Building a Heap with an Array of Data
- Assumption: Data already exists as an unsorted array in memory.
- Goal: Organize the data as a minHeap as fast as possible.

Solutions:
1. Sort the array, O(n lg(n))
2. Use Heap::insert for every element, O(n lg(n))
3. Use a heapifyDown strategy on half the array:

\[
\begin{array}{c}
B \\
U \\
I \\
L \\
D \\
H \\
E \\
A \\
P \\
N \\
O \\
W \\
\end{array}
\]

Theorem: The running time of buildHeap on array of size \(n\) is: ________.

Strategy:

Define \(S(h)\):
Let \(S(h)\) denote the sum of the heights of all nodes in a complete tree of height \(h\).

\[
\begin{align*}
S(0) &= \\
S(1) &= \\
S(2) &= \\
S(h) &= \\
\end{align*}
\]

Proof of \(S(h)\) by Induction:

Finally, finding the running time:

Heap Sort

Algorithm:
1. 
2. 
3. 

Running time?

Why do we care about another sort?
Disjoint Sets
Let $R$ be an equivalence relation on $us$ where $(s, t) \in R$ if $s$ and $t$ have the same favorite among:

\{
\text{___, ___, ___, ___, ___, ___}
\}

Examples:

\begin{center}
\begin{tabular}{ccc}
\text{2} & \text{5} & \text{9} \\
\text{0} & \text{1} & \text{4} & \text{8} \\
\end{tabular}
\end{center}

Building Disjoint Sets:
- Maintain a collection $S = \{s_0, s_1, \ldots, s_k\}$
- Each set has a representative member.
- ADT:
  \begin{itemize}
  \item void makeSet(const T & t);
  \item void union(const T & k1, const T & k2);
  \item T & find(const T & k);
  \end{itemize}

\begin{center}
\begin{tabular}{ccc}
\text{Operation: } & \text{find(k)} \\
\text{Operation: } & \text{union(k1, k2)}
\end{tabular}
\end{center}

Implementation #2:
- Continue to use an array where the index is the key
- The value of the array is:
  \begin{itemize}
  \item -1, if we have found the representative element
  \item The index of the parent, if we haven’t found the rep. element
  \end{itemize}

Example:

\begin{center}
\begin{tabular}{cccc}
\text{4} & \text{8} & \text{5} & \text{6} \\
\text{-1} & \text{-1} & \text{-1} & \text{-1} \\
\text{4} & \text{5} & \text{8} & \text{5} \\
\end{tabular}
\end{center}

...where is the error in this table?

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CS 225 – Things To Be Doing:
1. MP5 deadline tonight Monday, April 2\textsuperscript{nd}
2. Theory Exam 3 starts tomorrow (Tuesday, April 3\textsuperscript{rd})
3. lab_heap starts on Wednesday
4. Daily POTDs are ongoing!