Data Structures and Algorithms
Hashing 2

CS 225
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Learning Objectives

Review fundamentals of hash tables

Introduce closed hashing approaches to hash collisions

Determine when and how to resize a hash table

Justify when to use different index approaches

A Hash Table based Dictionary

User Code (is a map):

```
Dictionary<KeyType, ValueType> d;
d[k] = v;
```

A **Hash Table** consists of three things:

1. A hash function

2. A data storage structure

3. A method of addressing *hash collisions*

Open vs Closed Hashing

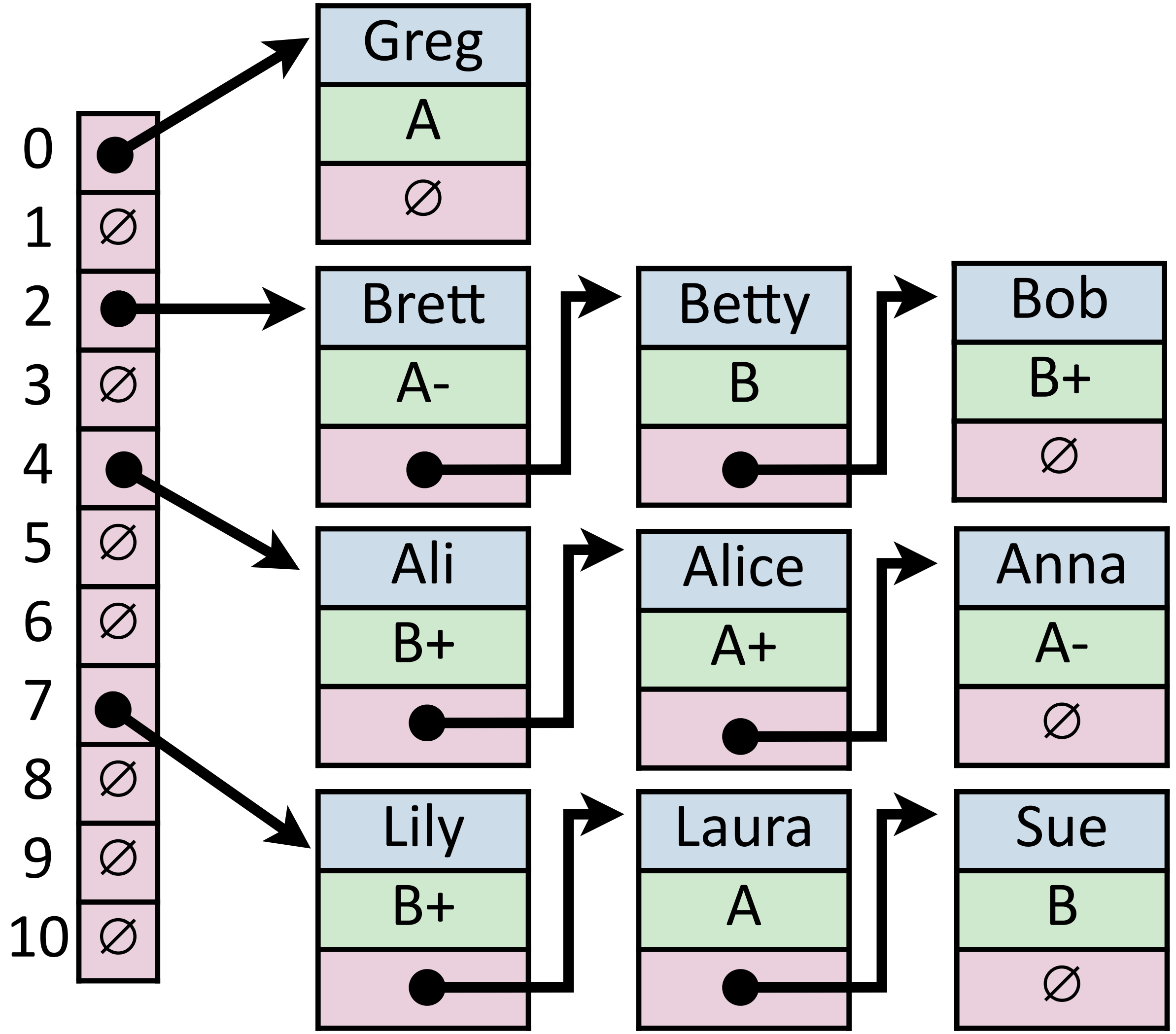
Addressing hash collisions depends on your storage structure.

- **Open Hashing:** store *k,v* pairs externally

- **Closed Hashing:** store *k,v* pairs in the hash table

Hash Table (Separate Chaining)

Key	Value	Hash
Bob	B+	2
Anna	A-	4
Alice	A+	4
Betty	B	2
Brett	A-	2
Greg	A	0
Sue	B	7
Ali	B+	4
Laura	A	7
Lily	B+	7



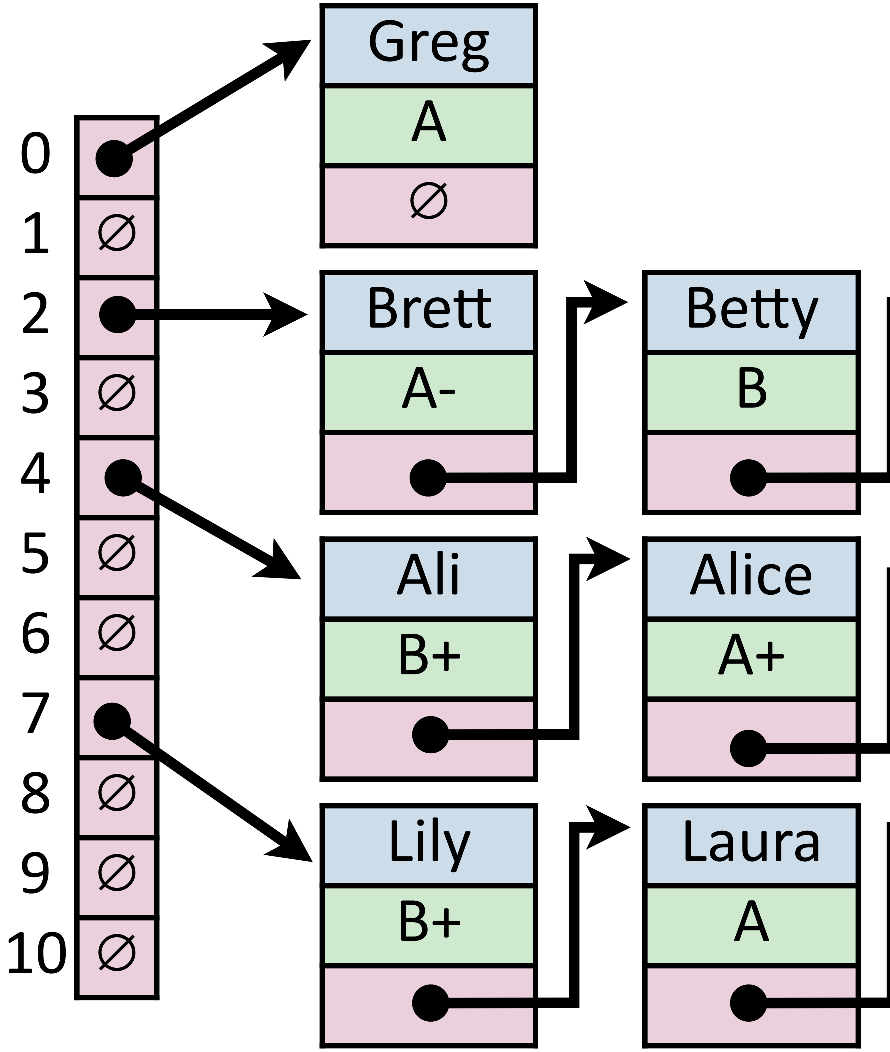
Hash Table (Separate Chaining)

For hash table of size *m* and *n* elements:

Find runs in: ____________________

Insert runs in: ____________________

Remove runs in: ____________________



Hash Table

Worst-Case behavior is bad — but what about randomness?

1) **Fix *h***, our hash, and assume it is good for ***all keys***:

2) Create a ***universal hash function family:***

Simple Uniform Hashing Assumption

Given table of size m, a simple uniform hash, h, implies

$$\forall k_1, k_2 \in U \text{ where } k_1 \neq k_2,\ Pr(h[k_1] = h[k_2]) = \frac{1}{m}$$

Uniform:

Independent:

Separate Chaining Under SUHA

Given table of size m and n inserted objects

Claim: Under SUHA, expected length of chain is $\frac{n}{m}$

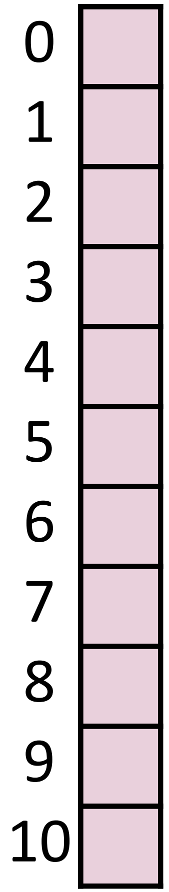
Separate Chaining Under SUHA

Under SUHA, a hash table of size *m* and *n* elements:

Find runs in: ___________.

Insert runs in: ___________.

Remove runs in: ___________.



(Example of closed hashing)

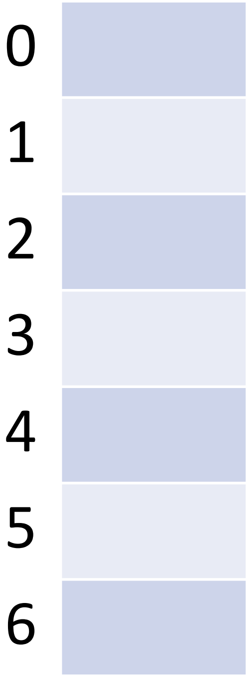
Collision Handling: Probe-based Hashing

S = { 1, 8 , 15}

h(k) = k % 7

|S| = n

|Array| = m



(Example of closed hashing)

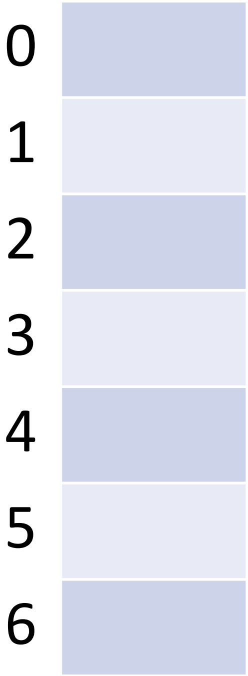
Collision Handling: Linear Probing

S = { 16, 8, 4, 13, 29, 11, 22 }

|S| = n

h(k) = k % 7

|Array| = m



h(k, i) = (k + i) % 7

Try h(k) = (k + 0) % 7, if full…

Try h(k) = (k + 1) % 7, if full…

Try h(k) = (k + 2) % 7, if full…

Try …

(Example of closed hashing)

Collision Handling: Linear Probing

S = { 16, 8, 4, 13, 29, 11, 22 }

|S| = n

h(k, i) = (k + i) % 7

|Array| = m

`_find(29)`

0	22
1	8
2	16
3	29
4	4
5	11
6	13

(Example of closed hashing)

Collision Handling: Linear Probing

S = { 16, 8, 4, 13, 29, 11, 22 }

h(k, i) = (k + i) % 7

|S| = n

|Array| = m

`_remove(16)`

0	22
1	8
2	16
3	29
4	4
5	11
6	13

A Problem w/ Linear Probing

Primary clustering:

0	
1	1_1
2	1_2
3	3_1
4	1_3
5	3_2
6	
7	
8	
9	

Description:

Remedy:

(Example of closed hashing)

Collision Handling: Quadratic Probing

S = { 16, 8, 4, 13, 29, 12, 22 }

|S| = n

h(k) = k % 7

|Array| = m

0	
1	8
2	16
3	
4	4
5	
6	13

h(k, i) = (k + i*i) % 7

Try h(k) = (k + 0) % 7, if full…

Try h(k) = (k + 1*1) % 7, if full…

Try h(k) = (k + 2*2) % 7, if full…

Try …

A Problem w/ Quadratic Probing

Secondary clustering:

0	0_1
1	0_2
2	
3	
4	0_3
5	
6	
7	
8	
9	0_4

Description:

Remedy:

(Example of closed hashing)

Collision Handling: Double Hashing

S = { 16, 8, 4, 13, 29, 11, 22 }

$|S| = n$

$h_1(k) = k \% 7$

$|Array| = m$

$h_2(k) = 5 - (k \% 5)$

$h(k, i) = (h_1(k) + i*h_2(k)) \% 7$

Try $h(k) = (k + 0*h_2(k)) \% 7$, if full...

Try $h(k) = (k + 1*h_2(k)) \% 7$, if full...

Try $h(k) = (k + 2*h_2(k)) \% 7$, if full...

Try ...

0	
1	**8**
2	**16**
3	
4	**4**
5	
6	**13**

Running Times

(Don't memorize these equations, no need.)

