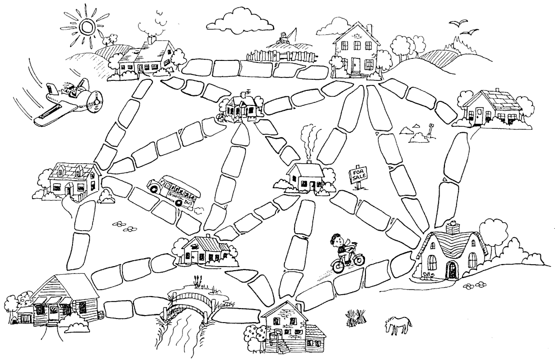
#31: MSTs: Kruskal + Prim's Algorithm

April 6, 2022 · *G Carl Evans*

Minimum Spanning Tree



"The Muddy City" by CS Unplugged, **Creative Commons BY-NC-SA 4.0**

A **Spanning Tree** on a connected graph **G** is a subgraph, **G'**, such that:

1. Every vertex is G is in G' and
2. G' is connected with the minimum number of edges

This construction will always create a new graph that is a __________ (connected, acyclic graph) that spans G.

A **Minimum Spanning Tree** is a spanning tree with the **minimal total edge weights** among all spanning trees.

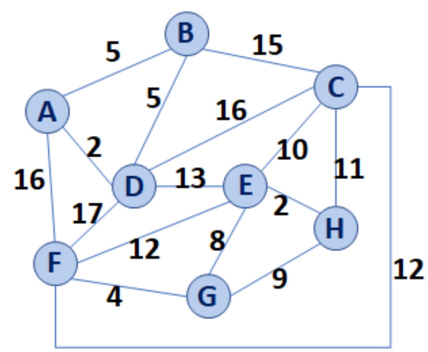
- Every edge must have a weight
  - The weights are unconstrained, except they must be additive *(eg: can be negative, can be non-integers)*
- Output of a MST algorithm produces G':
  - G' is a spanning graph of G
  - G' is a tree

G' has a minimal total weight among all spanning trees. There may be multiple minimum spanning trees, but they will have the same total weight.

Pseudocode for Kruskal's MST Algorithm

```
KruskalMST(G):
  DisjointSets forest
  foreach (Vertex v : G):
    forest.makeSet(v)

  PriorityQueue Q    // min edge weight
  foreach (Edge e : G):
    Q.insert(e)

  Graph T = (V, {})

  while |T.edges()| < n-1:
    Vertex (u, v) = Q.removeMin()
    if forest.find(u) == forest.find(v):
       T.addEdge(u, v)
       forest.union( forest.find(u),
                     forest.find(v) )

  return T
```

Kruskal's Algorithm



(A, D)
(E, H)
(F, G)
(A, B)
(B, D)
(G, E)
(G, H)
(E, C)
(C, H)
(E, F)
(F, C)
(D, E)
(B, C)
(C, D)
(A, F)
(D, F)

Kruskal's Running Time Analysis

We have multiple choices on which underlying data structure to use to build the Priority Queue used in Kruskal's Algorithm:

Priority Queue Implementations:	Heap	Sorted Array
Building :6-8		
Each removeMin :13		

Based on our algorithm choice:

Priority Queue Implementation:	Total Running Time
Heap	
Sorted Array	

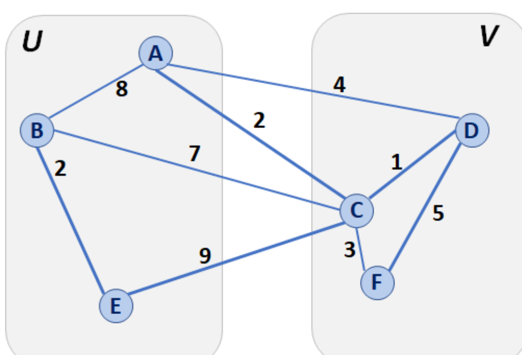
Reflections

Why would we prefer a Heap?

Why would be prefer a Sorted Array?

Partition Property

Consider an arbitrary partition of the vertices on **G** into two subsets **U** and **V**.

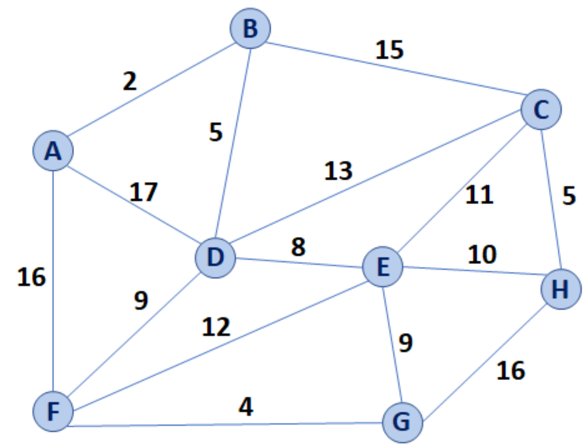


Let **e** be an edge of minimum weight across the partition.

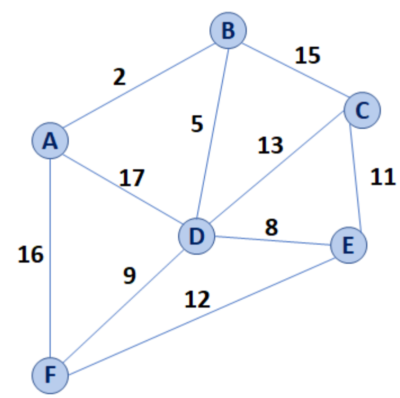
Then **e** is part of some minimum spanning tree.

Proof in CS 374!

Partition Property Algorithm



Prim's Minimum Spanning Tree Algorithm



CS 225 – Things To Be Doing:
1. **mp_mazes due Monday!** 2. If your final project has not been approved get it revised. 3. Daily POTDs are ongoing for +1 point /problem but pausing over break 4. No lecture Friday