String Algorithms and Data Structures

Tries

CS 199-225
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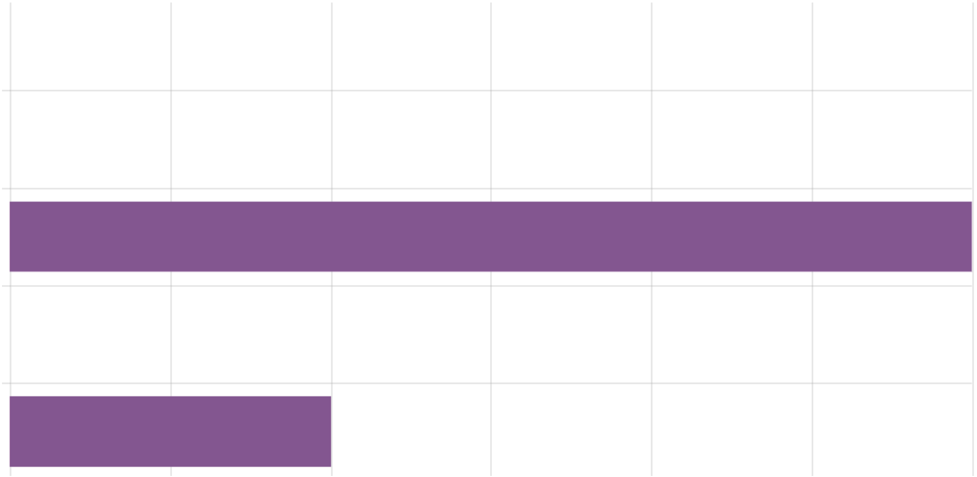
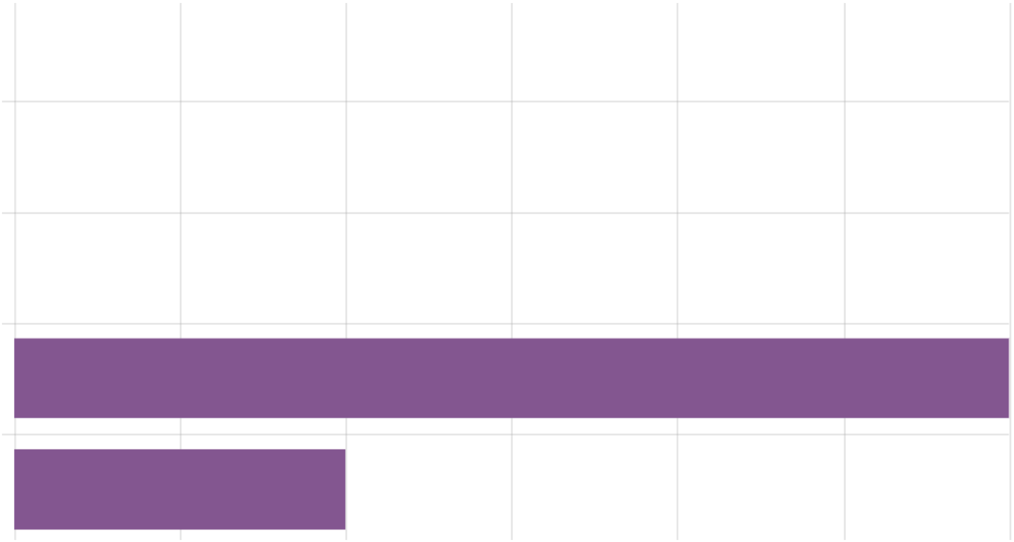
February 21, 2022

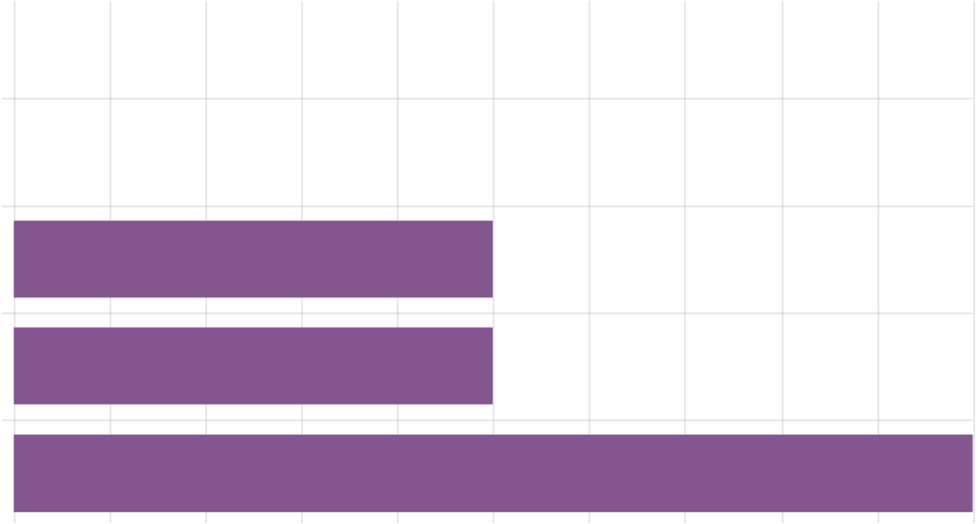
UNIVERSITY OF ILLINOIS
URBANA-CHAMPAIGN

Department of Computer Science

A_zalg reflection


Time


Lecture Helpfulness


Learning Objectives met

Want better labeling of values

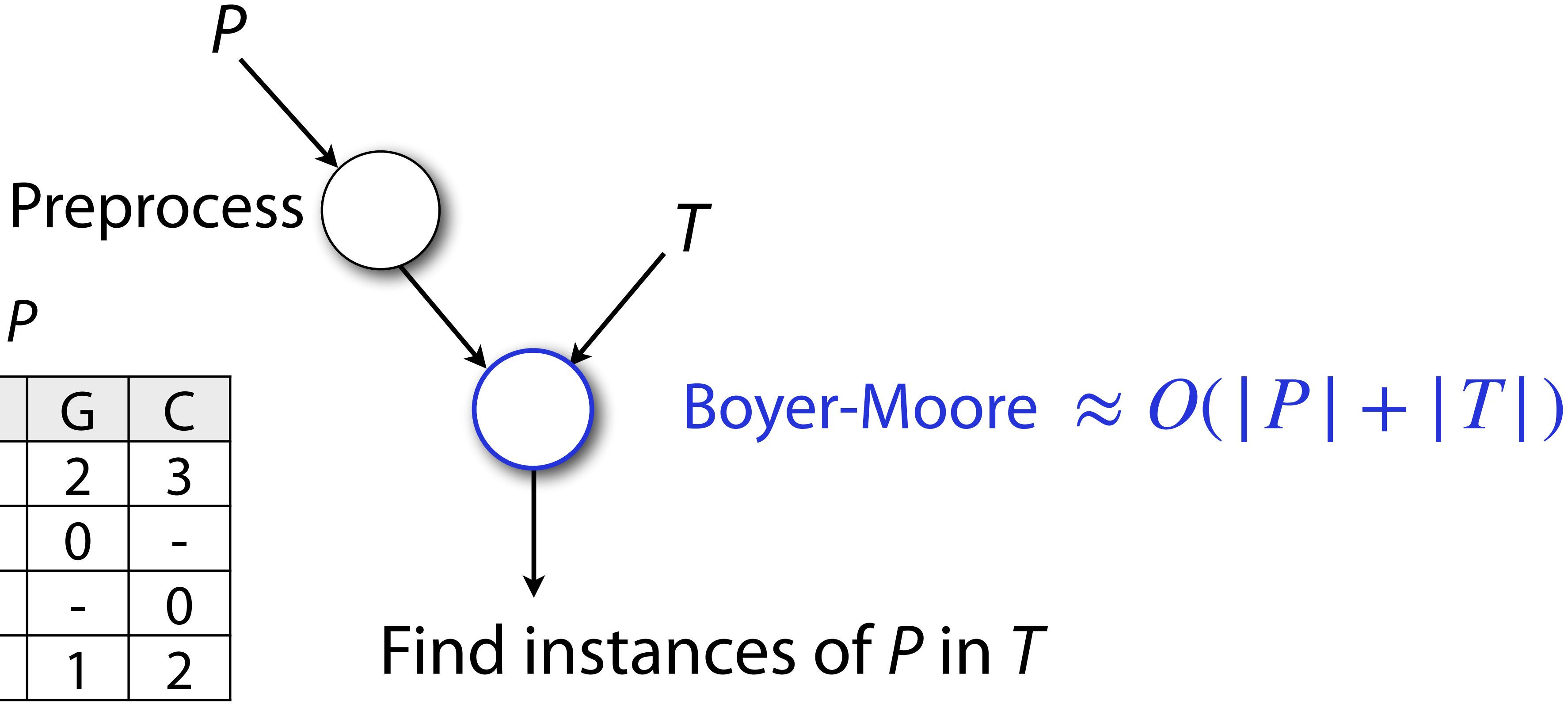
Want better intuition of how / why values work and are needed

A_bmoore due today!

Did you successfully implement preprocessing in linear time?

Exact pattern matching w/ Boyer-Moore

As seen in HW: sub-linear time *in practice*



P

Preprocess

T

Boyer-Moore $\approx O(|P| + |T|)$

Find instances of *P* in *T*

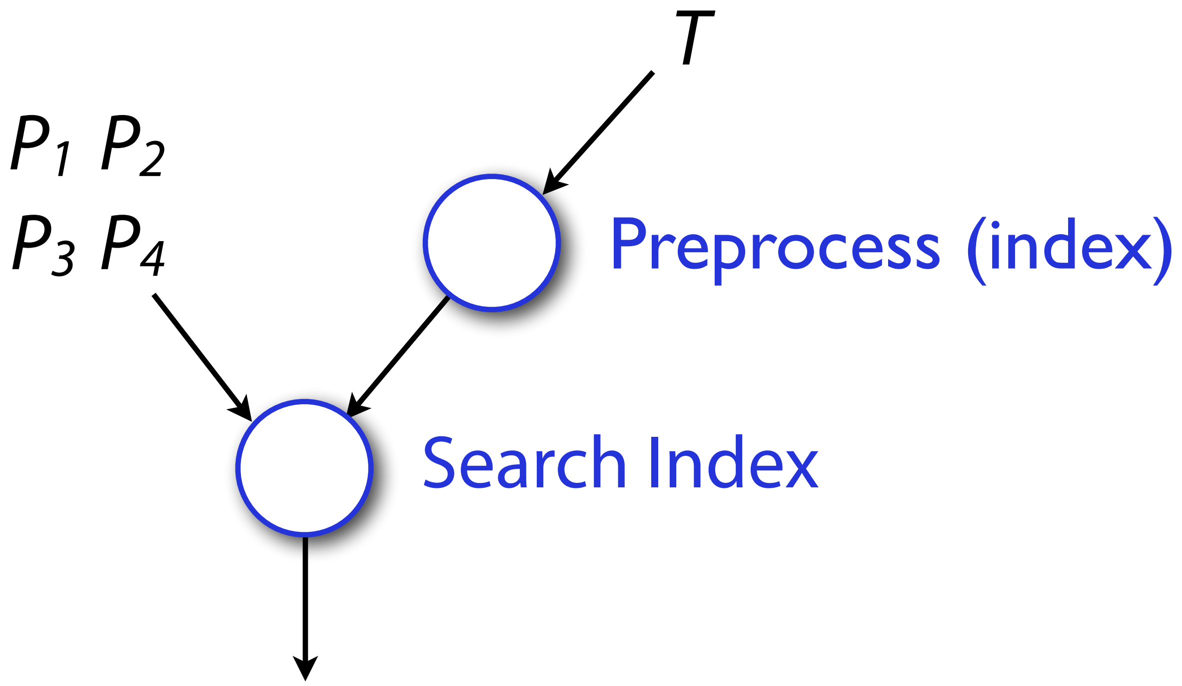
P

		T	C	G	C
	A	0	1	2	3
	C	0	-	0	-
Σ	G	0	1	-	0
	T	-	0	1	2

Make lookup tables

Exact pattern matching *w/ indexing*

Conventionally $T >> P$:



T

P_1 P_2

P_3 P_4

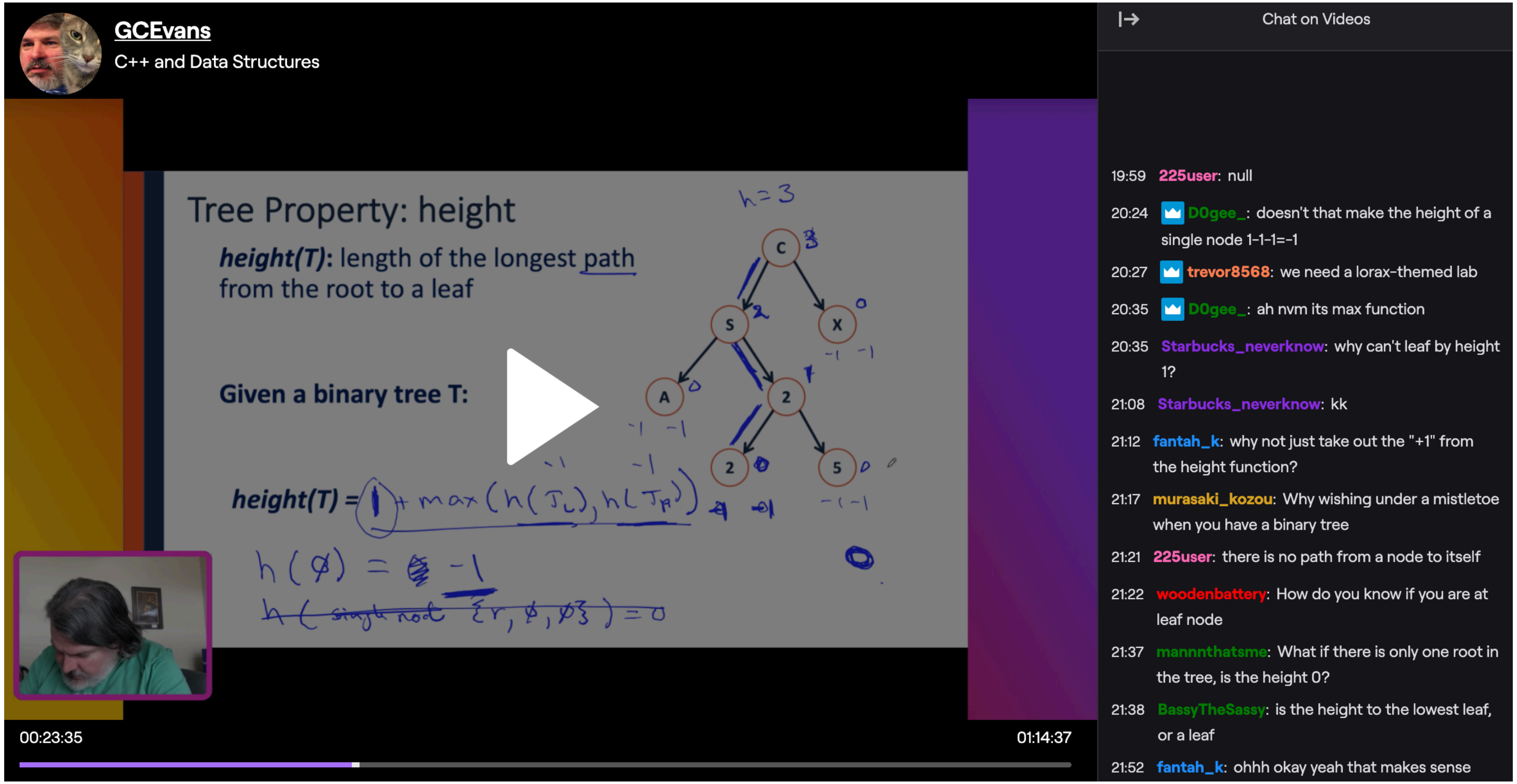
Preprocess (index)

Search Index

Find instances of P in T

Amortize cost of preprocessing T over many P

Preprocessing: Live chat streams



Tree Property: height

height(T)**:** length of the longest path from the root to a leaf

Given a binary tree T:

height(T) = 1 + max(h(T_L), h(T_R))

h(∅) = -1

~~h(single node~~ {r, ∅, ∅}) = 0

19:59 **225user**: null

20:24 **D0gee_**: doesn't that make the height of a single node 1-1-1=-1

20:27 **trevor8568**: we need a lorax-themed lab

20:35 **D0gee_**: ah nvm its max function

20:35 **Starbucks_neverknow**: why can't leaf by height 1?

21:08 **Starbucks_neverknow**: kk

21:12 **fantah_k**: why not just take out the "+1" from the height function?

21:17 **murasaki_kozou**: Why wishing under a mistletoe when you have a binary tree

21:21 **225user**: there is no path from a node to itself

21:22 **woodenbattery**: How do you know if you are at leaf node

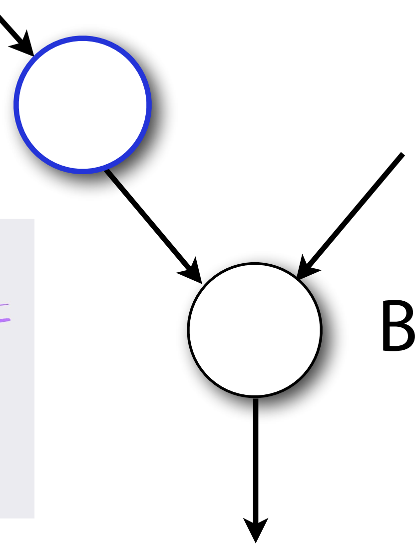
21:37 **mannnthatsme**: What if there is only one root in the tree, is the height 0?

21:38 **BassyTheSassy**: is the height to the lowest leaf, or a leaf

21:52 **fantah_k**: ohhh okay yeah that makes sense

Patterns: banned phrases **Text:** Chat messages

Preprocessing: Live chat streams



!@#$!%

Preprocess

Thanks for the help!

I don't understand that…

You are a !@#$!% teacher

Boyer-Moore

Find instances of *P* in *T*

Amortize cost of preprocessing *P* over many *T*

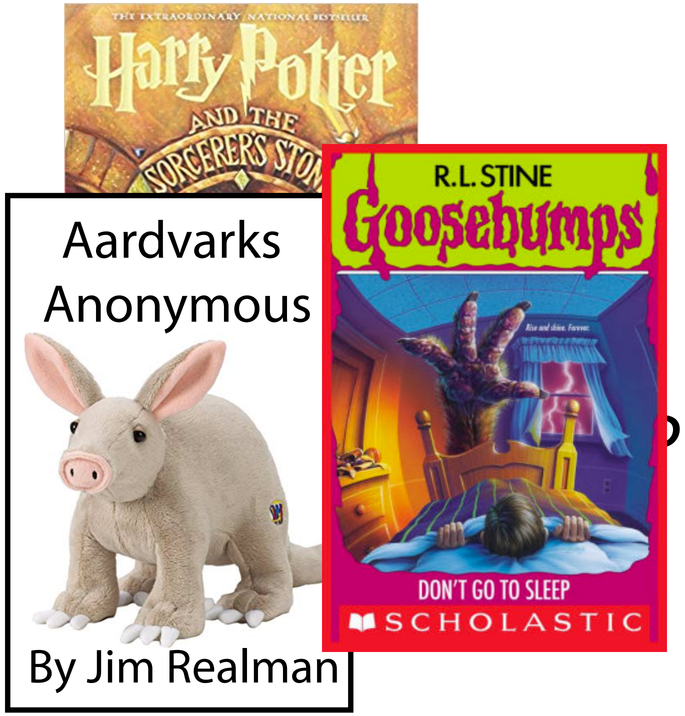
Preprocessing: Libraries

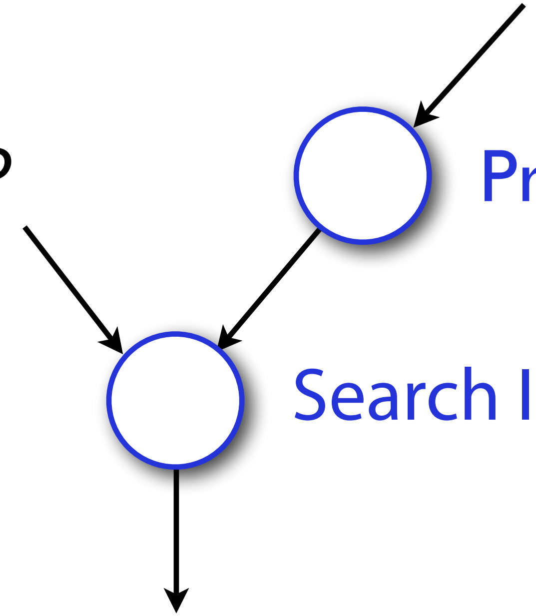
Patterns: Book of interest **Text:** All books in library

Preprocessing: Libraries

Preprocess the library by *indexing* all the books

Preprocessing: Libraries





List of all library books

Preprocess (index)

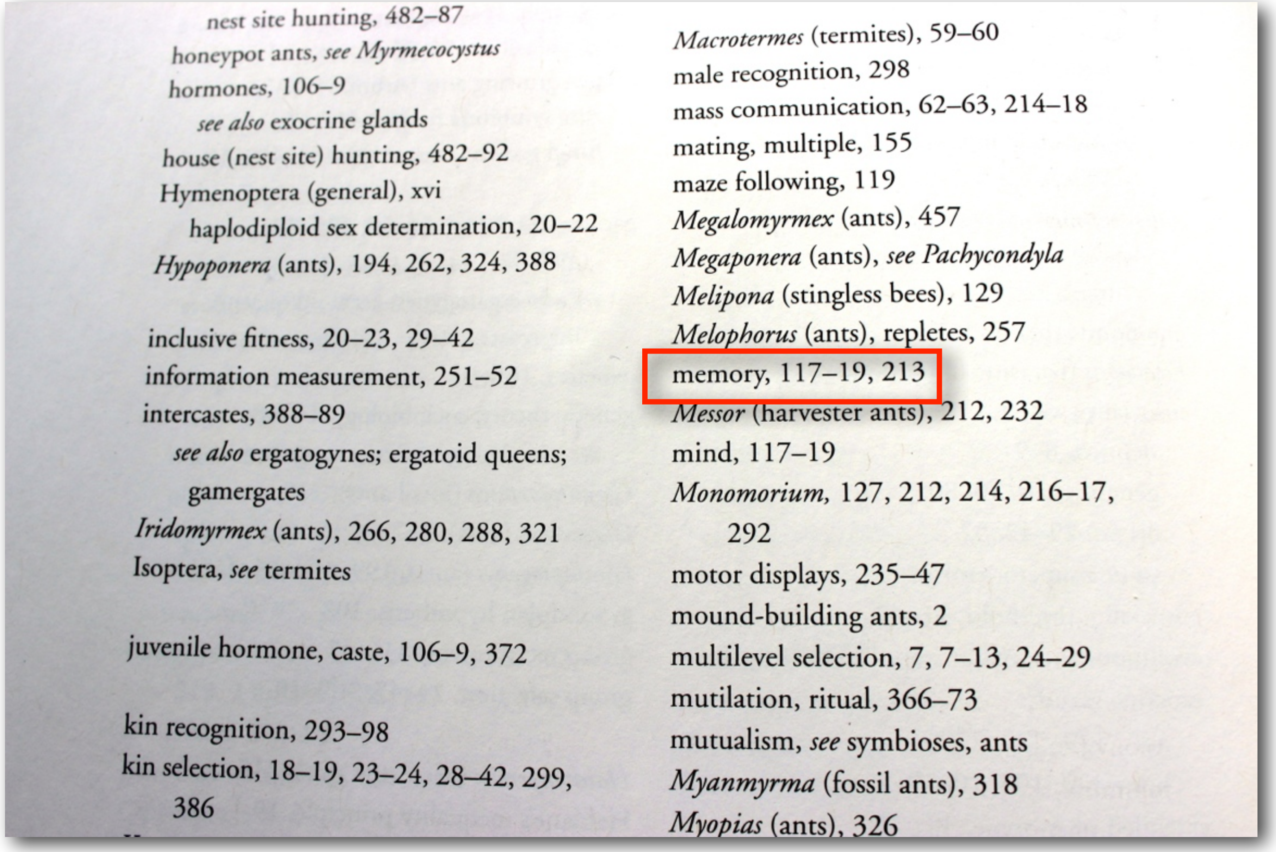
Search Index

Find instances of *P* in *T*

Given full library, built an index once* that is re-used

Preprocessing: Glossaries

What method of preprocessing is this?



nest site hunting, 482–87
honeypot ants, *see Myrmecocystus*
hormones, 106–9
  see also exocrine glands
house (nest site) hunting, 482–92
Hymenoptera (general), xvi
  haplodiploid sex determination, 20–22
Hypoponera (ants), 194, 262, 324, 388

inclusive fitness, 20–23, 29–42
information measurement, 251–52
intercastes, 388–89
  see also ergatogynes; ergatoid queens; gamergates
Iridomyrmex (ants), 266, 280, 288, 321
Isoptera, *see* termites

juvenile hormone, caste, 106–9, 372

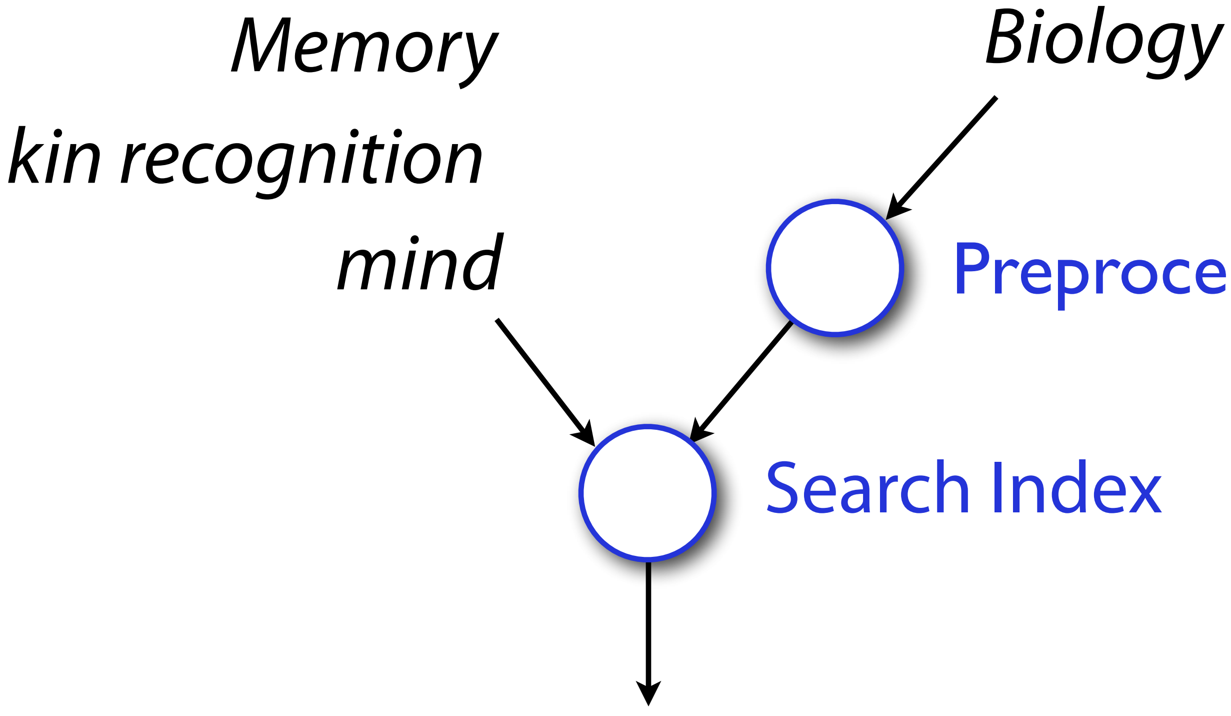
kin recognition, 293–98
kin selection, 18–19, 23–24, 28–42, 299, 386

Macrotermes (termites), 59–60
male recognition, 298
mass communication, 62–63, 214–18
mating, multiple, 155
maze following, 119
Megalomyrmex (ants), 457
Megaponera (ants), *see Pachycondyla*
Melipona (stingless bees), 129
Melophorus (ants), repletes, 257
memory, 117–19, 213
Messor (harvester ants), 212, 232
mind, 117–19
Monomorium, 127, 212, 214, 216–17, 292
motor displays, 235–47
mound-building ants, 2
multilevel selection, 7, 7–13, 24–29
mutilation, ritual, 366–73
mutualism, *see* symbioses, ants
Myanmyrma (fossil ants), 318
Myopias (ants), 326

Patterns: Key terms

Text: All text in the book

Preprocessing: Glossaries

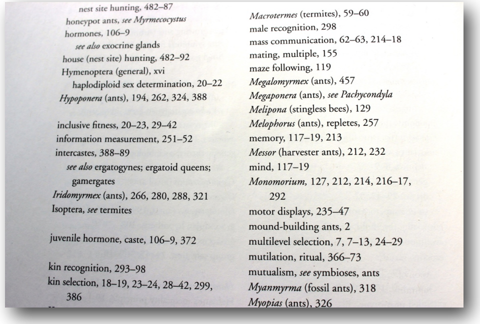


Memory
kin recognition
mind

Biology Textbook

Preprocess (index)

Search Index



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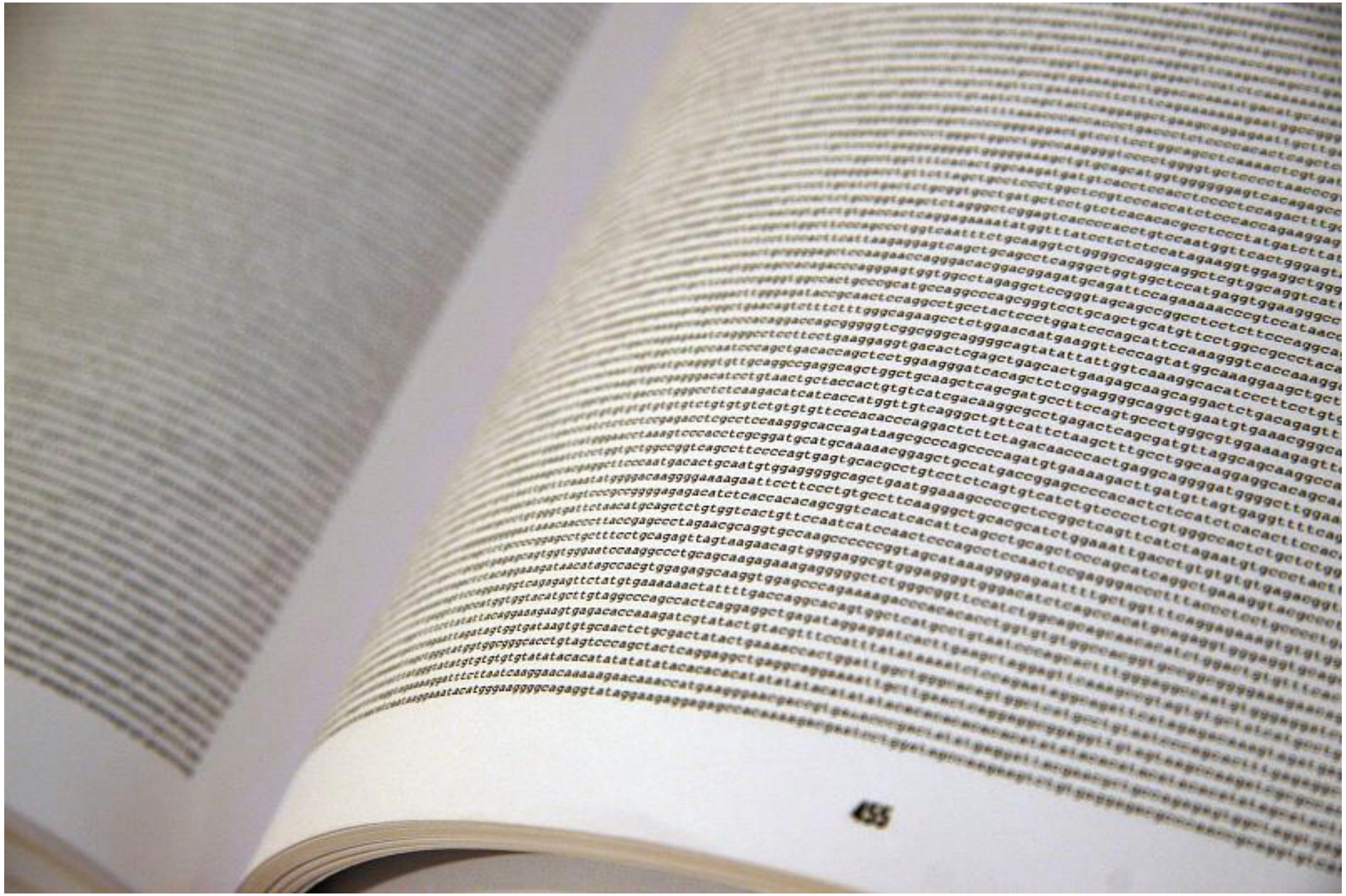
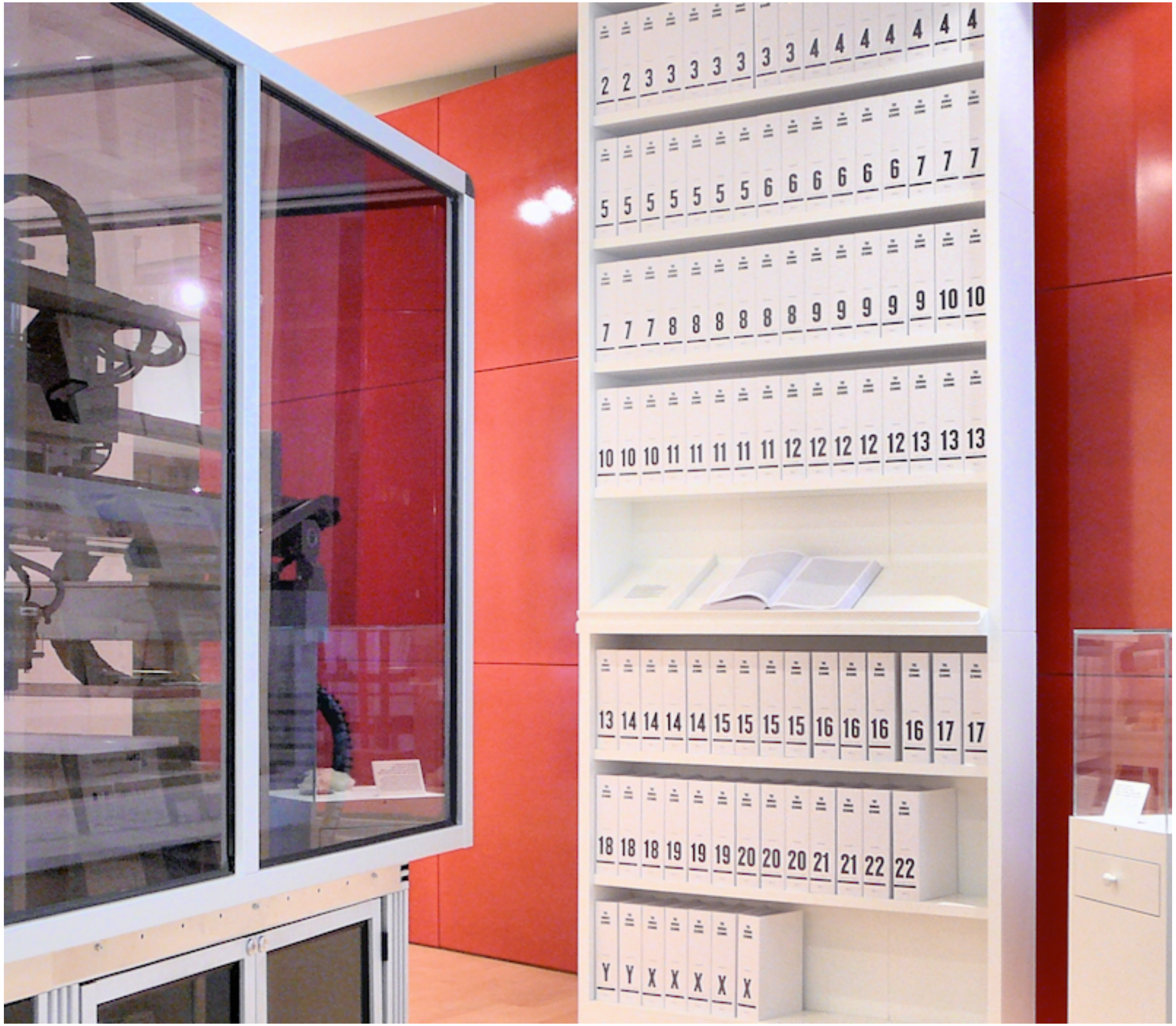
Myopias (ants), 326

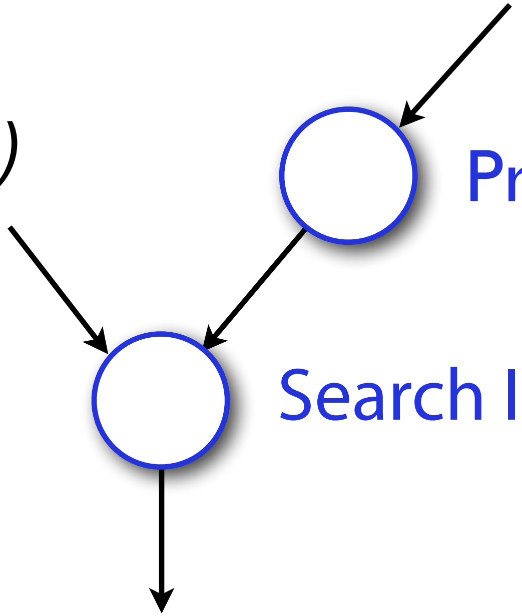
Find instances of *P* in *T*

Glossary built on total contents *T*, useful for multiple *P*


455


2 2 3 3 3 3 3 3 3 4 4 4 4 4 4 4
5 5 5 5 5 5 5 6 6 6 6 6 7 7 7
7 7 7 8 8 8 8 8 8 9 9 9 9 9 10 10
10 10 10 11 11 11 11 11 12 12 12 12 12 13 13 13
13 14 14 14 14 15 15 15 15 16 16 16 16 17 17
18 18 18 19 19 19 20 20 20 21 21 22 22
Y Y X X X X X

Exact pattern matching w/ *indexing*



The human genome (~3 billion)

1 Billion reads (~100)

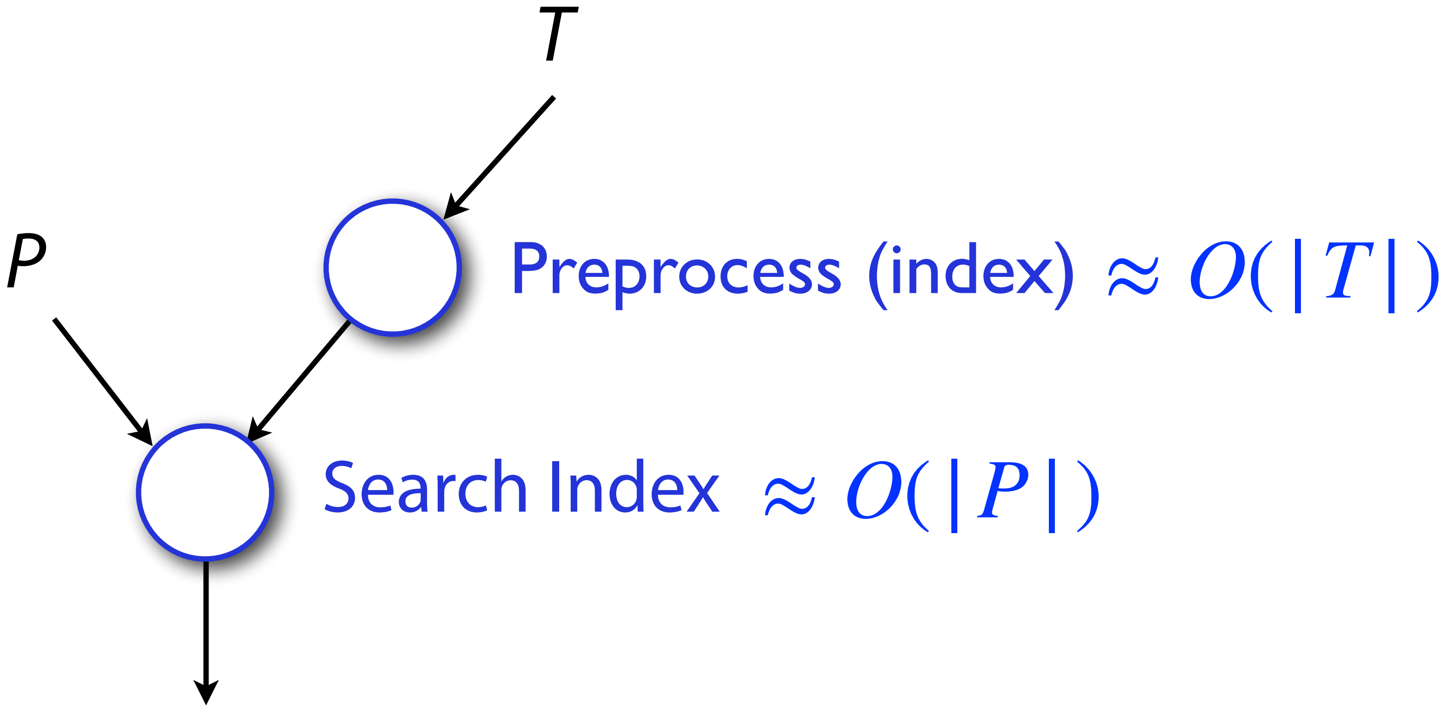
Preprocess (index) $\approx O(|T|)$

Search Index $\approx O(|P|)$

Find instances of *P* in *T*

Amortize cost of preprocessing *T* over many *P*

Exact pattern matching *w/ indexing*



Find instances of *P* in *T*

What information from *T* do we need to search for *P*?

