Data Structures

Graph Traversals and MST

CS 225

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Announcements

Project teams be sure to schedule your mid-project checkin soon!

This week's lab is extra credit lab

Exam 5 is happening now!

ICES Evaluations are open! If enough students submit, extra credit!

Learning Objectives

Review graph traversal algorithms

Introduce the minimum spanning tree (with weights)

Discuss Kruskal's MST Algorithm

$|V| = n, |E| = m$

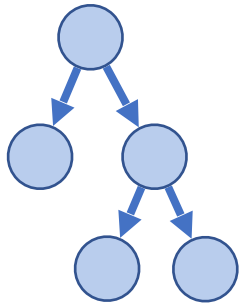
Expressed as O(f)	Edge List	Adjacency Matrix	Adjacency List
Space	n+m	n^2	n+m
insertVertex(v)	1*	n*	1*
removeVertex(v)	m**	n	deg(v)
insertEdge(u, v)	1	1	1*
removeEdge(u, v)	m	1	min(deg(u), deg(v))
incidentEdges(v)	m	n	deg(v)
areAdjacent(u, v)	m	1	min(deg(u), deg(v))

Graph Traversals

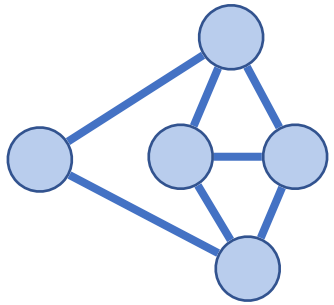
Objective: Visit every vertex and every edge in the graph.

How can we systematically go through a complex graph in the fewest steps?

Tree traversals won't work — lets compare:



- Rooted
- Acyclic
- A clear 'endpoint'



- No root (any start position valid)
- Cycles
- No obvious 'endpoint'

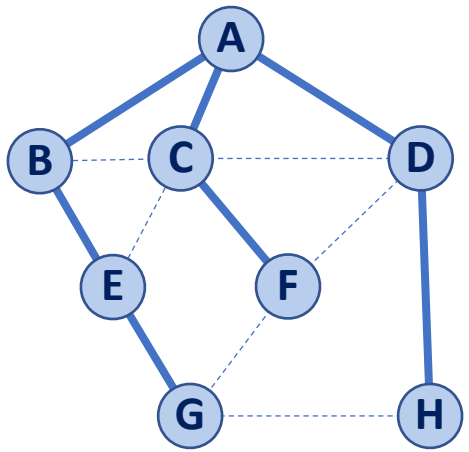
Input: Graph, G

Output: A labeling of the edges in G as discovery or cross

```
 1 BFS(G):
 2   foreach (Vertex v : G.vertices()):
 3    setPred(v, NULL)
 4    setDist(v, -1)
 5
 6   foreach (Edge e : G.edges()):
 7     setLabel(e, UNEXPLORED)
 8
 9   foreach (Vertex v : G.vertices()):
10     if getDist(v) == -1:
11        BFS(G, v)
```

```
12 BFS(G, v):
13   Queue q
14   setDist(v, 0)
15   q.enqueue(v)
16
17   while !q.empty():
18     v = q.dequeue()
19
20     foreach (Vertex w : G.adjacent(v)):
21        if( getDist(w) == -1):
22           setLabel((v, w), DISCOVERY)
23           setPred(w, v)
24           setDist(w, v + 1)
25           q.enqueue(w)
26        else:
27           setLabel((v, w), CROSS)
```

v	d	P	Adjacent Edges
A	0	-	B C D
B	1	A	A C E
C	1	A	A B D E F
D	1	A	A C F H
E	2	B	B C G
F	2	C	C D G
G	3	E	E F H
H	2	D	D G



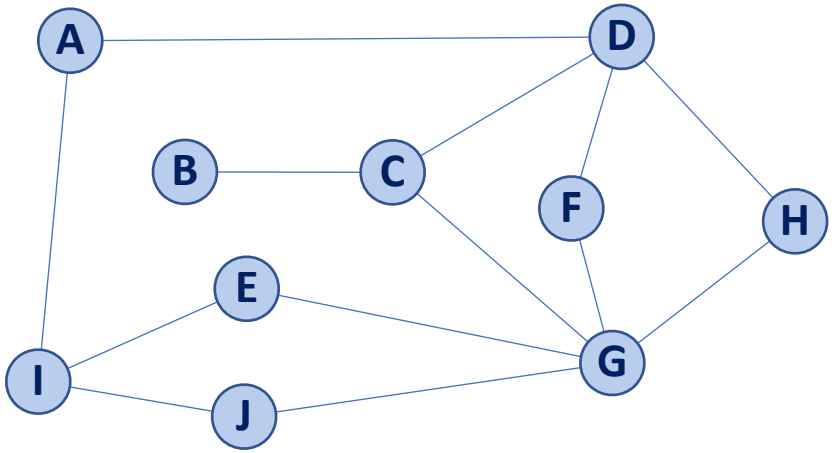
BFS Observations

1. BFS can be used to count components

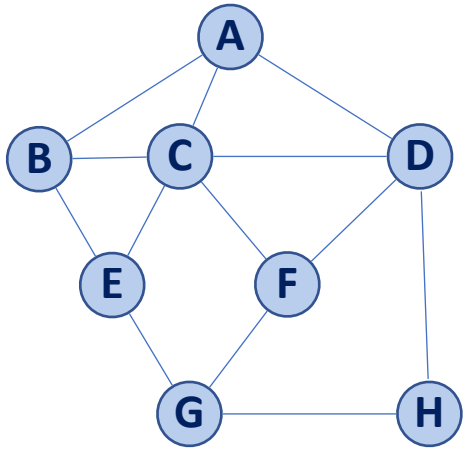
2. BFS can be used to detect cycles

3. The BFS ‘distance’ value is always the shortest distance from source to any vertex (and the discovery edges form a MST)

Traversal: DFS

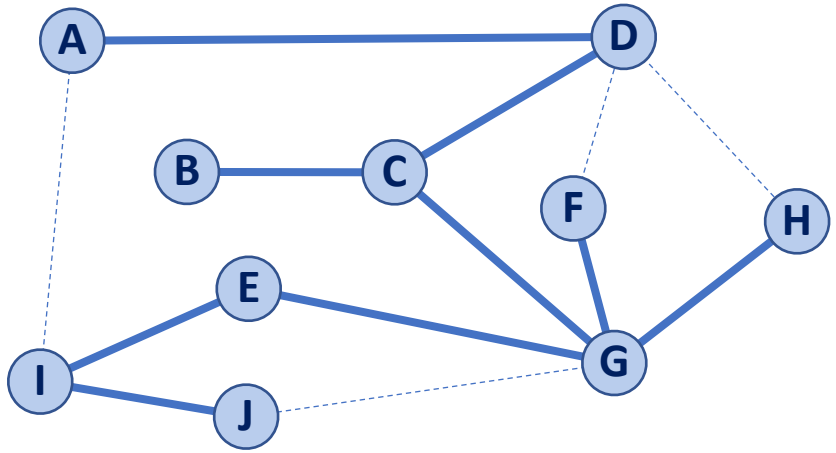


```
1  DFS(G):
2    foreach (Vertex v : G.vertices()):
3      setPred(v, NULL)
4      setDist(v, -1)
5
6    foreach (Edge e : G.edges()):
7      setLabel(e, UNEXPLORED)
8
9    foreach (Vertex v : G.vertices()):
10     if getDist(v) == -1:
11       DFS(G, v)
```




```
12 DFS(G, v):
13
14     foreach (Vertex w : G.adjacent(v)):
15         if( getDist(w) == -1):
16             setLabel((v, w), DISCOVERY)
17             setPred(w, v)
18             setDist(w, v + 1)
19             DFS(G, w)
20         else:
21             setLabel((v, w), BACK)
```

Traversal: DFS



Discovery Edge

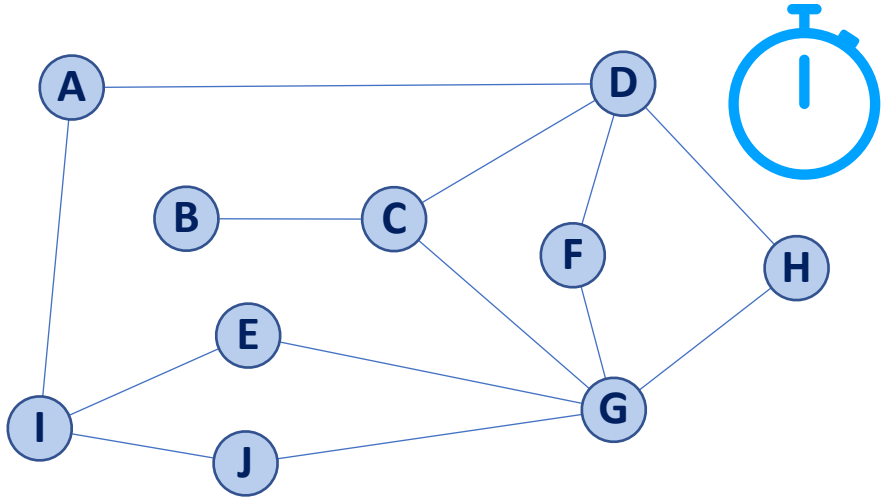
Back Edge

Do we still make a spanning tree?

Does distance have meaning here?

Do our edge labels have meaning here?

Running time of DFS



Labeling:

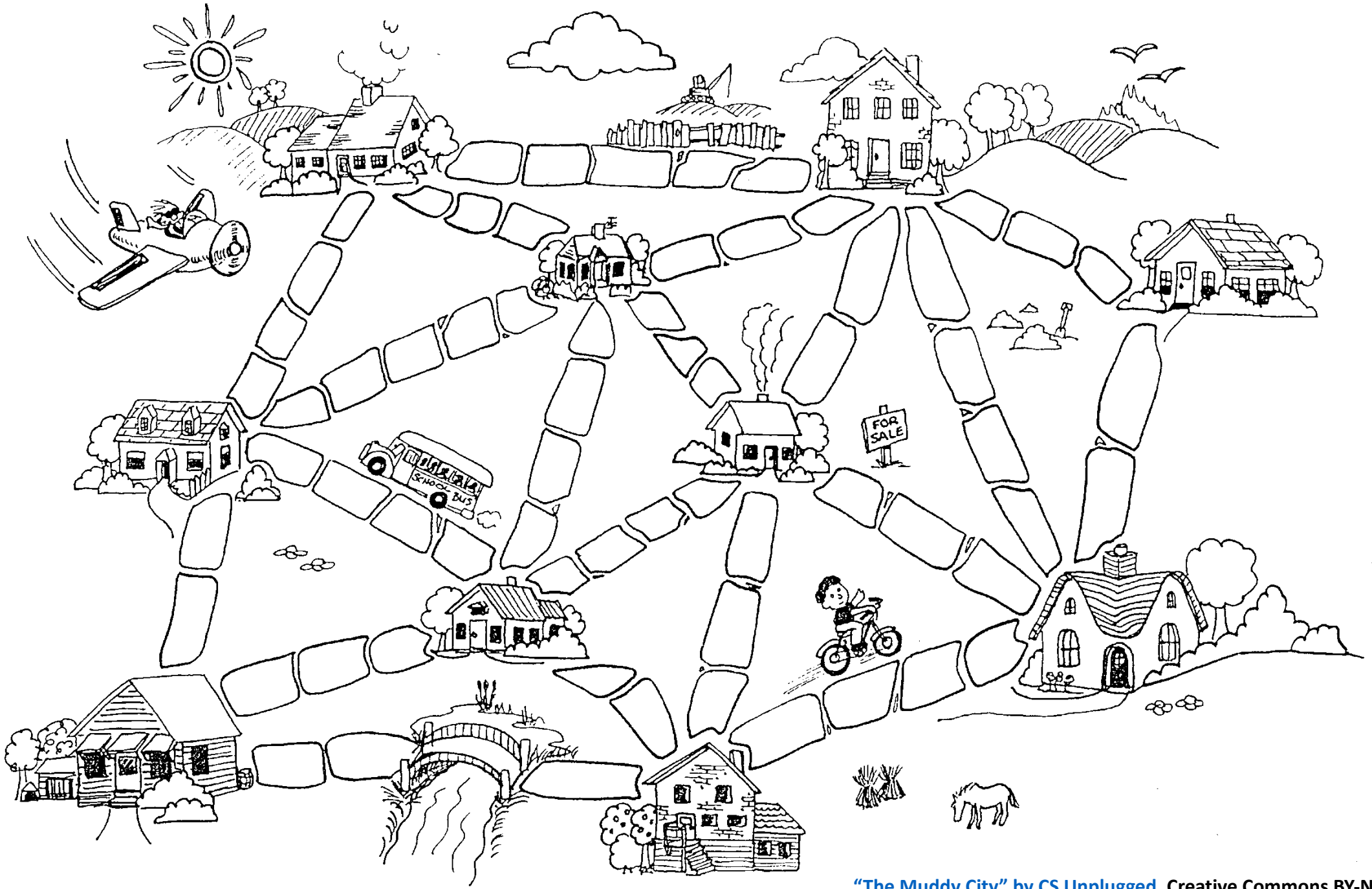
- Vertex:

- Edge:

Traversal:

- Vertex:

- Edge:

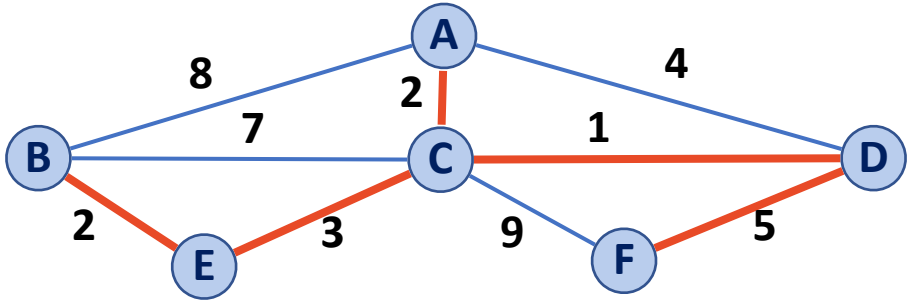


Minimum Spanning Tree Algorithms

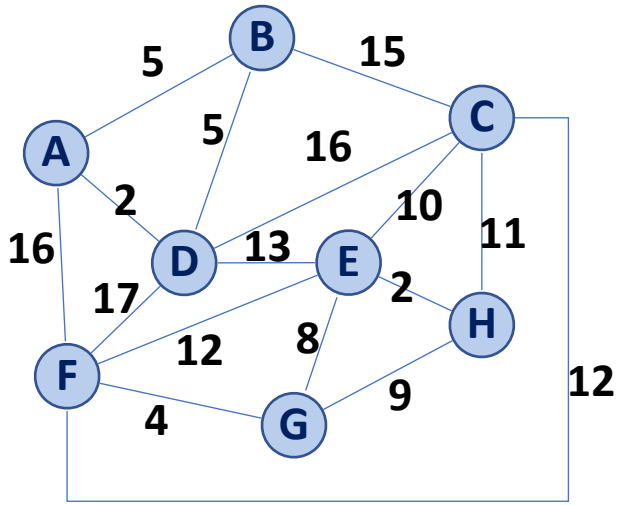
Input: Connected, undirected graph **G** with edge weights (unconstrained, but must be additive)

Output: A graph G’ with the following properties:

- G’ is a spanning graph of G
- G’ is a tree (connected, acyclic)
- G’ has a minimal total weight among all spanning trees

