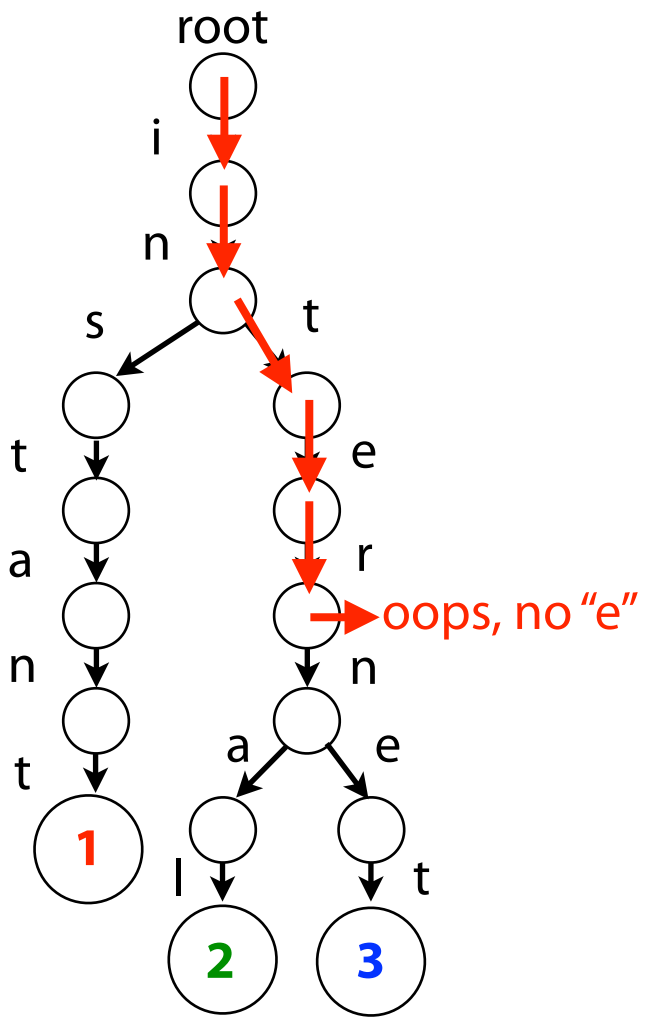
(1) Try to match front character

(2) If match, move to appropriate child

(2.5) Set pattern equal to remainder

(2.5) Go back to (1)

(3) If mismatch, *P* is not a key!



Searching a Trie

Given *P*, search the trie for keys and return values

Pattern: `i n s t a`

`i n s t a`

Lets break that down using *recursion*:

Starting at root:

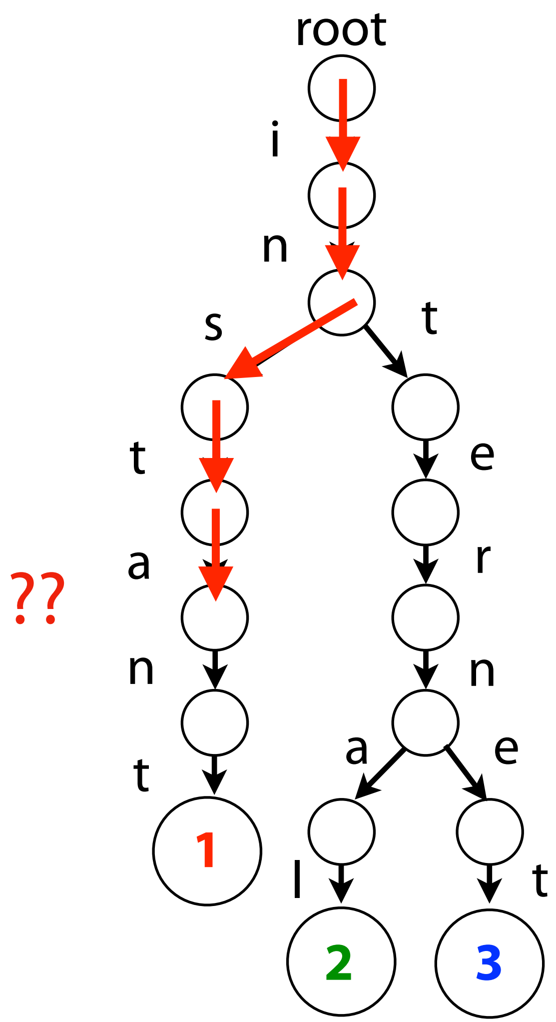
(1) Try to match front character

(2) If match, move to appropriate child

(2.5) Set pattern equal to remainder

(2.5) Go back to (1)

(3) If mismatch, *P* is not a key!



Searching a Trie

Given *P*, search the trie for keys and return values

Pattern: `i n s t a`

`i n s t a`

Lets break that down using *recursion*:

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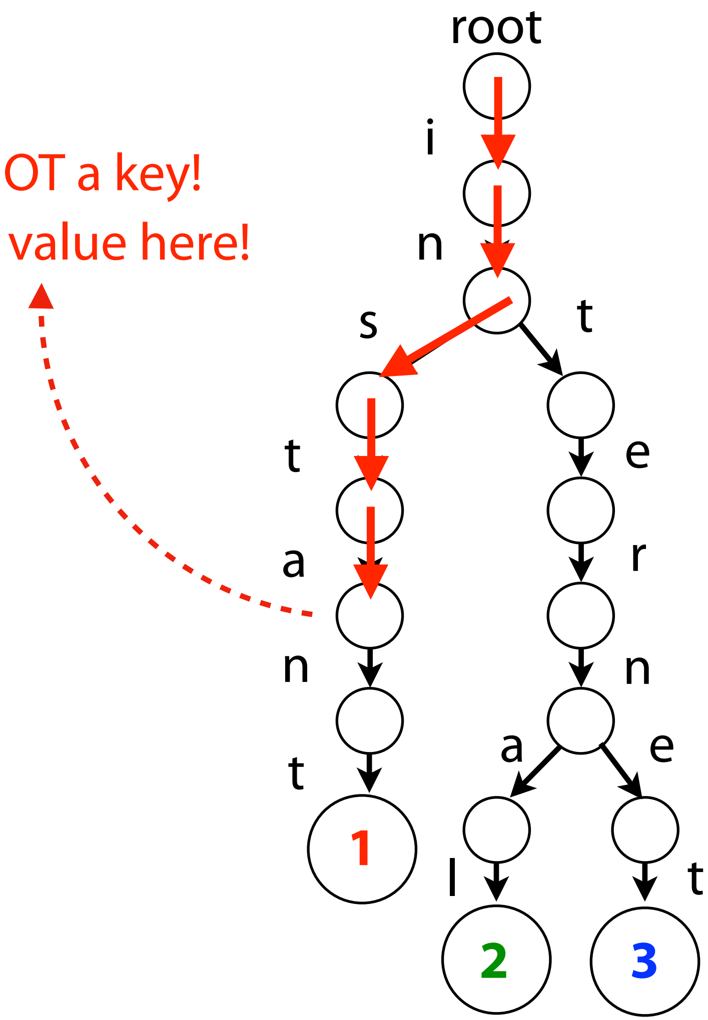
(2.5) Go back to (1)

(3) If mismatch, *P* is not a key!

"Insta" is NOT a key!

There's no value here!



String indexing with Tries

A rooted tree storing a collection of (key, value) pairs

Keys:	Values:
`instant`	1
`internal`	2
`internet`	3

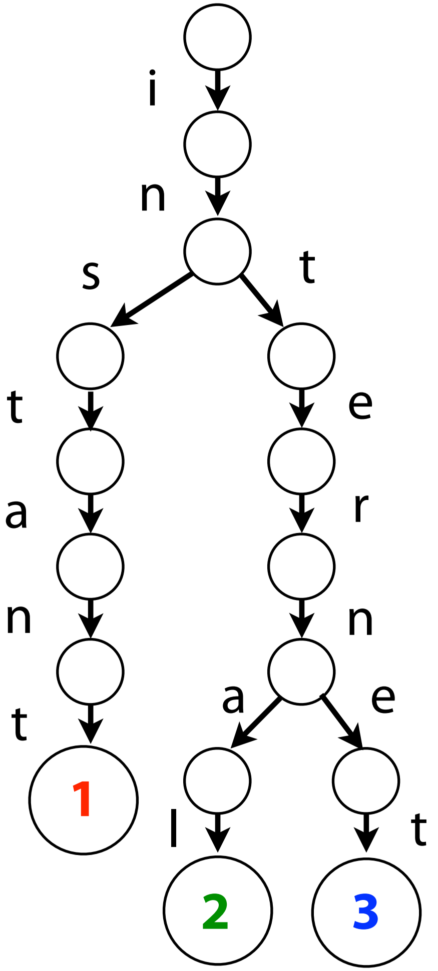
The trie is structured such that:

Each edge is labeled with a character $c \in \Sigma$

For given node, at most one child edge has label c, for any $c \in \Sigma$

Each key is "spelled out" along some path starting at root

Each key's value is stored at the last node in the path



Searching a Trie

Given *P*, search the trie for keys and return values

Pattern: `i n s t a`

`i n s t a` ⬚

Lets break that down using *recursion*:

Starting at root:

(0) If we have no 'front' char, check value

(0.5) If no value, *P* is not a key!

(0.5) If value, *P* is a key, return value(s).

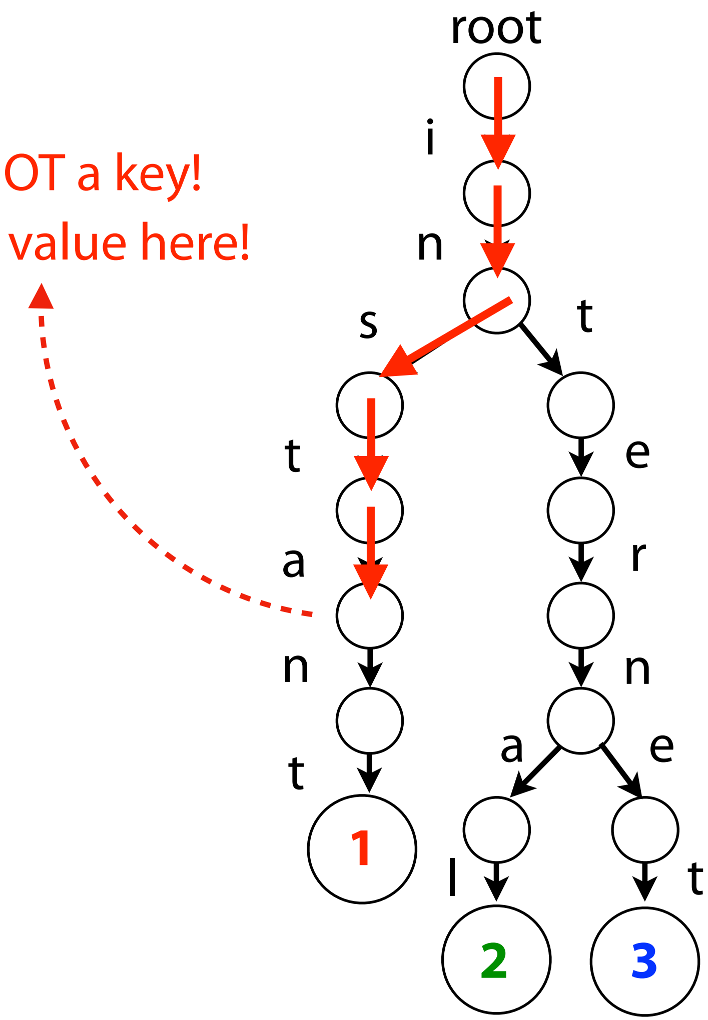
(1) Try to match front character

(2) If match, move to appropriate child

(2.5) Set pattern equal to remainder

(2.5) Go back to (1)

(3) If mismatch, *P* is not a key!



"Insta" is NOT a key!
There's no value here!

Searching a Trie

Given *P*, search the trie for keys and return values

Pattern: `i n s t a n t`

`instant`

Lets break that down using *recursion*:

Starting at root:

(0) If we have no 'front' char, check value

(0.5) If no value, *P* is not a key.

(0.5) If value, *P* is a key, return value(s).

