Binary Search Tree (BST) Finale

Q: How does our data determine the height?

1 3 2 4 5 7 6 vs. 4 2 3 6 7 1 5

Q: How many different ways are there to insert data into a BST?

Q: What is the average height of every arrangement?

....what’s the intuition for this argument?

Let us describe the balance \( b \) of a BST to be:

- If \( b \) is negative:
- If \( b \) is positive:

We define a BST tree \( T \) to be **height balanced** if:

A node is considered to be **out of balance** if it’s not height balanced. What is the lowest node that is out of balance?

<table>
<thead>
<tr>
<th>operation</th>
<th>BST Avg. Case</th>
<th>BST Worst Case</th>
<th>Sorted Array</th>
<th>Sorted List</th>
</tr>
</thead>
<tbody>
<tr>
<td>find</td>
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<tr>
<td>insert</td>
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<td>delete</td>
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<tr>
<td>traverse</td>
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**Brining a tree back into balance**

Goal: Create a strategy to bring a BST back into balance after an operation has caused the tree to be out of balance.

A **Tree Rotation** is an operation that maintains two properties:

1. 
2. 
Example 1: Defining a Rotation

1. Where is the deepest point of imbalance in the tree: ➔

2. Perform a left rotation to balance this tree:

Implementing a left rotation:

Example 2: A Complex Rotation

Rotation #1:

Rotation #2:

BST Rotation Summary:

1. Four kinds of rotations (L, R, LR, and RL)
2. All rotations are local
3. All rotations run in constant time, O(1)
4. BST property is maintained!

Overall Goal:

...and we call these trees:

<table>
<thead>
<tr>
<th>CS 225 – Things To Be Doing:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. mp_mosaic released EC due Monday.</td>
</tr>
<tr>
<td>2. lab_quacks due Sunday on PL</td>
</tr>
<tr>
<td>3. exam 1 reschedule window Saturday 2/26 – Monday 2/28.</td>
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<tr>
<td>4. Daily POTDs</td>
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