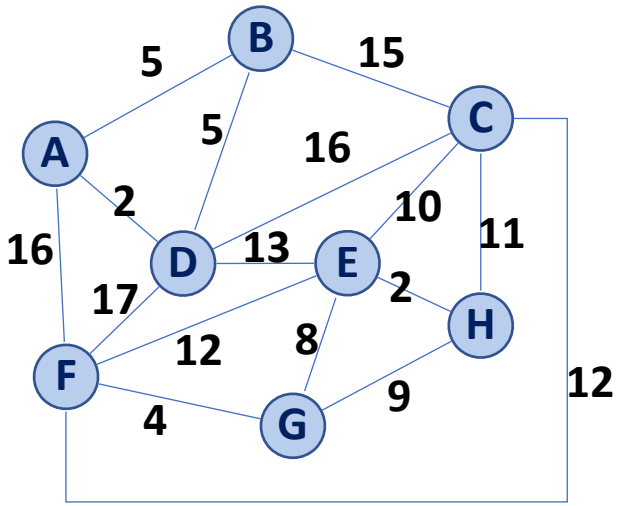


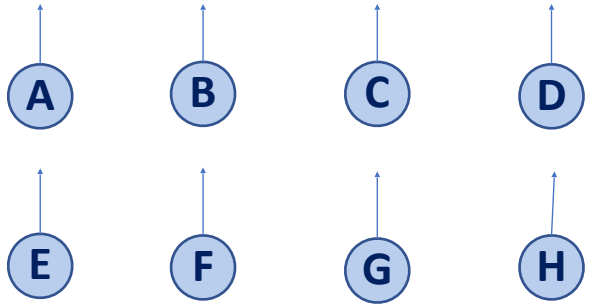
Kruskal's Algorithm



Kruskal's Algorithm

(A, D)
(E, H)
(F, G)
(A, B)
(B, D)
(G, E)
(G, H)
(E, C)
(C, H)
(E, F)
(F, C)
(D, E)
(B, C)
(C, D)
(A, F)
(D, F)



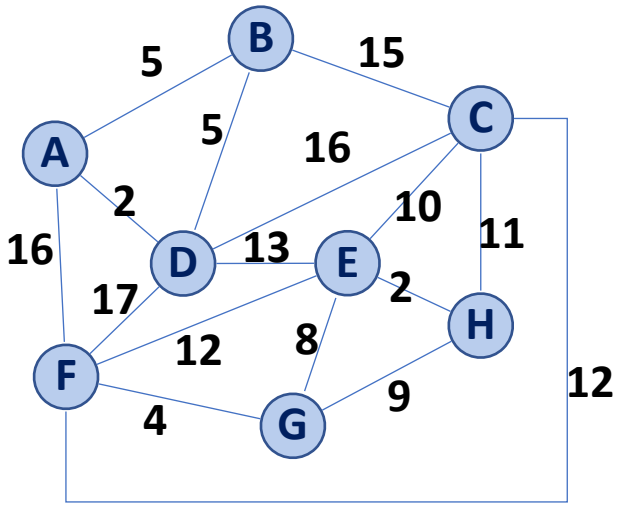


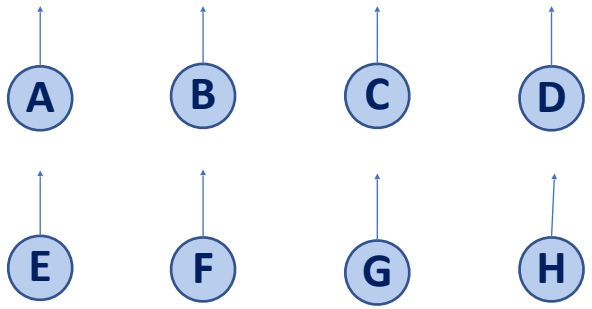
1) Build a **priority queue** on edges

2) Build a **disjoint set** on vertices

Kruskal's Algorithm

(A, D)
(E, H)
(F, G)
(A, B)
(B, D)
(G, E)
(G, H)
(E, C)
(C, H)
(E, F)
(F, C)
(D, E)
(B, C)
(C, D)
(A, F)
(D, F)






```
 1  KruskalMST(G):
 2    DisjointSets forest
 3    foreach (Vertex v : G.vertices()):
 4      forest.makeSet(v)
 5
 6    PriorityQueue Q     // min edge weight
 7    Q.buildFromGraph(G.edges())
 8
 9    Graph T = (V, {})
10
11    while |T.edges()| < n-1:
12      Vertex (u, v) = Q.removeMin()
13      if forest.find(u) != forest.find(v):
14         T.addEdge(u, v)
15         forest.union( forest.find(u),
16                       forest.find(v) )
17
18    return T
19
```

Kruskal's Algorithm

Priority Queue:	Heap	Sorted Array
Building :7		
Each removeMin :12		

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19
```

Kruskal's Algorithm

Priority Queue:	Total Running Time
Heap	
Sorted Array	

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