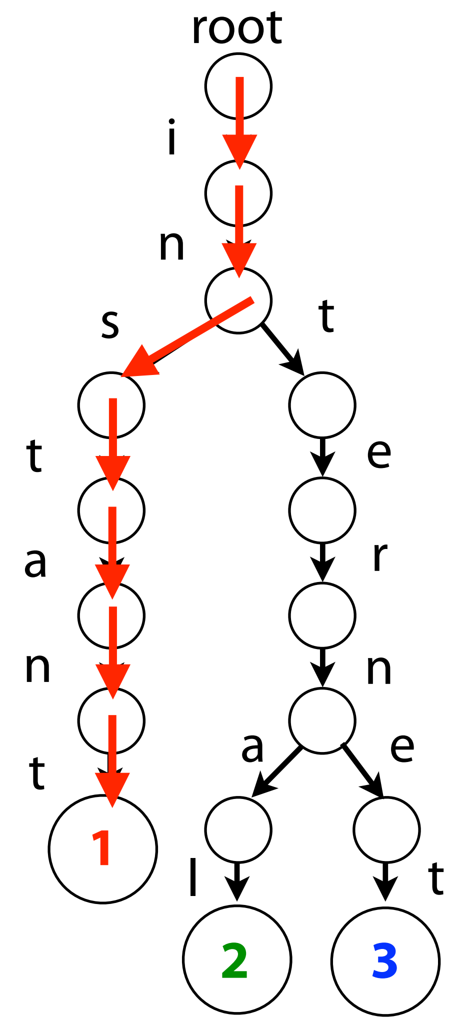
(1) Try to match front character

(2) If match, move to appropriate child

(2.5) Set pattern equal to remainder

(2.5) Go back to (1)

(3) If mismatch, *P* is not a key!



Assignment 5: a_narytree

Learning Objective:

Store all substrings in a trie using NaryTree implementation

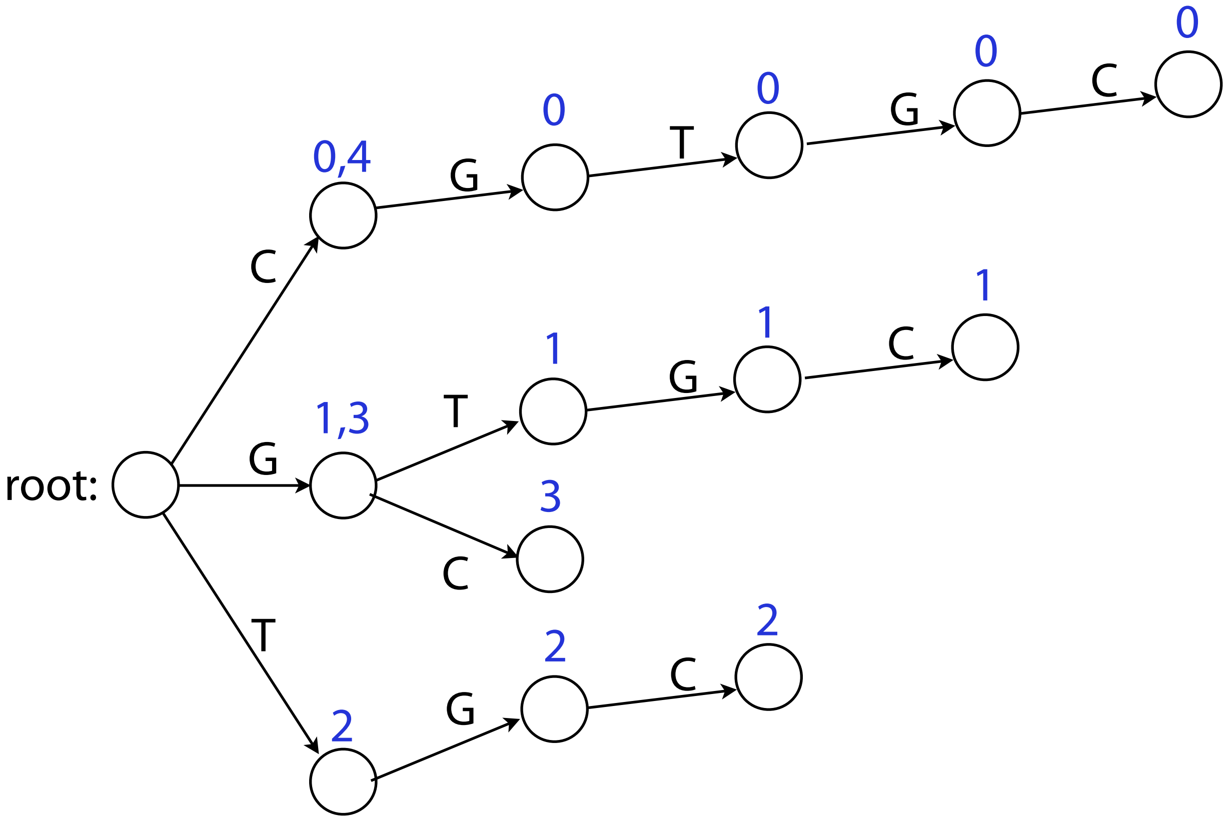
Implement exact pattern matching using this trie

Consider: How could we search the trie if we are only allowed to store one value in each node [instead of a vector of them]?

Preprocessing for exact pattern matching

T: **C G T G C**

Key	Value
C	0
G	1
T	2
G	3
C	4
CG	0
GT	1
TG	2
…	…



Preprocessing for exact pattern matching

T: **C G T G C**

Key	Value
C	0
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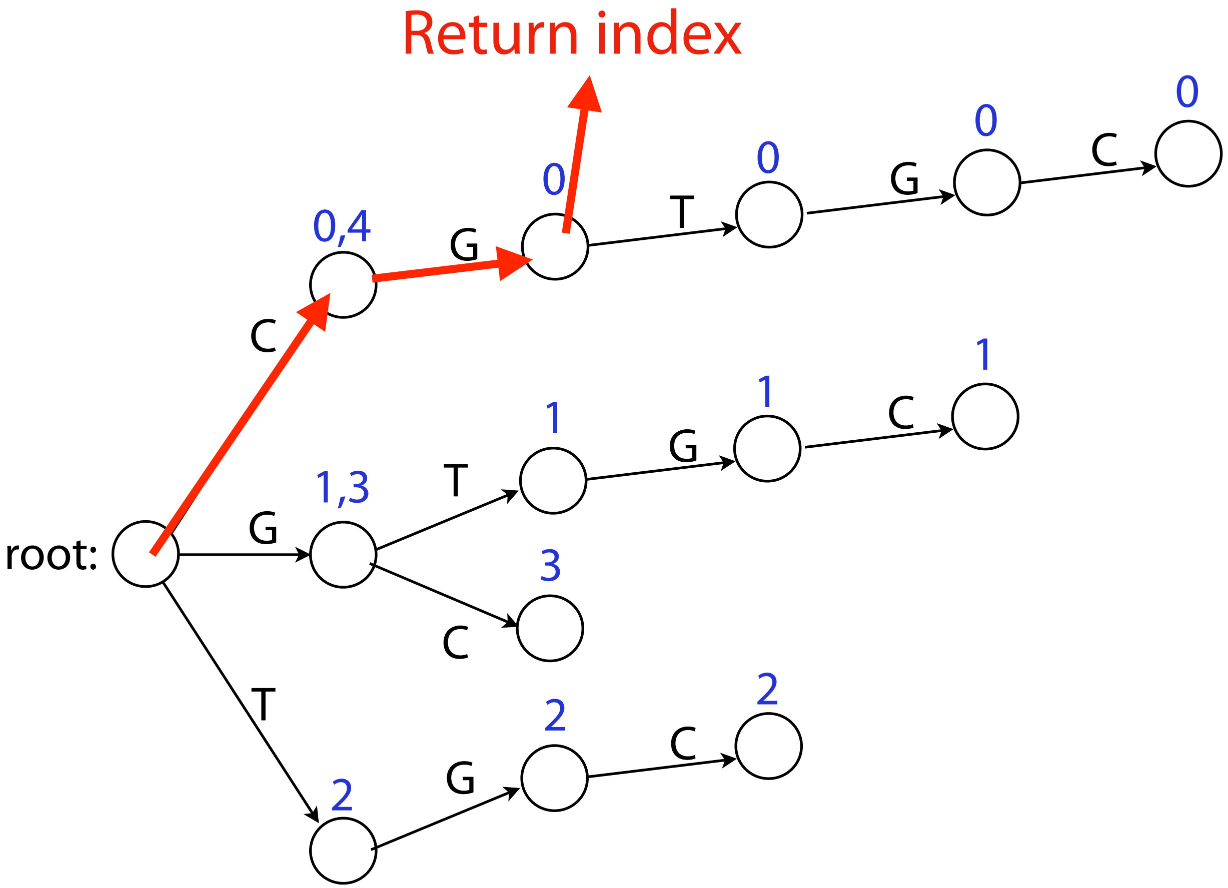
…	…


Return index

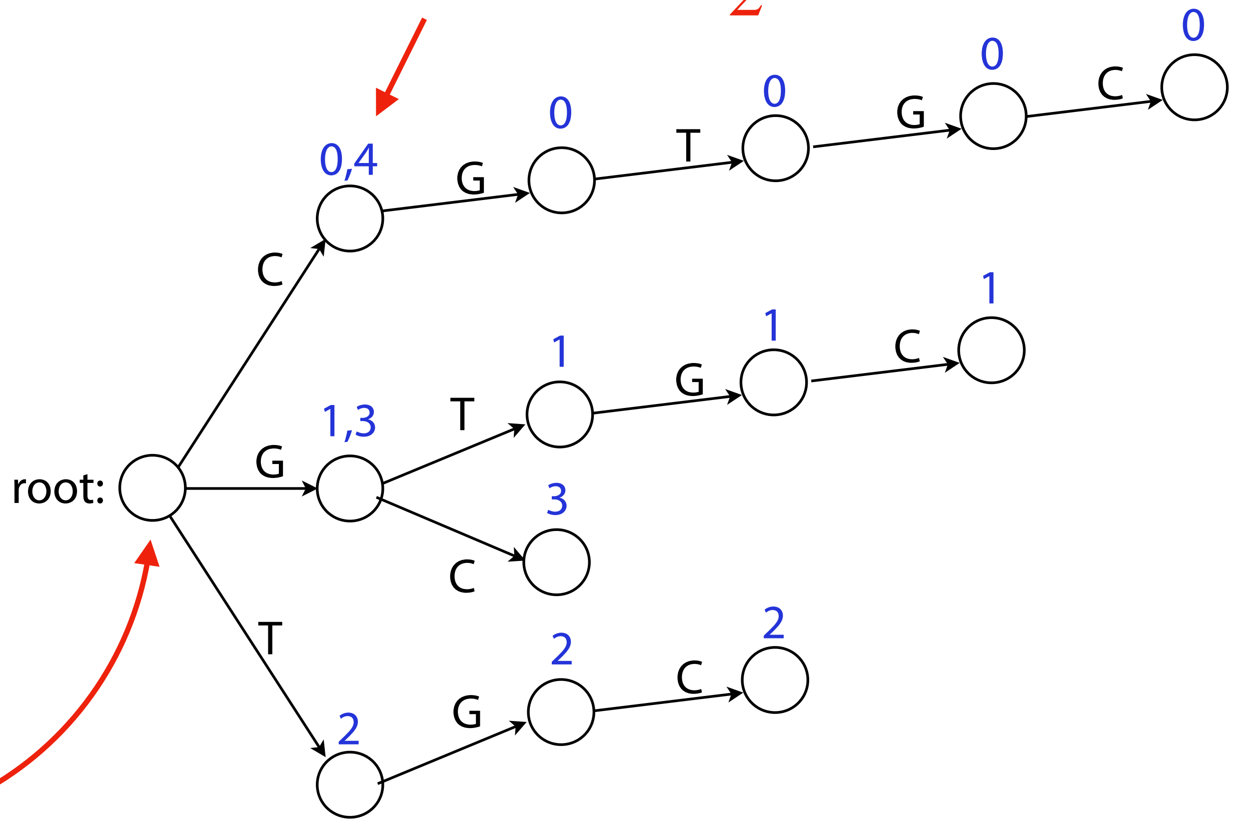
We can do exact pattern matching in $O(P)$ time!

Preprocessing for exact pattern matching

T: **C G T G C**

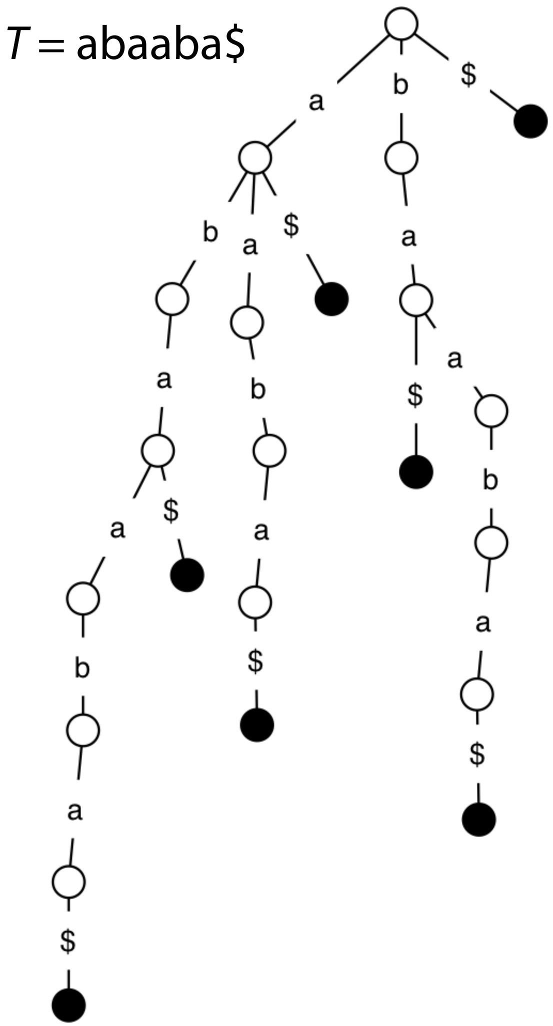
Key	Value
C	0
G	1
T	2
G	3
C	4
CG	0
GT	1
TG	2
…	…

We are storing $\frac{|T|(|T|+1)}{2}$ values



We had to do $\frac{|T|(|T|+1)}{2}$ insertions

Preprocessing for exact pattern matching

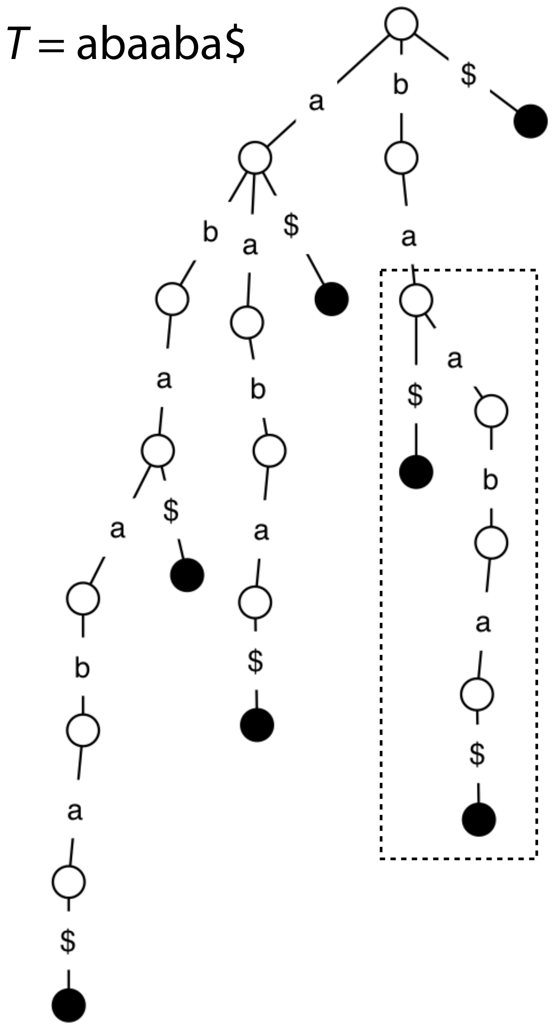


If only there was a way…

to insert fewer strings

to store fewer values

Preprocessing for exact pattern matching



If only there was a way…

to insert fewer strings

to store fewer values

to be even more efficient!