CS 225
Data Structures

February 24 – BST Balance
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5pm Today

https://illinois.zoom.us/j/84537367024?pwd=VDlSQnBjemedwSFhaVTdS
aENKU1JSUT09
BST Analysis

Therefore, for all BST:

**Lower bound:** \( h \geq O( \lg(n) ) \)

**Upper bound:** \( h \leq O(n) \)
BST Analysis

The height of a BST depends on the order in which the data is inserted into it.

ex: 1 3 2 4 5 7 6 vs. 4 2 3 6 7 1 5

Q: How many different ways are there to insert keys into a BST?

Q: What is the average height of all the arrangements?
# BST Analysis – Running Time

<table>
<thead>
<tr>
<th>Operation</th>
<th>BST Average case</th>
<th>BST Worst case</th>
<th>Sorted array</th>
<th>Sorted List</th>
</tr>
</thead>
<tbody>
<tr>
<td>find</td>
<td></td>
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</tr>
<tr>
<td>insert</td>
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<tr>
<td>delete</td>
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<tr>
<td>traverse</td>
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</tbody>
</table>
Height-Balanced Tree

What tree makes you happier?

Height balance: \[ b = \text{height}(T_R) - \text{height}(T_L) \]

A tree is height balanced if:
BST Rotation

We will perform a rotation that maintains two properties:

1.

2.
BST Rotation Summary

- Four kinds of rotations (L, R, LR, RL)
- All rotations are local (subtrees are not impacted)
- All rotations are constant time: O(1)
- BST property maintained

GOAL:

We call these trees:
AVL Trees

Three issues for consideration:
- Rotations
- Maintaining Height
- Detecting Imbalance