Preprocessing for exact pattern matching

T: **C G T G C**

P:

Search(*P*, *T*):

P:

Search(*P*, *T*):

P:

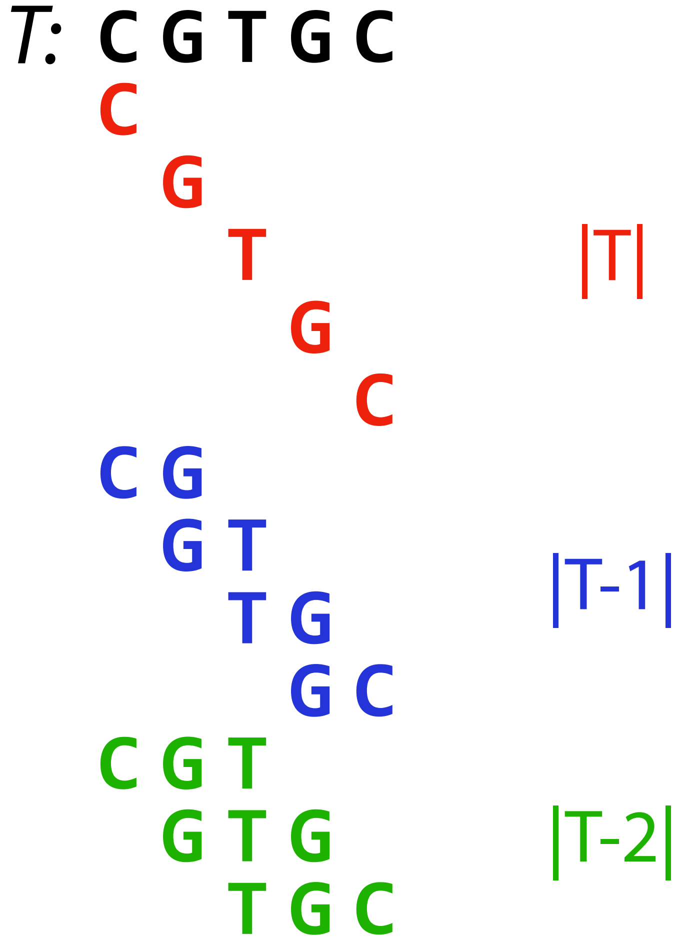
Search(*P*, *T*):

Preprocessing for exact pattern matching

T: **C G T G C**

A substring *S*	The position of *S* in *T*
C	0
G	1
T	2
G	3
C	4
C G	0
G T	1
T G	2
G C	3
C G T	0
G T G	1
T G C	2

Preprocessing for exact pattern matching



T: **C G T G C**

C
G
T |T|
G
C

C G
G T |T-1|
T G
G C

C G T
G T G |T-2|
T G C

Key	Value
C	0
G	1
T	2
G	3
C	4
CG	0
GT	1
TG	2
…	…

?

Preprocessing for exact pattern matching

T: **C G T G C**

C
G
T
G
C

|T|

C G
G T
T G
G C

|T-1|

C G T
G T G
T G C

|T-2|

Key	Value
C	0
G	1
T	2
G	3
C	4
CG	0
GT	1
TG	2
…	…

$$\frac{|T|(|T|+1)}{2}$$

Preprocessing for exact pattern matching

Key	Value
C	0
G	1
T	2
G	3
C	4
CG	0
GT	1
TG	2
…	…

Because our keys are strings, this is sometimes possible!

↑

$$\frac{|T|(|T|+1)}{2}$$

↓

We want to search in $O(|P|)$ without $O(|T|^2)$ space!

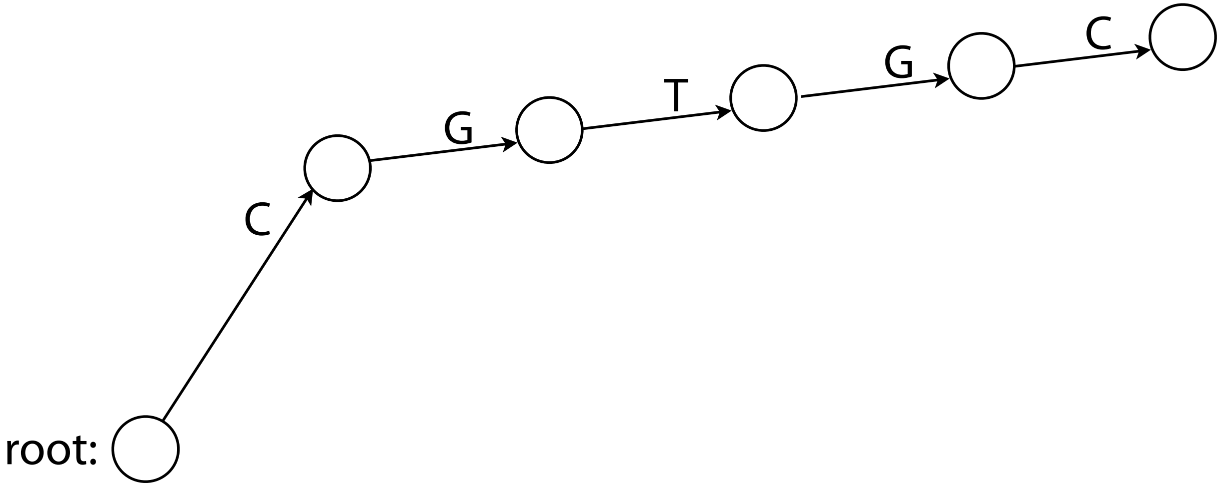
Preprocessing for exact pattern matching

Strings consist of individual characters!

… and these characters can overlap:

T: **C G T G C**

C
C G
C G T
C G T G
C G T G C



Preprocessing for exact pattern matching

Strings consist of individual characters!

… and these characters can overlap:

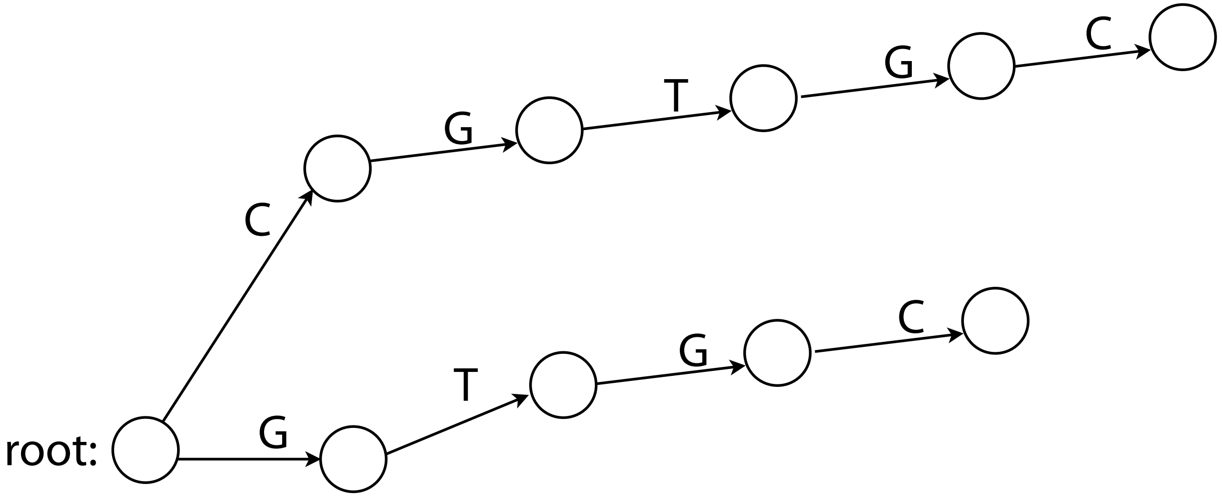
T: **C G T G C**

G

G T

G T G

G T G C



Preprocessing for exact pattern matching

Strings consist of individual characters!

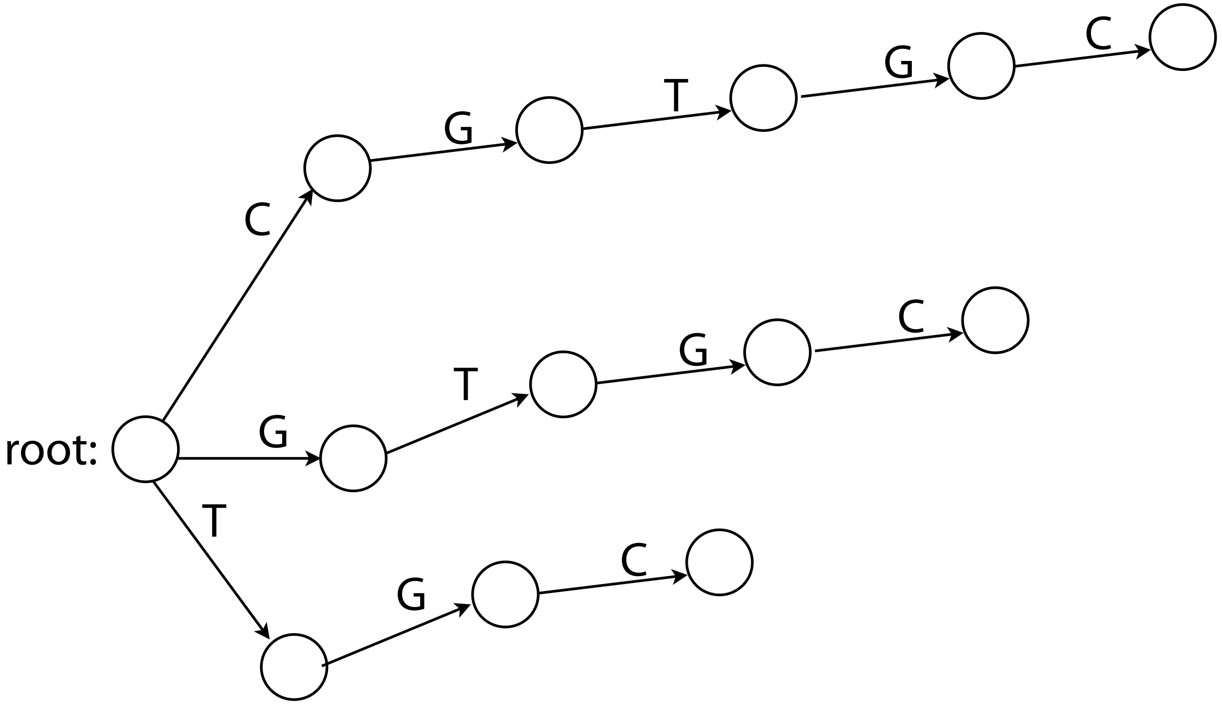
… and these characters can overlap:

T: **C G T G C**

T

T G

T G C



Preprocessing for exact pattern matching

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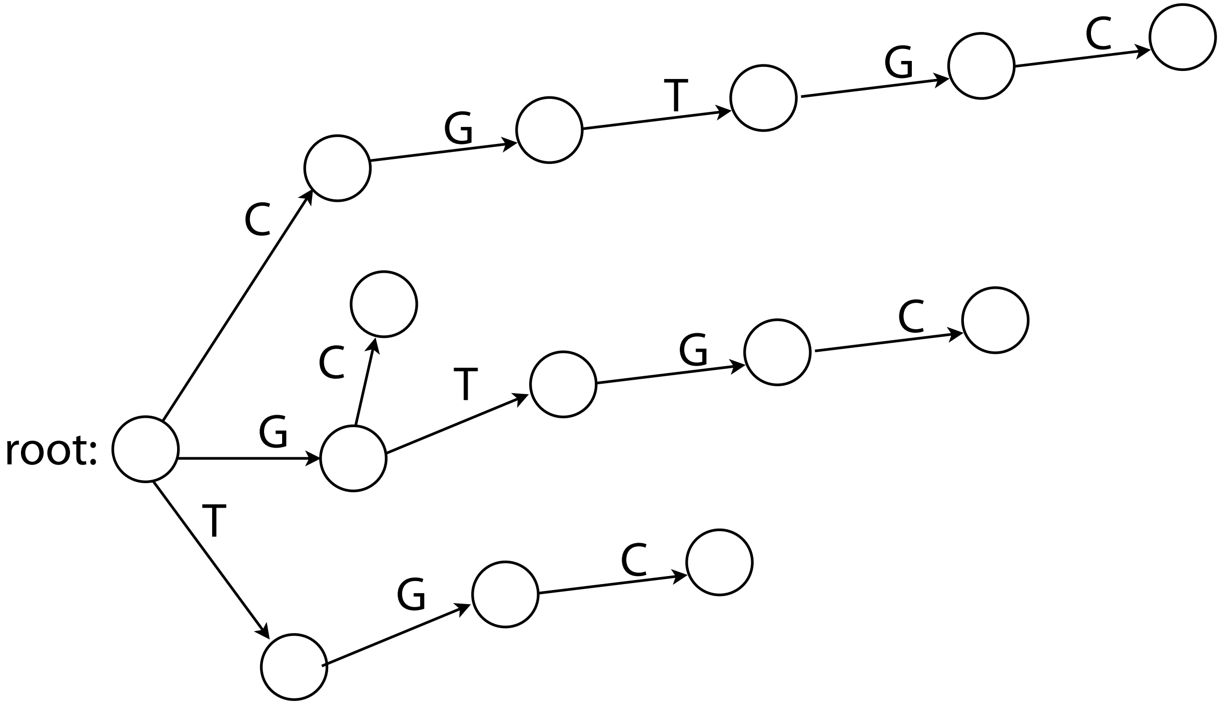
T: **C G T G C**

G

G C



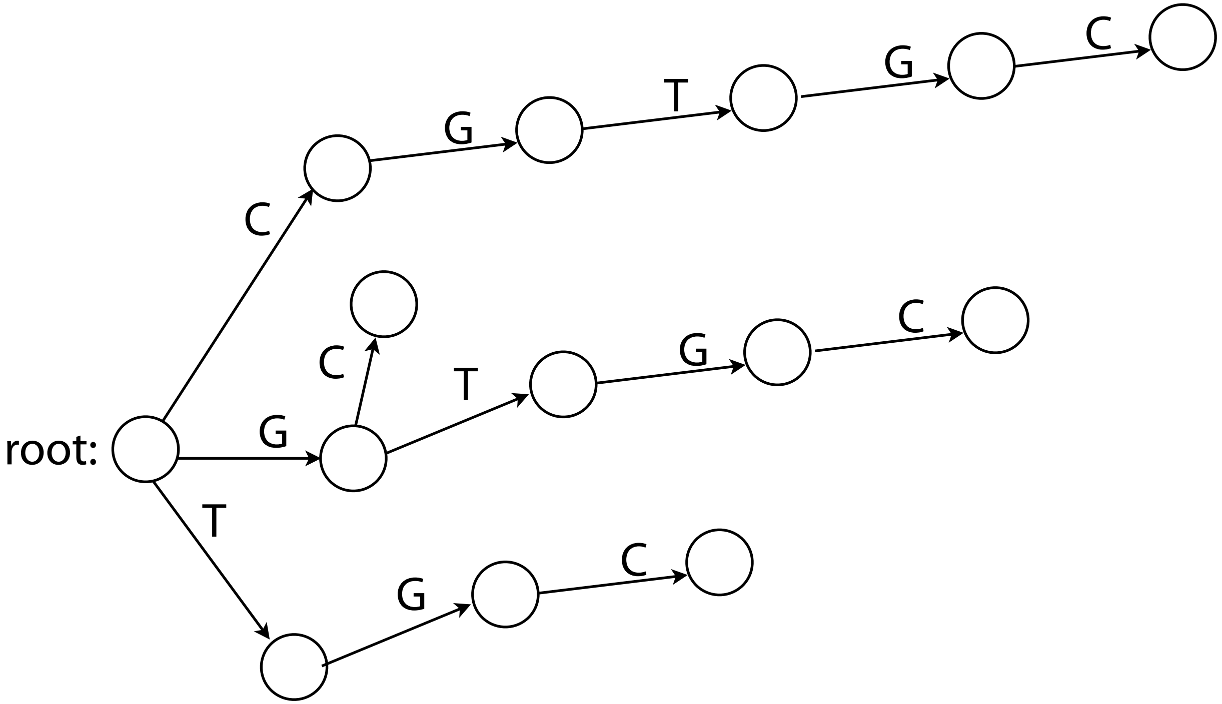
Preprocessing for exact pattern matching

Strings consist of individual characters!

… and these characters can overlap:

T: **C G T G C**

C



String indexing with Tries

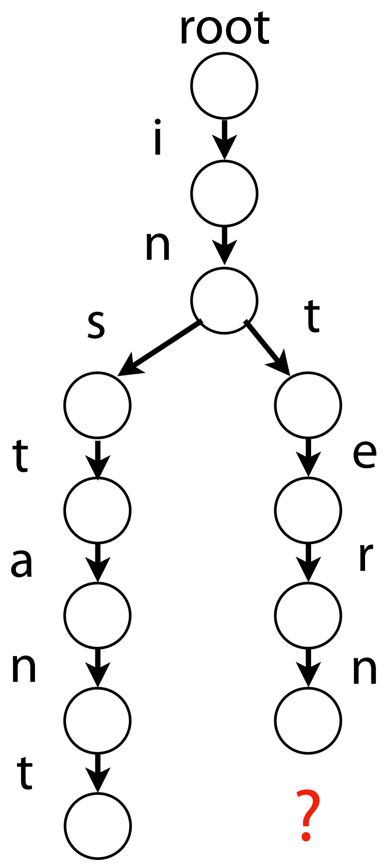
Trie: A rooted tree storing a collection of (key, value) pairs

Keys:	Values:
`instant`	1
`internal`	2
`internet`	3

Each edge is labeled with a character $c \in \Sigma$

For given node, at most one child edge has label c, for any $c \in \Sigma$

Each key is “spelled out” along some path starting at root



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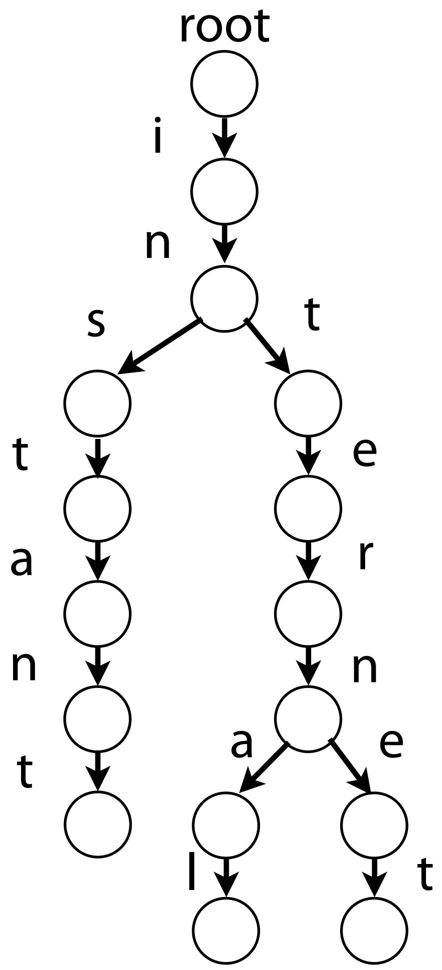
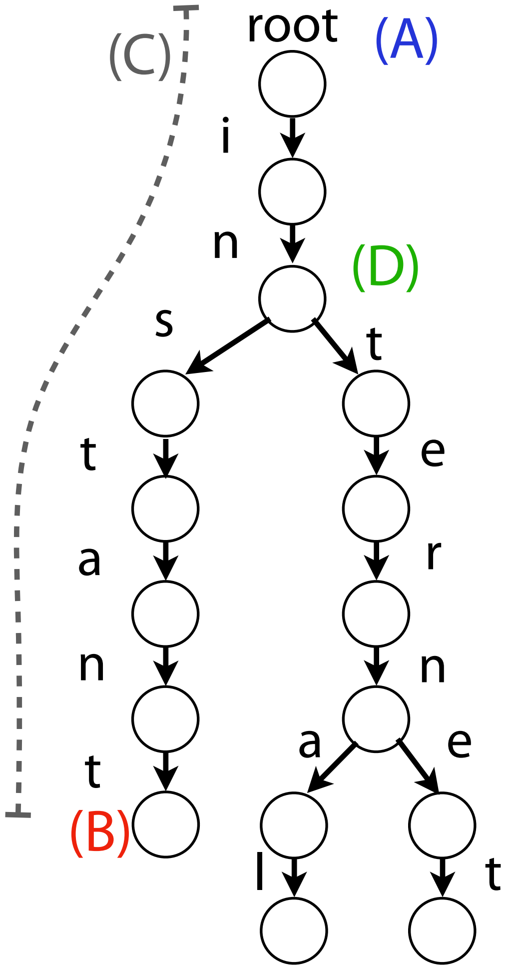


String indexing with Tries

Trie: A rooted tree storing a collection of (key, value) pairs

Keys:	Values:
`instant`	1
`internal`	2
`internet`	3

Where should I store the value 1?

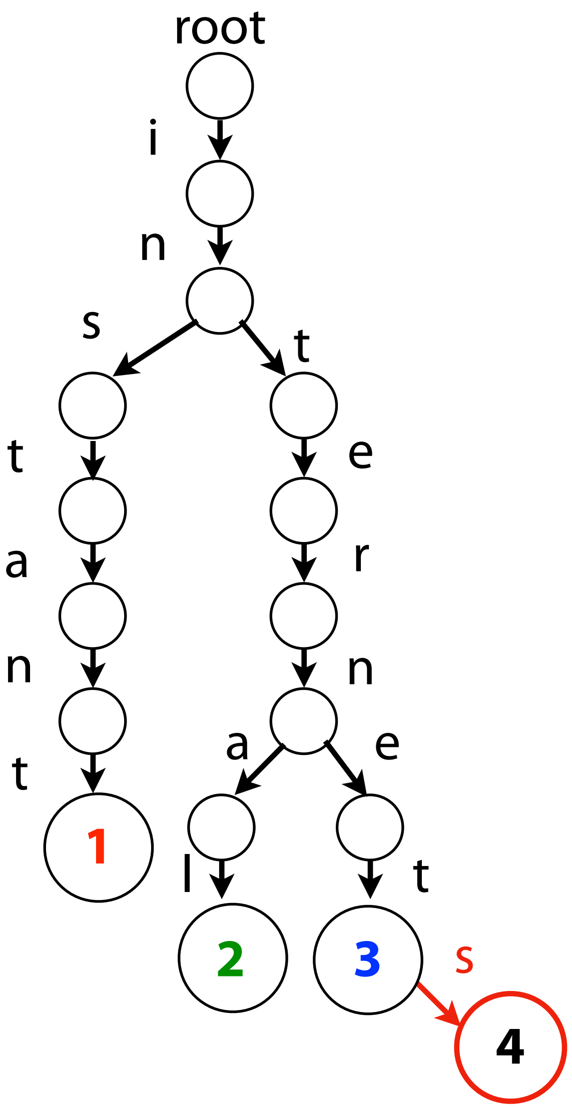


String indexing with Tries

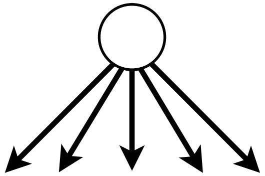
Trie: A rooted tree storing a collection of (key, value) pairs

Keys:	Values:
`instant`	1
`internal`	2
`internet`	3
`internets`	4

Each key's value is stored at the last node in the path



The Node Implementation



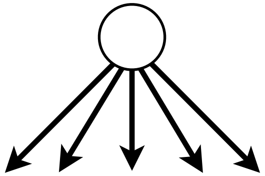
Each node in my trie has $\leq |\Sigma|$ edges!

Each edge is a (potentially NULL) pointer.

How can we encode this?

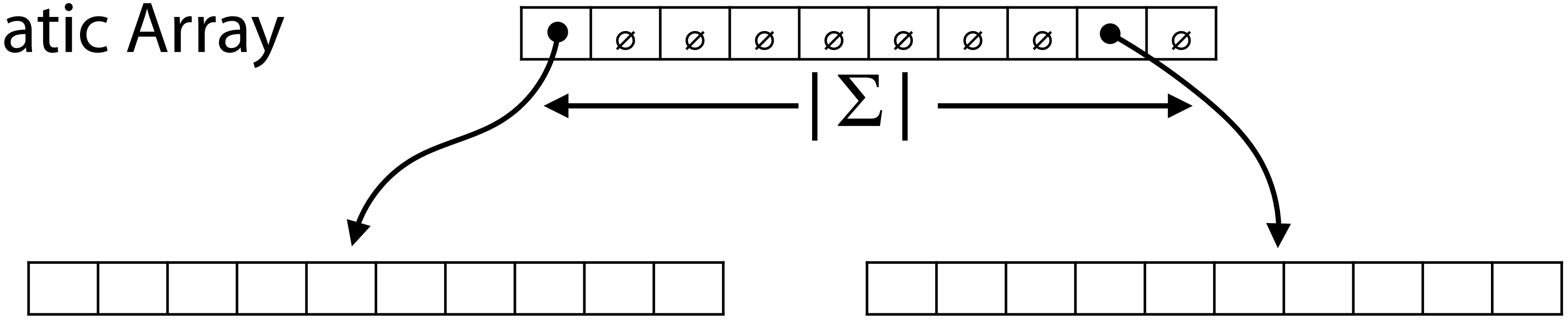
The Node Implementation

Each node in my trie has $\leq |\Sigma|$ edges!

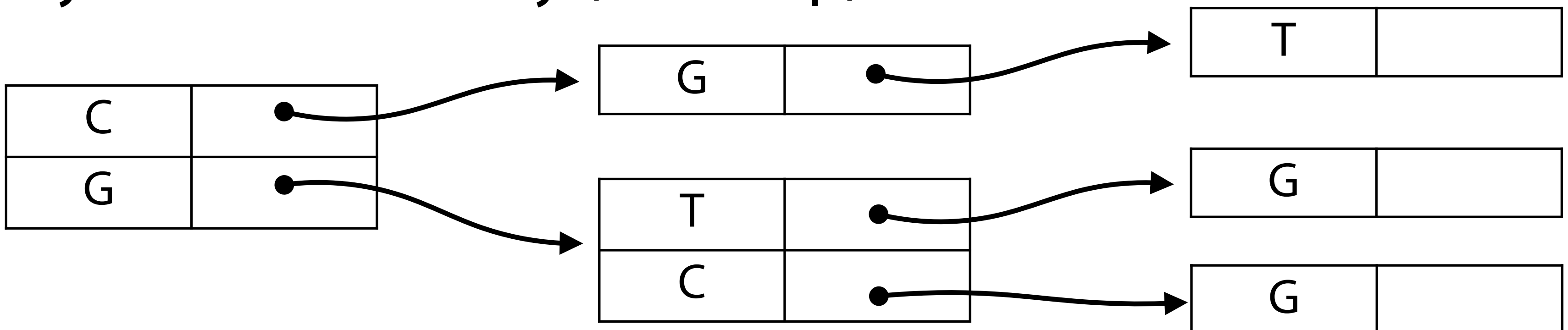


Each edge is a (potentially NULL) pointer.

1) Static Array



2) Dynamically-sized Dictionary (std::map)



Trie Node Implementation

NaryTree.h

```
class NaryTree
{
  public:
    struct Node {
      std::vector<int> index;
      std::map<char, Node*> children;

      Node(std::string s, int i)
      {
        if(s.length() > 0 ){
          children[s[0]] = new Node(s.substr(1), i);
        } else {
          index.push_back(i);
        }
      }
    };
  protected:
    Node* root;
…
}
```

Trie Node Implementation

NaryTree.h

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          index.push_back(i);
        }
      }
    };
  protected:
    Node* root;
…
}
```

C G T G C

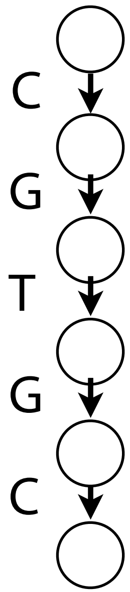
G T G C

T G C

G C

C

i here ->



What if we have more than one string?

Trie Node Implementation

main.cpp

```
NaryTree myT;
myTree.print();

myTree.insert("AB",0);
myTree.print();

myTree.insert("ABA",1);
myTree.print();

myTree.insert(“ABB",2);
myTree.print();

myTree.insert("BAB",3);
myTree.print();

myTree.insert(“BBB",4);
myTree.print();
```

```
x
```

Trie Node Implementation

main.cpp

```
NaryTree myT;
myTree.print();

myTree.insert("AB",0);
myTree.print();

myTree.insert("ABA",1);
myTree.print();

myTree.insert(“ABB",2);
myTree.print();

myTree.insert("BAB",3);
myTree.print();

myTree.insert(“BBB",4);
myTree.print();
```

```
o
+--A--o      0
      +--B--o
```

Trie Node Implementation

main.cpp

```
NaryTree myT;
myTree.print();

myTree.insert("AB",0);
myTree.print();

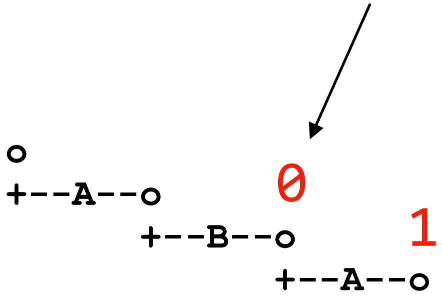
myTree.insert("ABA",1);
myTree.print();

myTree.insert(“ABB",2);
myTree.print();

myTree.insert("BAB",3);
myTree.print();

myTree.insert(“BBB",4);
myTree.print();
```

Former leaf node, still holds value



```
o
+--A--o
      +--B--o
            +--A--o
```

```
struct Node {
        std::vector<int> index;
        std::map<char, Node*> children;
}
```

Trie Node Implementation

main.cpp

```
NaryTree myT;
myTree.print();

myTree.insert("AB",0);
myTree.print();

myTree.insert("ABA",1);
myTree.print();

myTree.insert(“ABB",2);
myTree.print();

myTree.insert("BAB",3);
myTree.print();

myTree.insert(“BBB",4);
myTree.print();
```

```
o
+--A--o      0
      +--B--o      1
            +--A--o
            +--B--o  2
```

Trie Node Implementation

main.cpp

```
NaryTree myT;
myTree.print();

myTree.insert("AB",0);
myTree.print();

myTree.insert("ABA",1);
myTree.print();

myTree.insert(“ABB",2);
myTree.print();

myTree.insert("BAB",3);
myTree.print();

myTree.insert(“BBB",4);
myTree.print();




```

```
o
+--A--o      0
|     +--B--o      1
|           +--A--o
|           +--B--o  2
+--B--o
      +--A--o
            +--B--o  3
```

Trie Node Implementation

main.cpp

```
NaryTree myT;
myTree.print();

myTree.insert("AB",0);
myTree.print();

myTree.insert("ABA",1);
myTree.print();

myTree.insert(“ABB",2);
myTree.print();

myTree.insert("BAB",3);
myTree.print();

myTree.insert(“BBB",4);
myTree.print();
```

```
o
+--A--o      0
|     +--B--o      1
|           +--A--o
|           +--B--o  2
+--B--o
      +--A--o
      |     +--B--o  3
      +--B--o
            +--B--o  4
```

Trie Node Implementation

NaryTree.h

```
void NaryTree::insert(const std::string& s, int i)
{
    insert(root, s, int i);
}

void NaryTree::insert(Node*& node, const std::string & s, int i)
{
    // If we're at a NULL pointer, we make a new Node
    if (node == NULL) {
        node = new Node(s, i);
    } else {
        if(s.length() > 0 ){
            if(node->children.count(s[0]) > 0){
               insert(node->children[s[0]],s.substr(1), i);
            }else{
                node->children[s[0]] = new Node(s.substr(1), i);
            }
        } else{
            node->index = i;
        }

    }
}
```

Assignment 5: a_narytree

Learning Objective:

Store all substrings in a trie using NaryTree implementation

Implement exact pattern matching using this trie

Due: February 28th 11:59 PM

Consider: How many insertions are we doing for each string?
Is there a better or faster way to do this?

Trie Node Implementation

main.cpp

```
NaryTree myT;

myTree.insert("AB",0);

myTree.insert(“AB",2);

myTree.print();
```

```
o
+--A--o        ??
      +--B--o
```

Trie Node Implementation

main.cpp

```
NaryTree myT;

myTree.insert("AB",0);

myTree.insert(“AB",2);

myTree.print();
```

```
o
+--A--o
      +--B--o
```

0,2

```
struct Node {
    std::vector<int> index;
    std::map<char, Node*> children;
}
```

```
if(s.length() > 0 ){
    if(node->children.count(s[0]) > 0){
        insert(node->children[s[0]],s.substr(1), i);
    }else{
        node->children[s[0]] = new Node(s.substr(1), i);
    }
} else{
    node->index.push_back(i);
}
```

Searching a Trie

Given *P*, search the trie for keys and return values

Pattern: `infer`

`infer`

`infer`

`infer`

Lets break that down using *recursion*:

Starting at root:

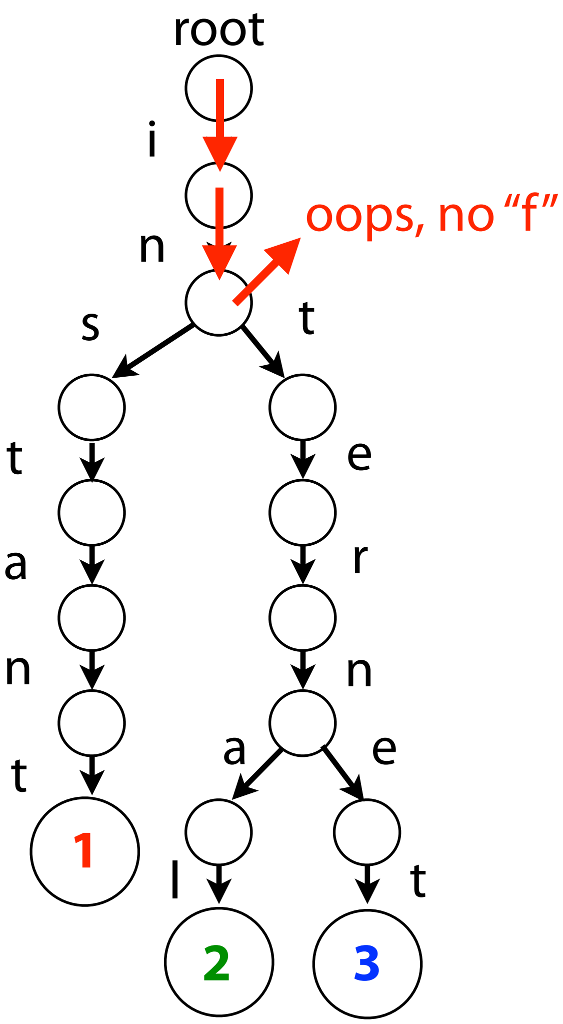
(1) Try to match front character

(2) If match, move to appropriate child

(2.5) Set pattern equal to remainder

(2.5) Go back to (1)

(3) If mismatch, *P* is not a key!



Searching a Trie

Given *P*, search the trie for keys and return values

Pattern: `i n t e r e s t i n g`

`i n t e r e s t i n g`

`i n t e r e s t i n g`

Lets break that down using *recursion*:

Starting at root:

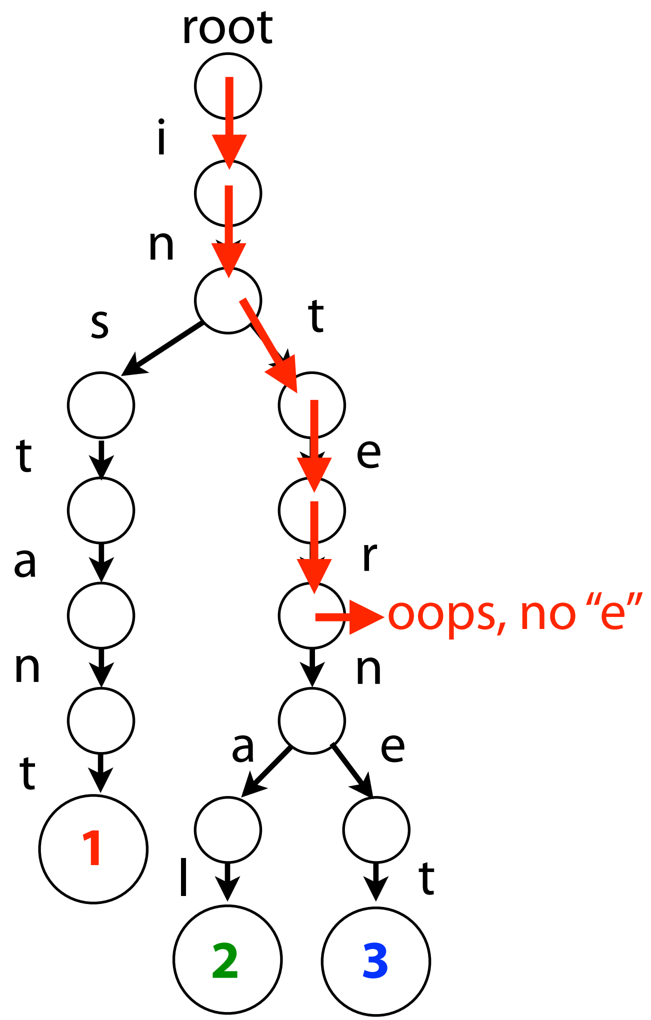
(1) Try to match front character

(2) If match, move to appropriate child

(2.5) Set pattern equal to remainder

(2.5) Go back to (1)

(3) If mismatch, *P* is not a key!



Searching a Trie

Given *P*, search the trie for keys and return values

Pattern: `i n s t a`

`i n s t a`

Lets break that down using *recursion*:

Starting at root:

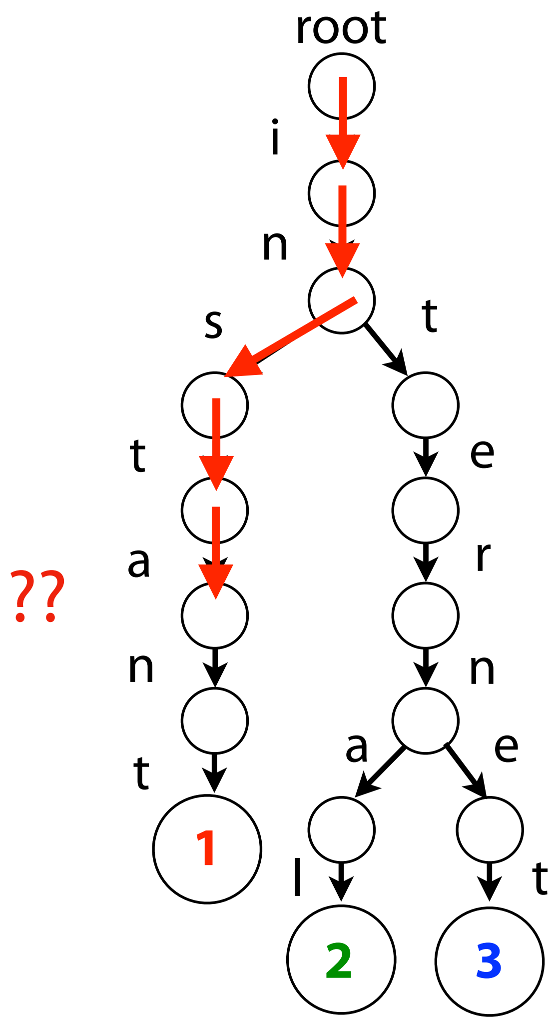
(1) Try to match front character

(2) If match, move to appropriate child

(2.5) Set pattern equal to remainder

(2.5) Go back to (1)

(3) If mismatch, *P* is not a key!



Searching a Trie

Given *P*, search the trie for keys and return values

Pattern: `i n s t a`

`i n s t a`

Lets break that down using *recursion*:

Starting at root:

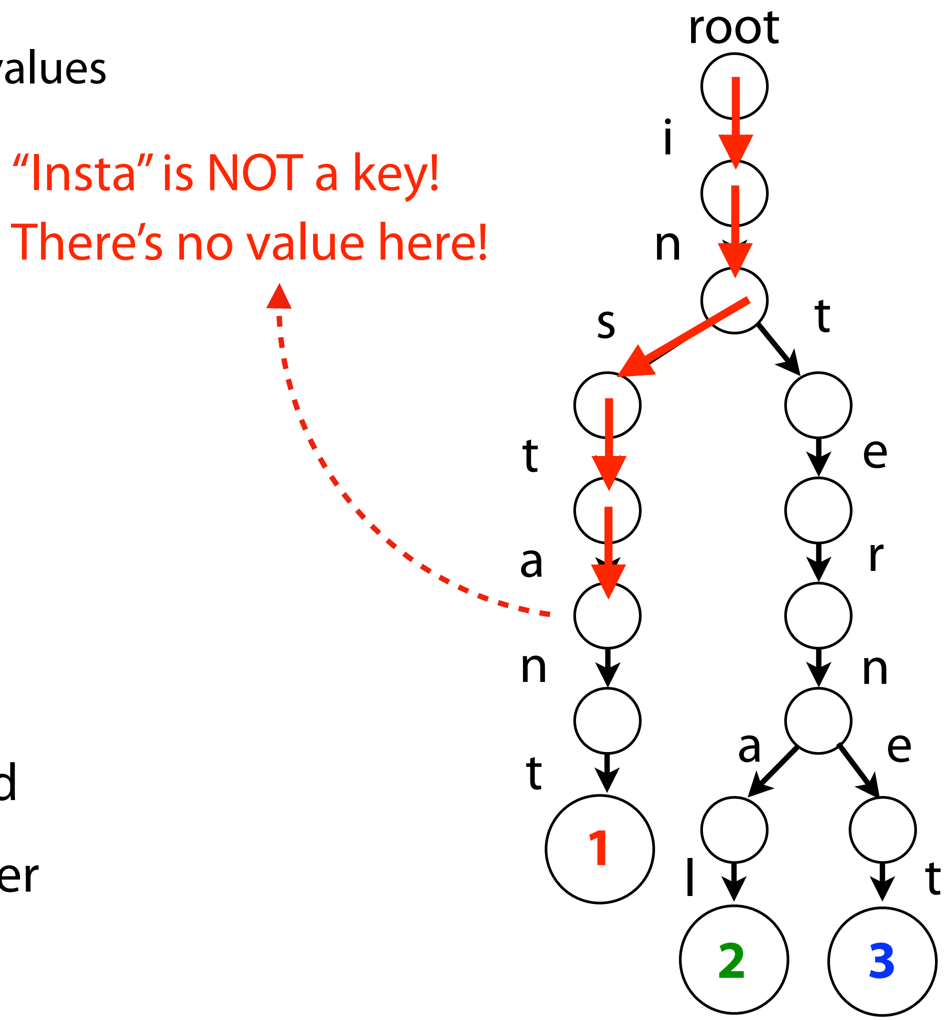
(1) Try to match front character

(2) If match, move to appropriate child

(2.5) Set pattern equal to remainder

(2.5) Go back to (1)

(3) If mismatch, *P* is not a key!


“Insta” is NOT a key!

There’s no value here!

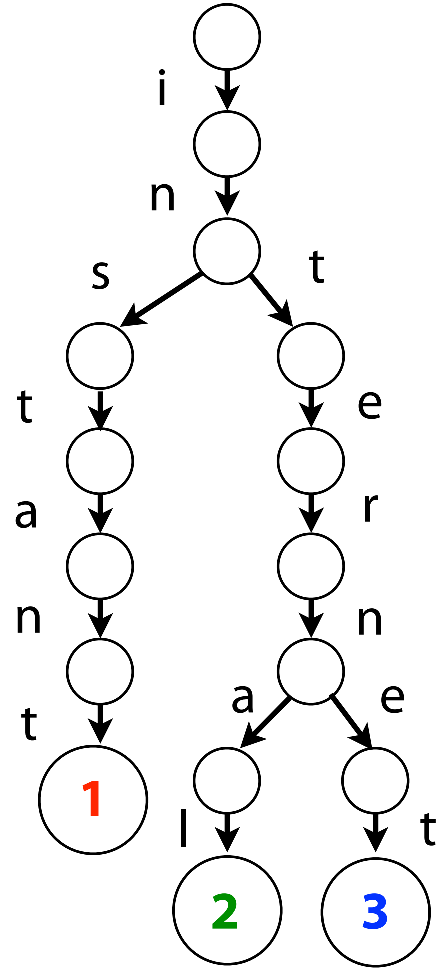
String indexing with Tries

A rooted tree storing a collection of (key, value) pairs

Keys:	Values:
`instant`	`1`
`internal`	`2`
`internet`	`3`

The trie is structured such that:

- Each edge is labeled with a character $c \in \Sigma$
- For given node, at most one child edge has label c, for any $c \in \Sigma$
- Each key is "spelled out" along some path starting at root
- Each key's value is stored at the last node in the path



Searching a Trie

