Feb. 23 – BST Remove
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Interactive Lecture Questions

- Ask Questions: Ask in-lecture questions using this Google Form! Questions are reviewed and answered live during lecture.
- Detailed Answers After Lecture: If we didn’t get to answer your question in lecture, we provide detailed answers to common questions here.
- You must be logged in with an active U. Illinois email address to be asked to log in.

Lecture Videos

- Recorded on echo360.org, log in to view.

Schedule

<table>
<thead>
<tr>
<th>Monday</th>
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<tbody>
<tr>
<td>January 15</td>
</tr>
<tr>
<td>MLK Day</td>
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<tr>
<td>January 22</td>
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<tr>
<td>Memory</td>
</tr>
</tbody>
</table>

Slides | Handout | Pointers.pdf | Code | TA Notes |...
template<typename K, typename V>
void BST::_insert(TreeNode *& root, K & key, V & value) {
    TreeNode *t = _find(root, key);
    t = new TreeNode(key, value);
}
template<typename K, typename V>
void BST::_insert(TreeNode * & root, K & key, V & value) {
    TreeNode * t = _find(root, key);
    t = new TreeNode(key, value);
}

root_
HAPPY Valentine's Day
Tonight's the night when we forget about the deadlines, it's time, uh oh
I don't know about you, but
I'm FAILING 225

segmentation fault
segmentation fault
segmentation fault
segmentation fault
template<typename K, typename V>

TreeNode* _remove(TreeNode* & root, const K & key) {
}
remove(40);
remove(25);
remove(10);
remove(13);
## BST Analysis – Running Time

<table>
<thead>
<tr>
<th>Operation</th>
<th>BST Worst Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>find</td>
<td></td>
</tr>
<tr>
<td>insert</td>
<td></td>
</tr>
<tr>
<td>delete</td>
<td></td>
</tr>
<tr>
<td>traverse</td>
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BST Analysis

Every operation that we have studied on a BST depends on the height of the tree: \( O(h) \).

...what is this in terms of \( n \), the amount of data?

We need a relationship between \( h \) and \( n \):

\[
 f(n) \leq h \leq g(n)
\]
Q: What is the maximum number of nodes in a tree of height $h$?
BST Analysis

**Q:** What is the minimum number of nodes in a tree of height $h$?

What is the maximum height for a tree of $n$ nodes?
BST Analysis

Therefore, for all BST:

Lower bound:

Upper bound:
The height of a BST depends on the order in which the data is inserted into it.

ex: 1 3 2 4 5 7 6  vs. 4 2 3 6 7 1 5

Q: How many different ways are there to insert keys into a BST?

Q: What is the average height of all the arrangements?
BST Analysis

Q: How many different ways are there to insert keys into a BST?

Q: What is the average height of all the arrangements?
# BST Analysis – Running Time

<table>
<thead>
<tr>
<th>Operation</th>
<th>BST Average case</th>
<th>BST Worst case</th>
<th>Sorted array</th>
<th>Sorted List</th>
</tr>
</thead>
<tbody>
<tr>
<td>find</td>
<td></td>
<td></td>
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<tr>
<td>insert</td>
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</table>
Height-Balanced Tree

What tree makes you happier?

Height balance: $b = \text{height}(T_L) - \text{height}(T_R)$

A tree is height balanced if: