Smart Union Options:
- Union by Height (root := -h - 1)
- Union by Size (root := -n)
- Union by Rank (root := #union ops)

In all smart unions:
...height of UpTree: ____________.

How do we improve this?

Running Time:
- Worst case running time of find(k):
- Worst case running time of union(r1, r2), given roots:
- New function: “Iterated Log”:
  \[ \log^*(n) := \]
- Overall running time:
  - A total of \( m \) union/find operation runs in:

A Review of Major Data Structures so Far
<table>
<thead>
<tr>
<th>Array-based</th>
<th>List/Pointer-based</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Sorted Array</td>
<td>- Singly Linked List</td>
</tr>
<tr>
<td>- Unsorted Array</td>
<td>- Doubly Linked List</td>
</tr>
<tr>
<td>- Stacks</td>
<td>- Trees</td>
</tr>
<tr>
<td>- Queues</td>
<td>- BTree</td>
</tr>
<tr>
<td>- Hashing</td>
<td>- Binary Tree</td>
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<tr>
<td>- Heaps</td>
<td>- Huffman Encoding</td>
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<td>- Priority Queues</td>
<td>- kd-Tree</td>
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<tr>
<td>- UpTrees</td>
<td>- AVL Tree</td>
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<tr>
<td>- Disjoint Sets</td>
<td></td>
</tr>
</tbody>
</table>

An Introduction to Graphs

HAMLET

TROILUS AND CRESSIDA
Motivation:
Graphs are awesome data structures that allow us to represent an enormous range of problems. To study these problems, we need:
1. A common vocabulary to talk about graphs
2. Implementation(s) of a graph
3. Traversals on graphs
4. Algorithms on graphs

Graph Vocabulary
Consider a graph $G$ with vertices $V$ and edges $E$, $G=(V,E)$.

- Incident Edges:
  $$I(v) = \{(x, v) \in E\}$$

- Degree(v):
  $$|I|$$

- Adjacent Vertices:
  $$A(v) = \{x : (x, v) \in E\}$$

- Path($G_2$): Sequence of vertices connected by edges

- Cycle($G_1$): Path with a common begin and end vertex.

- Simple Graph($G$): A graph with no self loops or multi-edges.

- Subgraph($G$): $G' = (V', E')$: $V' \in V$, $E' \in E$, and $(u, v) \in E \Rightarrow u \in V'$, $v \in V'$

CS 225 – Things To Be Doing:
1. Theory Exam 3 is ongoing!
2. lab_heap due Sunday, April 7th
3. MP6 released; Extra Credit +7 deadline April 8th
4. Daily POTDs are ongoing!