Given *P*, search the trie for keys and return values

Pattern: `i n s t a`

`i n s t a`

Lets break that down using *recursion*:

Starting at root:

(0) If we have no 'front' char, check value

(0.5) If no value, *P* is not a key!

(0.5) If value, *P* is a key, return value(s).

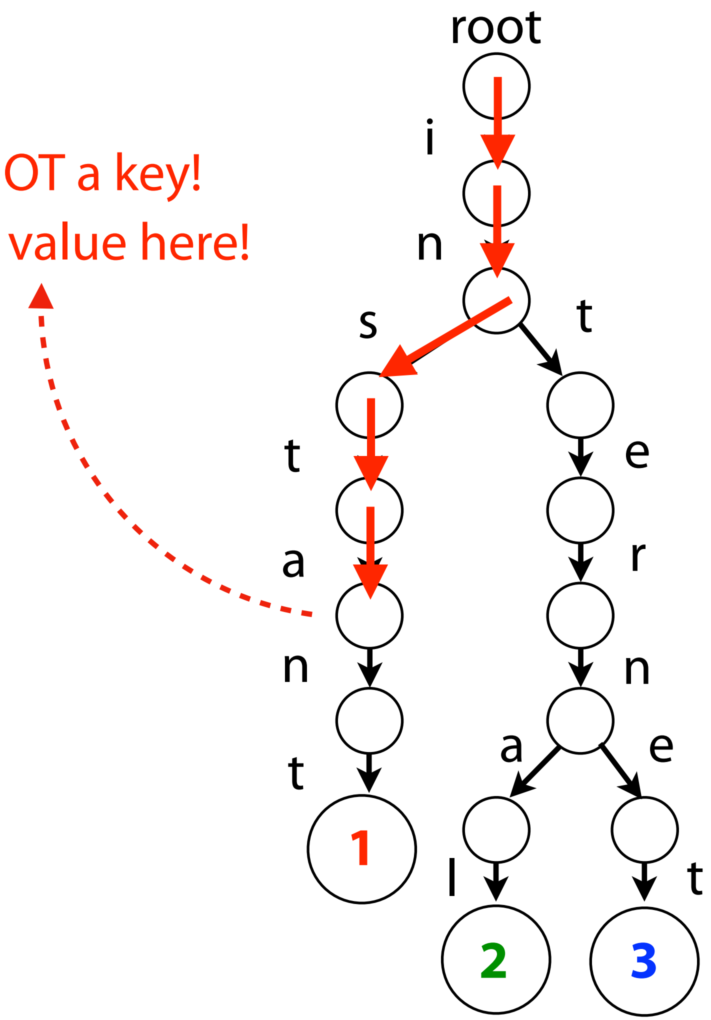
(1) Try to match front character

(2) If match, move to appropriate child

(2.5) Set pattern equal to remainder

(2.5) Go back to (1)

(3) If mismatch, *P* is not a key!



"Insta" is NOT a key!

There's no value here!

Searching a Trie

Given *P*, search the trie for keys and return values

Pattern: `i n s t a n t`

`i n s t a n t`

Lets break that down using *recursion*:

Starting at root:

(0) If we have no 'front' char, check value

(0.5) If no value, *P* is not a key.

(0.5) If value, *P* is a key, return value(s).

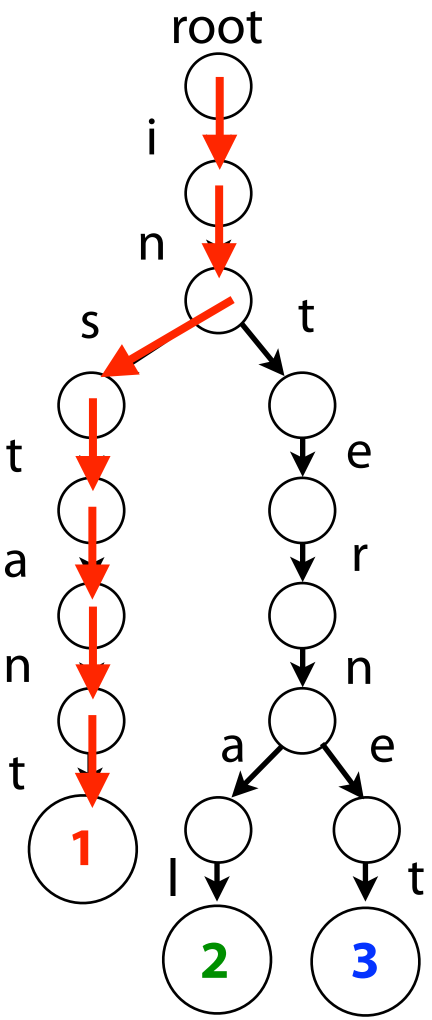
(1) Try to match front character

(2) If match, move to appropriate child

(2.5) Set pattern equal to remainder

(2.5) Go back to (1)

(3) If mismatch, *P* is not a key!



Assignment 5: a_narytree

Learning Objective:

Store all substrings in a trie using NaryTree implementation

Implement exact pattern matching using this trie

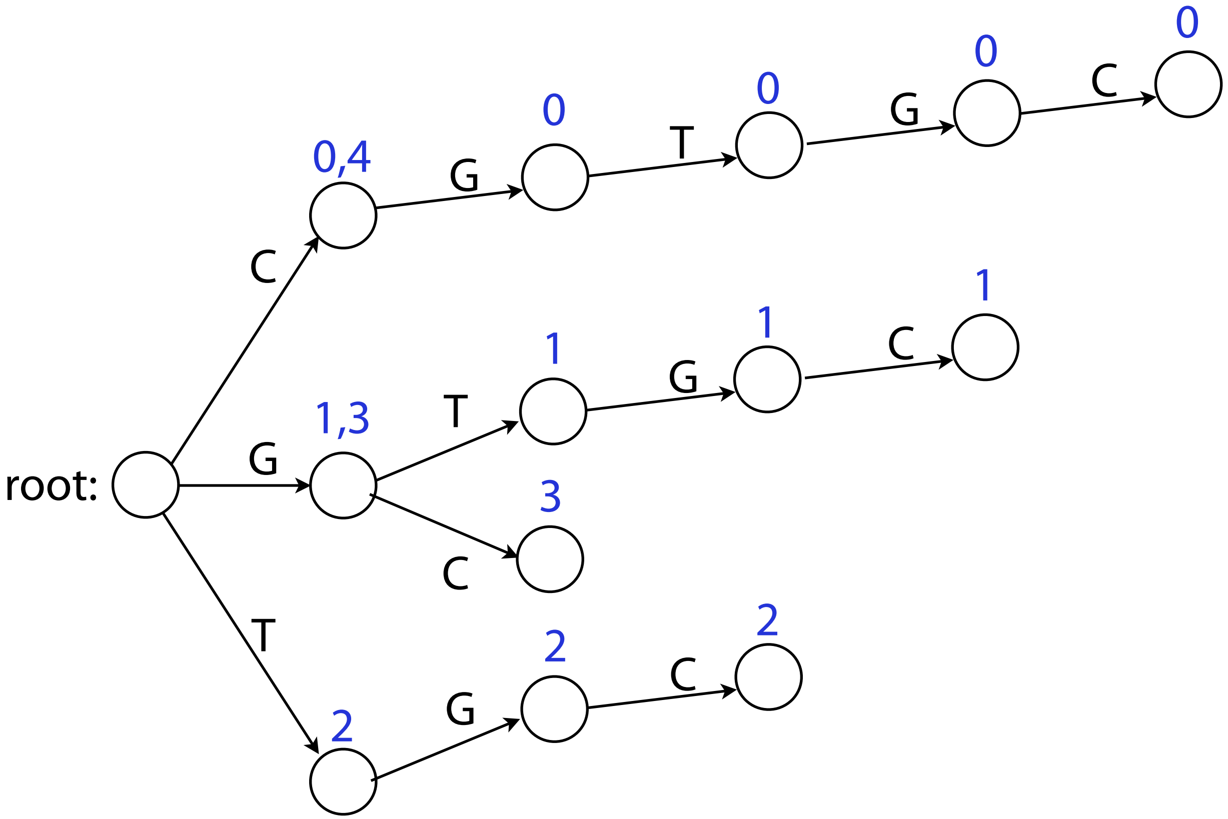
Due: February 28th 11:59 PM

Consider: How could we search the trie if we are only allowed to store one value in each node [instead of a vector of them]?

Preprocessing for exact pattern matching

T: **C G T G C**

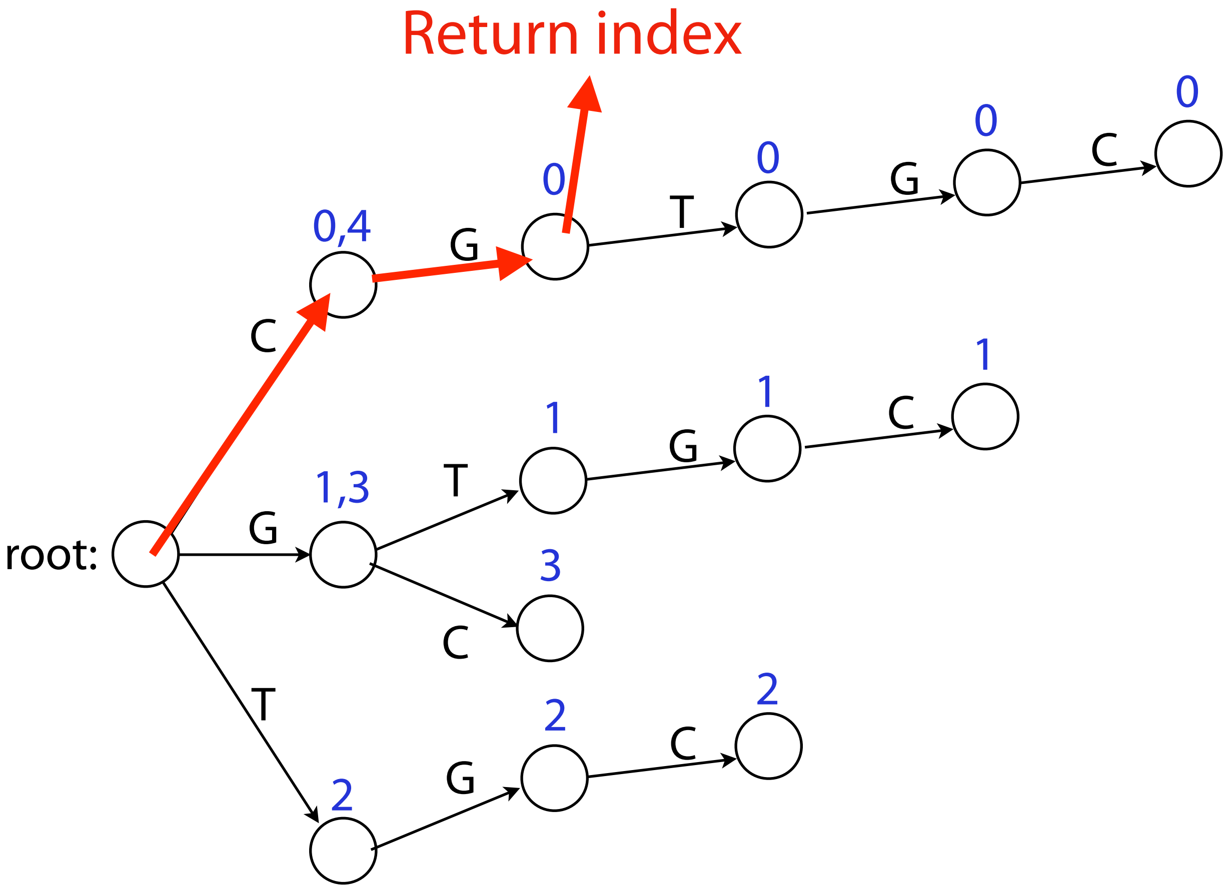
Key	Value
C	0
G	1
T	2
G	3
C	4
CG	0
GT	1
TG	2
…	…



Preprocessing for exact pattern matching

T: **C G T G C**

Key	Value
C	0
G	1
T	2
G	3
C	4
CG	0
GT	1
TG	2
…	…

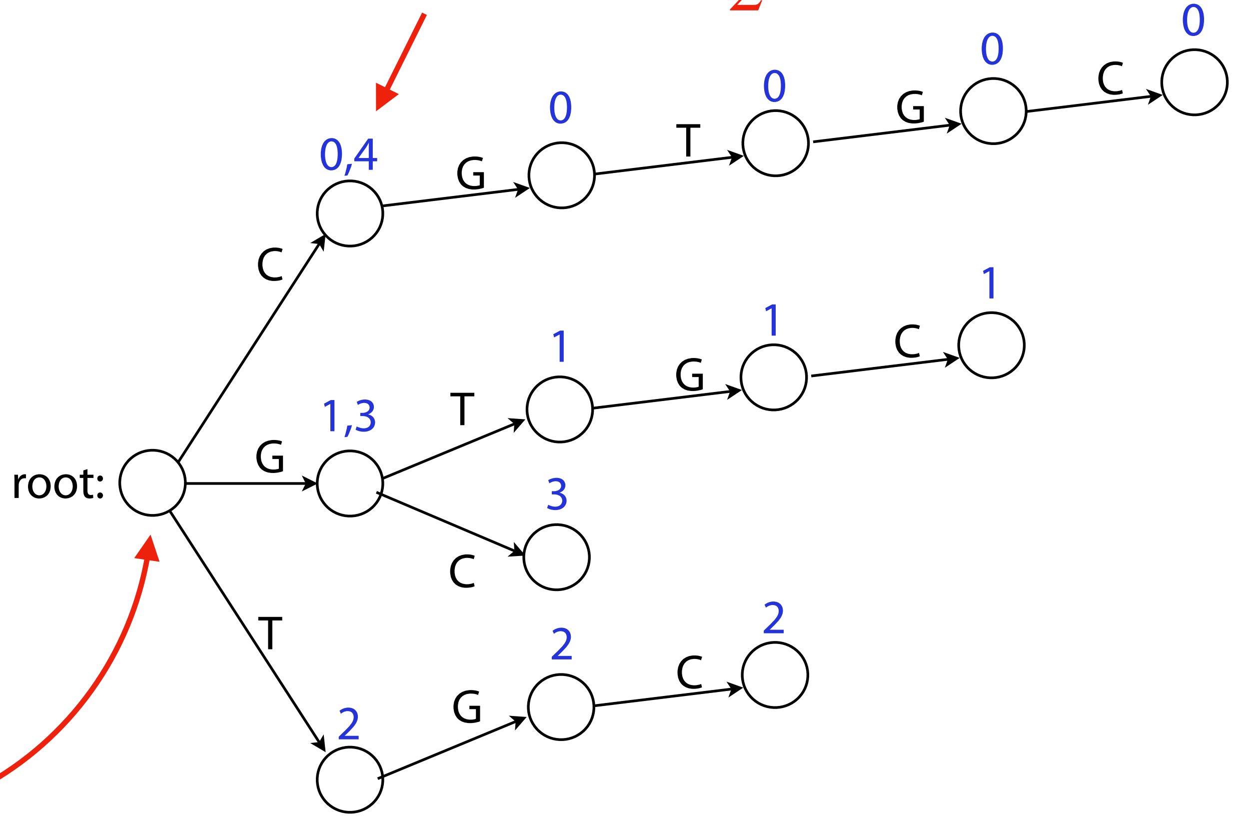


We can do exact pattern matching in $O(P)$ time!

Preprocessing for exact pattern matching

T: **C G T G C**

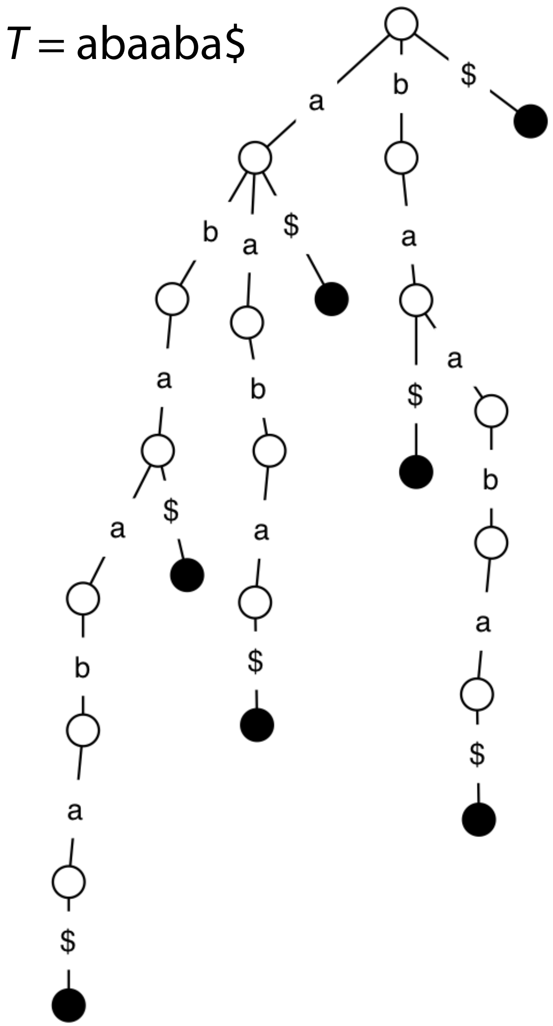
Key	Value
C	0
G	1
T	2
G	3
C	4
CG	0
GT	1
TG	2
…	…



We are storing $\frac{|T|(|T|+1)}{2}$ values

We had to do $\frac{|T|(|T|+1)}{2}$ insertions

Preprocessing for exact pattern matching

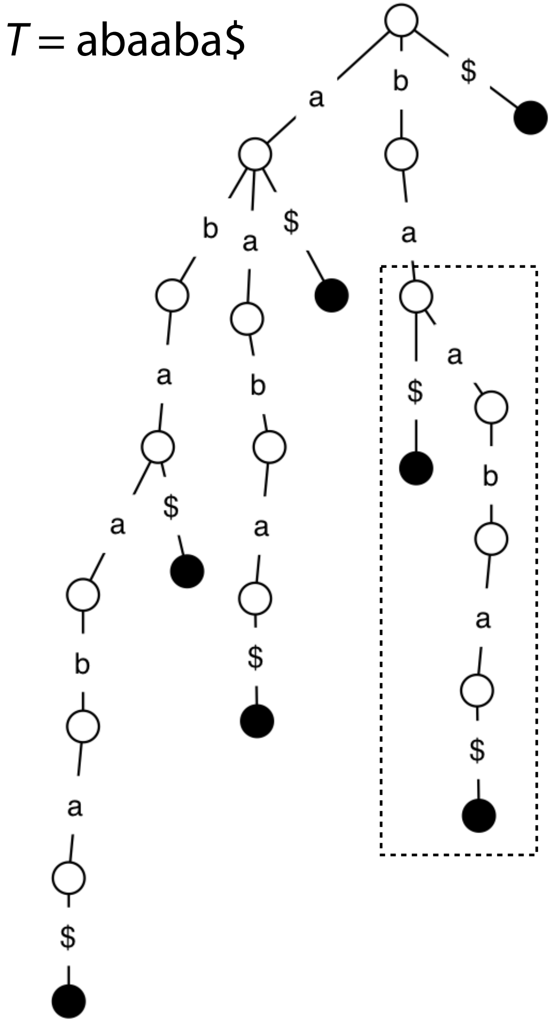


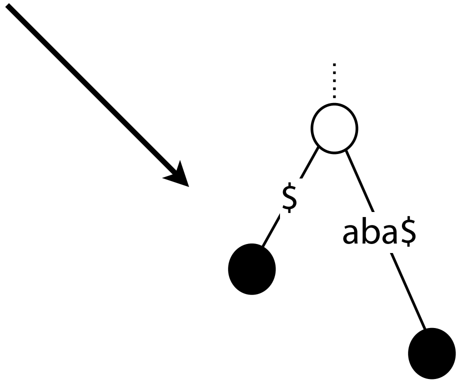
If only there was a way…

to insert fewer strings

to store fewer values

Preprocessing for exact pattern matching





If only there was a way…

to insert fewer strings

to store fewer values

to be even more efficient!