March 4 – AVL Applications
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On Friday, we proved an upper-bound on the height of an AVL tree is $2 \times \lg(n)$ or $O(\lg(n))$:

$N(h) :=$ Minimum # of nodes in an AVL tree of height $h$

$N(h) = 1 + N(h-1) + N(h-2)$

$> N(h-1) + N(h-2)$

$> 2 \times N(h-2)$

$> 2^{h/2}$

**Theorem #1:**

Every AVL tree of height $h$ has at least $2^{h/2}$ nodes.
AVL Runtime Proof

On Friday, we proved an upper-bound on the height of an AVL tree is $2 \times \lg(n)$ or $O(\lg(n))$:

$$\text{# of nodes } (n) \geq N(h) > 2^{h/2}$$
$$n > 2^{h/2}$$
$$\lg(n) > h/2$$
$$2 \times \lg(n) > h$$
$$h < 2 \times \lg(n) \quad , \text{for } h \geq 1$$

Proved: The maximum number of nodes in an AVL tree of height $h$ is less than $2 \times \lg(n)$. 
Summary of Balanced BST

AVL Trees
- Max height: $1.44 \times \log(n)$
- Rotations:
Summary of Balanced BST

AVL Trees
- Max height: $1.44 \times \lg(n)$
- Rotations:
  - Zero rotations on find
  - One rotation on insert
  - $O(h) = O(\lg(n))$ rotations on remove

Red-Black Trees
- Max height: $2 \times \lg(n)$
- Constant number of rotations on insert (max 2), remove (max 3).
Why AVL?
Summary of Balanced BST

Pros:
- Running Time:
  - Improvement Over:

- Great for specific applications:
Summary of Balanced BST

Cons:
- Running Time:

- In-memory Requirement:
Red-Black Trees in C++

C++ provides us a balanced BST as part of the standard library:

```cpp
std::map<K, V> map;
```
Red-Black Trees in C++

V & std::map<K, V>::operator[]( const K & )
Red-Black Trees in C++

\[
V & \text{std::map}<K, V>::\text{operator[]}(\text{const } K &)
\]

\[
\text{std::map}<K, V>::\text{erase}(\text{const } K &)
\]
Red-Black Trees in C++

iterator std::map<K, V>::lower_bound(const K &);
iterator std::map<K, V>::upper_bound(const K &);
CS 225 -- Course Update

Over the next two days, your grades will be uploaded into Compass for our first “grade update” where we will calculate your current course grade for you.

We will discuss the grades for the course as a whole (ex: average, etc) in lecture on Wednesday.
Iterators

Why do we care?

```cpp
DFS dfs(...);
for ( ImageTraversal::Iterator it = dfs.begin(); it != dfs.end(); ++it ) {
    std::cout << (*it) << std::endl;
}
```
Iterators

Why do we care?

```cpp
1  DFS dfs(...);
2  for ( ImageTraversal::Iterator it = dfs.begin(); it != dfs.end(); ++it ) {
3      std::cout << (*it) << std::endl;
4  }
```

```cpp
1  DFS dfs(...);
2  for ( const Point & p : dfs ) {
3      std::cout << p << std::endl;
4  }
```
Iterators

Why do we care?

```
1  DFS dfs(...);
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```
1  DFS dfs(...);
2  for ( const Point & p : dfs ) {
3      std::cout << p << std::endl;
4  }
```

```
1  ImageTraversal & traversal = /* ... */;
2  for ( const Point & p : traversal ) {
3      std::cout << p << std::endl;
4  }
```
Iterators

```cpp
ImageTraversal *traversal = /* ... */;
for ( const Point & p : traversal ) {
    std::cout << p << std::endl;
}
```
## Every Data Structure So Far

<table>
<thead>
<tr>
<th></th>
<th>Unsorted Array</th>
<th>Sorted Array</th>
<th>Unsorted List</th>
<th>Sorted List</th>
<th>Binary Tree</th>
<th>BST</th>
<th>AVL</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Find</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td><strong>Insert</strong></td>
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<tr>
<td><strong>Remove</strong></td>
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</tr>
<tr>
<td><strong>Traverse</strong></td>
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</tr>
</tbody>
</table>
Range-based Searches

**Q:** Consider points in 1D: \( p = \{p_1, p_2, ..., p_n\} \).
...what points fall in \([11, 42]\)?

**Tree construction:**
Range-based Searches

Balanced BSTs are useful structures for range-based and nearest-neighbor searches.

Q: Consider points in 1D: \( p = \{p_1, p_2, \ldots, p_n\} \).
...what points fall in [11, 42]?

Ex:

3 6 11 33 41 44 55
Range-based Searches

**Q:** Consider points in 1D: \( p = \{ p_1, p_2, \ldots, p_n \} \).

...what points fall in [11, 42]?

**Ex:**

3  6  11  33  41  44  55
Range-based Searches

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...what points fall in $[11, 42]$?

Tree construction:
Range-based Searches
Range-based Searches
Range-based Searches

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...what points fall in \([11, 42]\)?
Range-based Searches
Running Time