Tree Terminology

• Find an **edge** that is not on the longest **path** in the tree. Give that edge a reasonable name.

• One of the vertices is called the **root** of the tree. Which one?

• Identify the vertices that have a **parent** but no **sibling**.

• How many parents does each vertex have?

• Which vertex has the fewest **children**?

• Which vertex has the most **ancestors**?

• Which vertex has the most **descendants**?

• List all the vertices in **b**’s left **subtree**.

• List all the **leaves** in the tree.
Binary Tree – Defined

A binary tree $T$ is either:

•

OR

•

$\begin{align*}
C &\quad S &\quad X \\
&\quad A &\quad 2 &\quad S \\
&\quad 2 &\quad 5
\end{align*}$
Tree Property: height

*height(T):* length of the longest path from the root to a leaf

Given a binary tree $T$:

*$height(T) = \boxed{2}$*
Tree Property: full

A tree $F$ is **full** if and only if:

1. 
2. 
Tree Property: perfect

A **perfect** tree $P$ is defined in terms of the tree’s height.

Let $P_h$ be a perfect tree of height $h$, and:

1. 
2. 

![Diagram of a perfect tree with nodes labeled C, S, X, A, 2, 2, 5]
Tree Property: complete

**Conceptually:** A perfect tree for every level except the last, where the last level is “pushed to the left”.

**Slightly more formal:** For all levels $k$ in $[0, h-1]$, $k$ has $2^k$ nodes. For level $h$, all nodes are “pushed to the left”.
Tree Property: complete

A complete tree $C$ of height $h$, $C_h$:

1. $C_{-1} = \{\}$
2. $C_h$ (where $h > 0$) = \{r, T_L, T_R\} and either:

   - $T_L$ is __________ and $T_R$ is __________

   OR

   - $T_L$ is __________ and $T_R$ is __________
Tree Property: complete

Is every full tree complete?

If every complete tree full?
Open Office Hours
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1. Understand the problem, don’t just give up.
   - “I segfaulted” is not enough. *Where? Any idea why?*
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2. Your topic must be specific to one function, one test case, or one exam question.
   - Helps us know what to focus on before we see you!
   - Helps your peers to ensure all get questions answered!
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3. Get stuck, get help – not the other way around.
- If you immediately re-add yourself, you’re setting yourself up for failure.
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4. Be awesome.
Tree ADT
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**insert**, inserts an element to the tree.

**remove**, removes an element from the tree.

**traverse**,
#pragma once

template <class T>
class BinaryTree {
    public:
        /* ... */
    private:
        
};
Trees aren’t new:
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How many NULLs?

**Theorem:** If there are \( n \) data items in our representation of a binary tree, then there are \( \) NULL pointers.
How many NULLs?

Base Cases:

n = 0:

n = 1:

n = 2:
How many NULLs?

Induction Hypothesis:
How many NULLs?

Consider an arbitrary tree $T$ containing $n$ data elements: