March 11 – BTrees
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B-Tree Motivation

Big-O assumes uniform time for all operations, but this isn’t always true.

However, seeking data from disk may take 40ms+.
...an O(lg(n)) AVL tree no longer looks great:
BTree (of order m)

-3 8 23 25 31 42 43 55

Goal: Minimize the number of reads!
Build a tree that uses __________________________ / node
[1 network packet]
[1 disk block]
BTree Insertion

A **BTree**s of order \( m \) is an \( m \)-way tree:
- All keys within a node are ordered
- All nodes hold no more than \( m-1 \) keys.

\[ m=5 \]
BTree Insertion

When a BTree node reaches m keys:

\[ m = 5 \]
BTree Recursive Insert

```
-3  8  25  31  43  55
```

```
23  42

m=3
```
BTree Recursive Insert

m=3

-3 8
25 31
43 55

23 42
BTree Visualization/Tool

https://www.cs.usfca.edu/~galles/visualization/BTree.html
Btree Properties

A **BTrees** of order **m** is an m-way tree:
- All keys within a node are ordered
- All leaves contain hold no more than **m-1** keys.

- All internal nodes have exactly one more child than key
- Root nodes can be a leaf or have \([2, m]\) children.
- All non-root, internal nodes have \([\lceil \text{m}/2 \rceil, m]\) children.

- All leaves are on the same level
BTree
BTree Search
bool Btree::_exists(BTreeNode & node, const K & key) {
    unsigned i;
    for ( i = 0; i < node.keys_ct_ && key < node.keys_[i]; i++ ) { }
    if ( i < node.keys_ct_ && key == node.keys_[i] ) {
        return true;
    }
    if ( node.isLeaf() ) {
        return false;
    } else {
        BTreeNode nextChild = node._fetchChild(i);
        return _exists(nextChild, key);
    }
}
BTree Analysis

The height of the BTree determines maximum number of ______________ possible in search data.

...and the height of the structure is: ________________.

Therefore: The number of seeks is no more than ____________.

...suppose we want to prove this!
BTree Analysis

In our AVL Analysis, we saw finding an upper bound on the height (given $n$) is the same as finding a lower bound on the nodes (given $h$).

We want to find a relationship for BTrees between the number of keys ($n$) and the height ($h$).
BTree Analysis

Strategy:
We will first count the number of nodes, level by level.

Then, we will add the minimum number of keys per node (n).

The minimum number of nodes will tell us the largest possible height (h), allowing us to find an upper-bound on height.
BTree Analysis

The minimum number of nodes for a BTree of order m at each level:

root:

level 1:

level 2:

level 3:

...

level h:
BTree Analysis

The total number of nodes is the sum of all of the levels:
BTree Analysis

The **total number of keys**: 
BTree Analysis

The **smallest total number of keys** is:

So an inequality about \( n \), the total number of keys:

Solving for \( h \), since \( h \) is the number of seek operations:
BTree Analysis

Given $m=101$, a tree of height $h=4$ has:

Minimum Keys:

Maximum Keys: