



# CS 225

## Data Structures

*February 14 – Circular Lists and Trees*

*G Carl Evans*

## Queue.h

```
1 #pragma once
2
3 template <typename T>
4 class Queue {
5     public:
6         void enqueue(T e);
7         T dequeue();
8         bool isEmpty();
9
10    private:
11        T *items_;
12        unsigned capacity_;
13        unsigned size_;
14 };
15
16
17
18
19
20
21
22
```

What type of implementation is this Queue?

How is the data stored on this Queue?

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What type of implementation is this Queue?

How is the data stored on this Queue?



```
Queue<int> q;
q.enqueue(3);
q.enqueue(8);
q.enqueue(4);
q.dequeue();
q.enqueue(7);
q.dequeue();
q.dequeue();
q.enqueue(2);
q.enqueue(1);
q.enqueue(3);
q.enqueue(5);
q.dequeue();
q.enqueue(9);
```

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```



`Queue<char> q;`

...

`q.enqueue(m);`

`q.enqueue(o);`

`q.enqueue(n);`

...

`q.enqueue(d);`

`q.enqueue(a);`

`q.enqueue(y);`

`q.enqueue(i);`

`q.enqueue(s);`

`q.dequeue();`

`q.enqueue(h);`

`q.enqueue(a);`

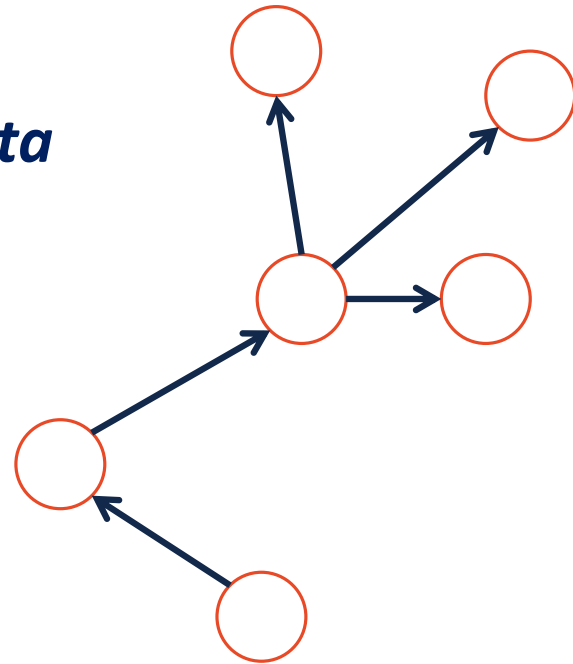
# Trees

*“The most important non-linear data structure in computer science.”*

*- David Knuth, The Art of Programming, Vol. 1*

**A tree is:**

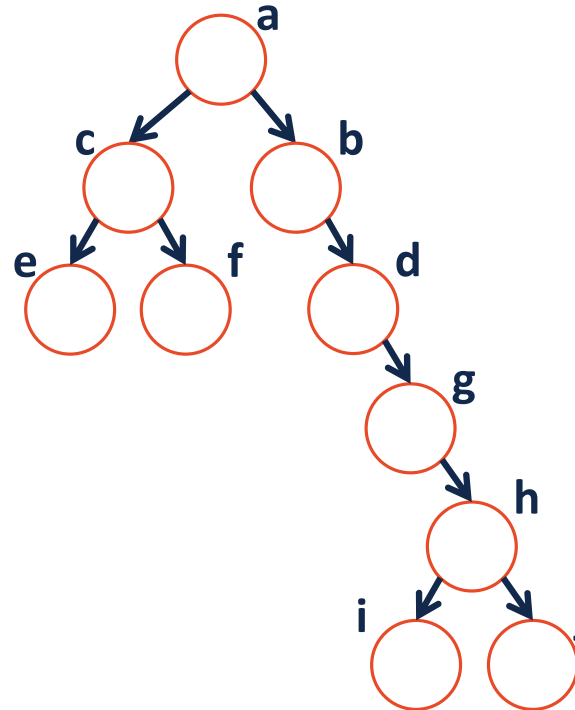
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# More Specific Trees

We'll focus on **binary trees**:

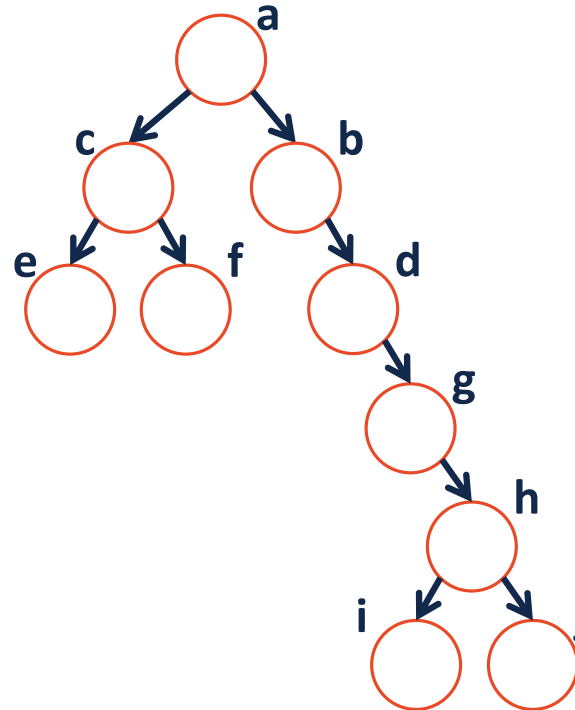
- A binary tree is **rooted** – every node can be reached via a path from the root



# More Specific Trees

We'll focus on **binary trees**:

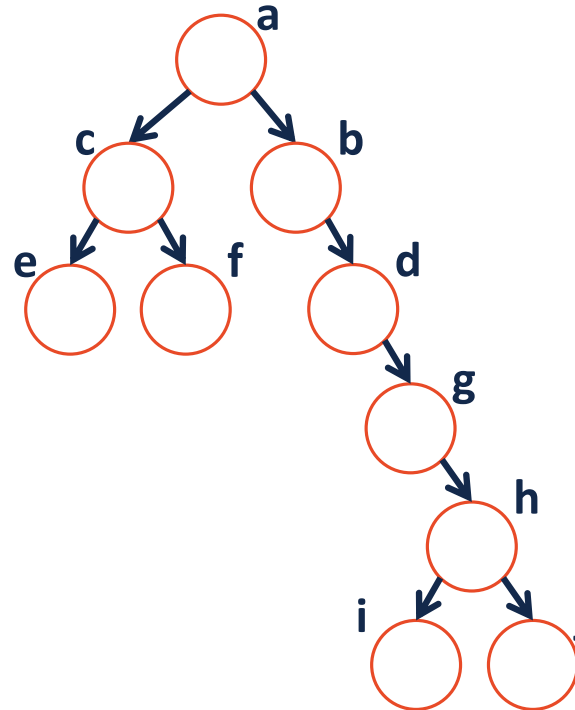
- A binary tree is **acyclic** – there are no cycles within the graph



# More Specific Trees

We'll focus on **binary trees**:

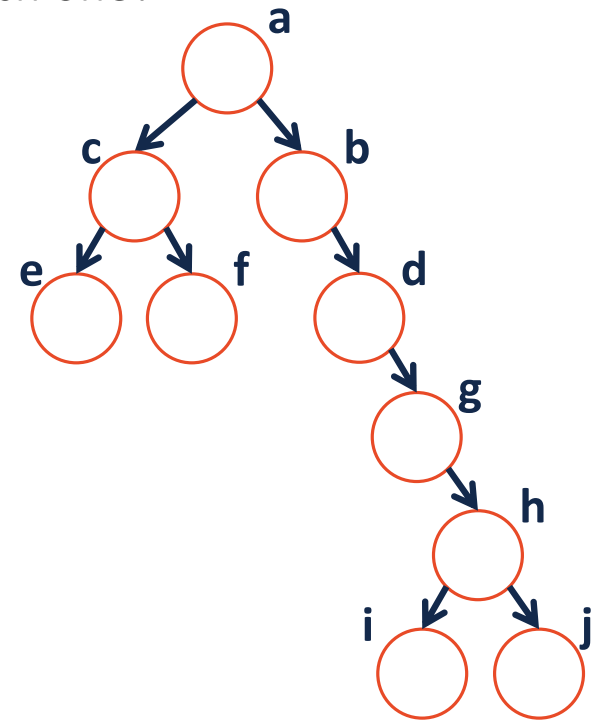
- A binary tree contains **two or fewer children** – where one is the “left child” and one is the “right child”:





# Tree Terminology

- Find an **edge** that is not on the longest **path** in the tree. Give that edge a reasonable name.
- One of the vertices is called the **root** of the tree. Which one?
- How many parents does each vertex have?
- Which vertex has the fewest **children**?
- Which vertex has the most **ancestors**?
- Which vertex has the most **descendants**?
- List all the vertices in b's left **subtree**.
- List all the **leaves** in the tree.



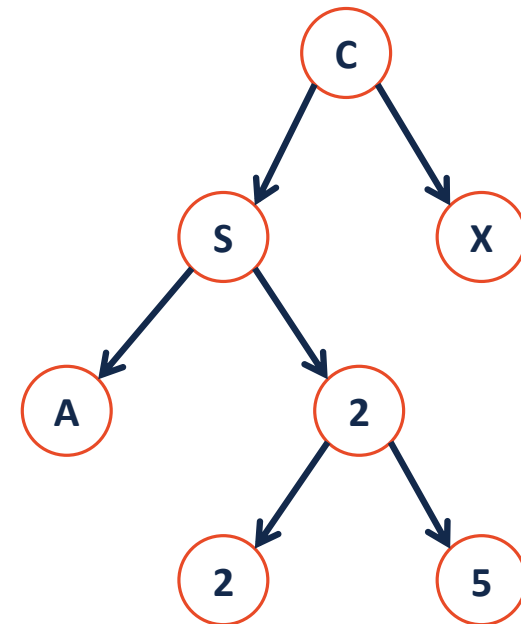
# Binary Tree – Defined

*A binary tree T is either:*

- 

**OR**

- 

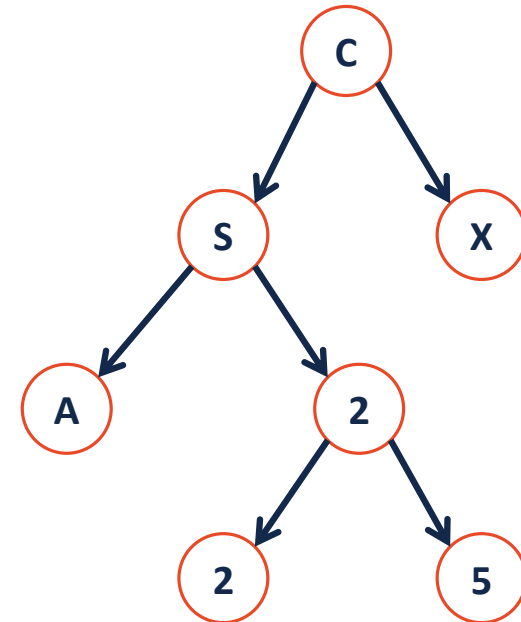


# Tree Property: height

***height(T)***: length of the longest path from the root to a leaf

**Given a binary tree T:**

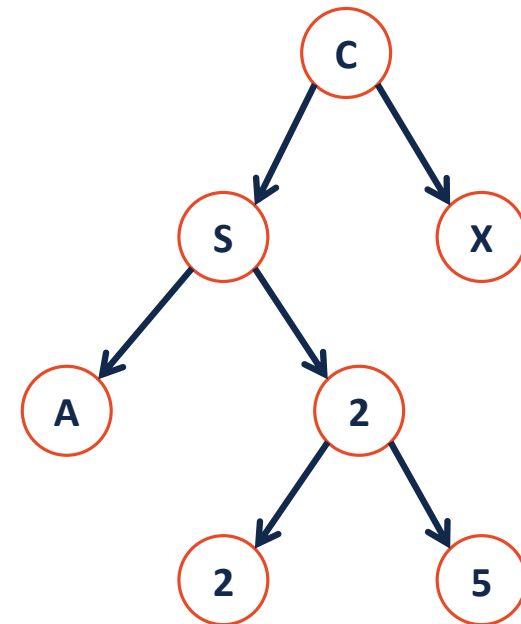
***height(T) =***



# Tree Property: full

A tree  $F$  is **full** if and only if:

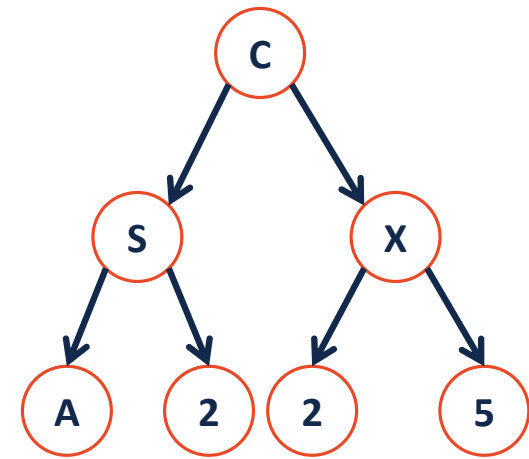
- 1.
- 2.



# Tree Property: perfect

A **perfect** tree  $P$  is:

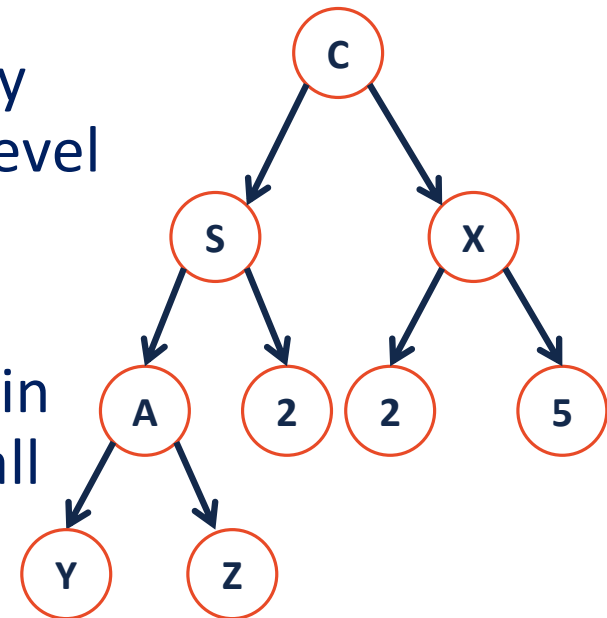
- 1.
- 2.



# Tree Property: complete

**Conceptually:** A perfect tree for every level except the last, where the last level is “pushed to the left”.

**Slightly more formal:** For any level  $k$  in  $[0, h-1]$ ,  $k$  has  $2^k$  nodes. For level  $h$ , all nodes are “pushed to the left”.



# Tree Property: complete

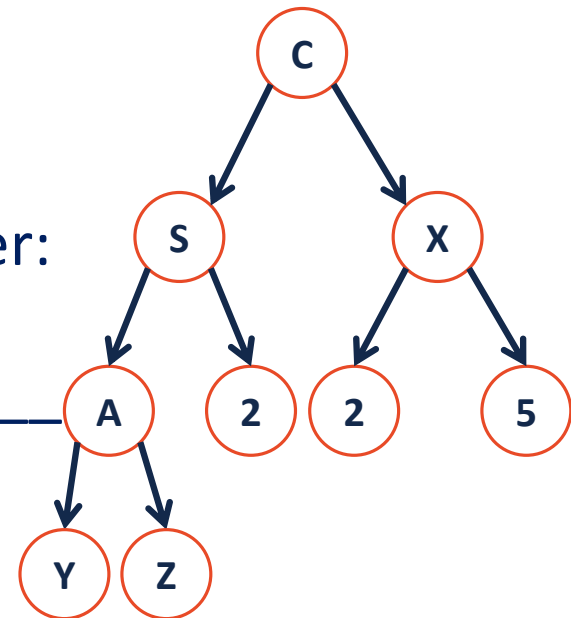
A **complete** tree  $C$  of height  $h$ ,  $C_h$ :

1.  $C_{-1} = \{\}$
2.  $C_h$  (where  $h > 0$ ) =  $\{r, T_L, T_R\}$  and either:

$T_L$  is \_\_\_\_\_ and  $T_R$  is \_\_\_\_\_

**OR**

$T_L$  is \_\_\_\_\_ and  $T_R$  is \_\_\_\_\_



# Tree Property: complete

Is every **full** tree **complete**?

If every **complete** tree **full**?