(Expectation under SUHA)

Open Hashing:

insert: ___________.

find/ remove: ___________.

Closed Hashing:

insert: ___________.

find/ remove: ___________.

Running Times *(Don't memorize these equations, no need.)*

The expected number of probes for find(key) under SUHA

Linear Probing:

- Successful: **$\frac{1}{2}(1 + 1/(1-\alpha))$**
- Unsuccessful: **$\frac{1}{2}(1 + 1/(1-\alpha))^2$**

Double Hashing:

- Successful: **$1/\alpha * \ln(1/(1-\alpha))$**
- Unsuccessful: **$1/(1-\alpha)$**

Separate Chaining:

- Successful: **$1 + \alpha/2$**
- Unsuccessful: **$1 + \alpha$**

Instead, observe:

- As α increases:

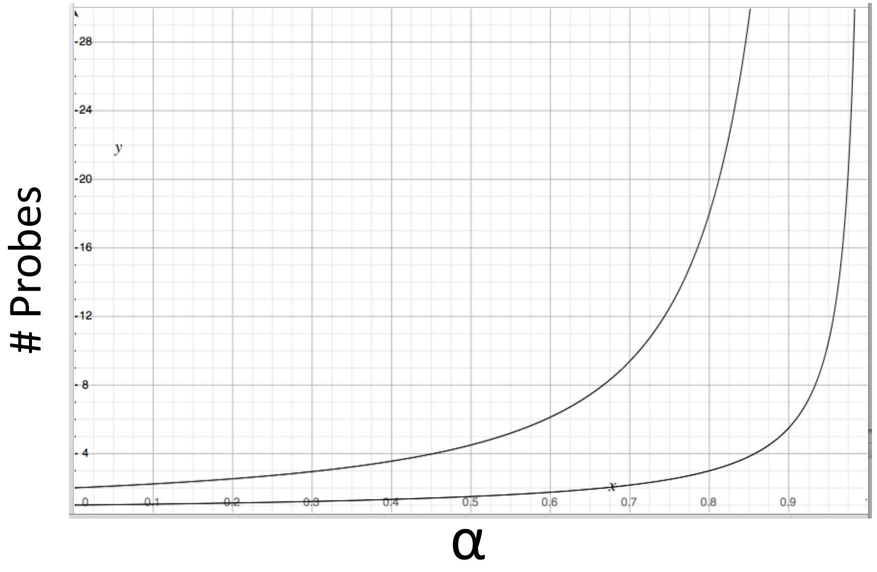
- If α is constant:

Running Times

The expected number of probes for find(key) under SUHA

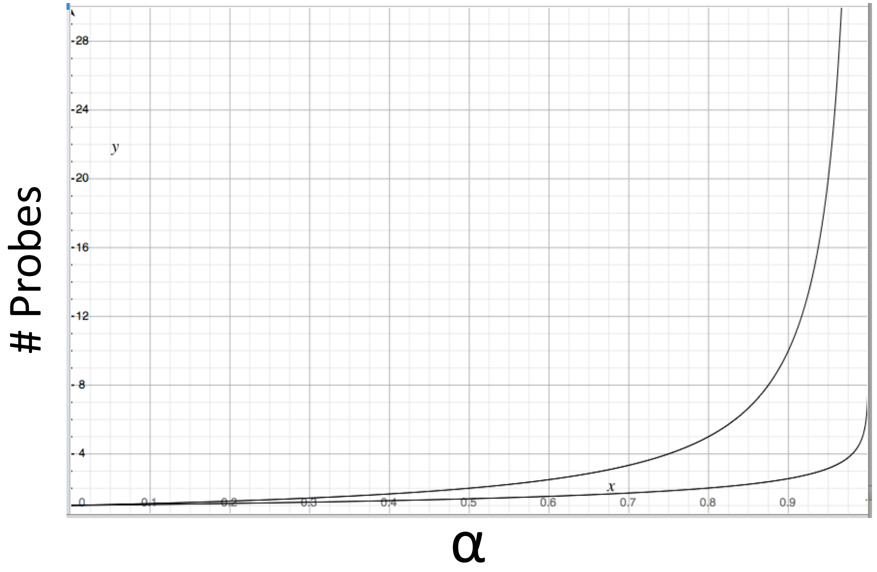
Linear Probing:

- Successful: **½(1 + 1/(1-α))**
- Unsuccessful: **½(1 + 1/(1-α))²**



Double Hashing:

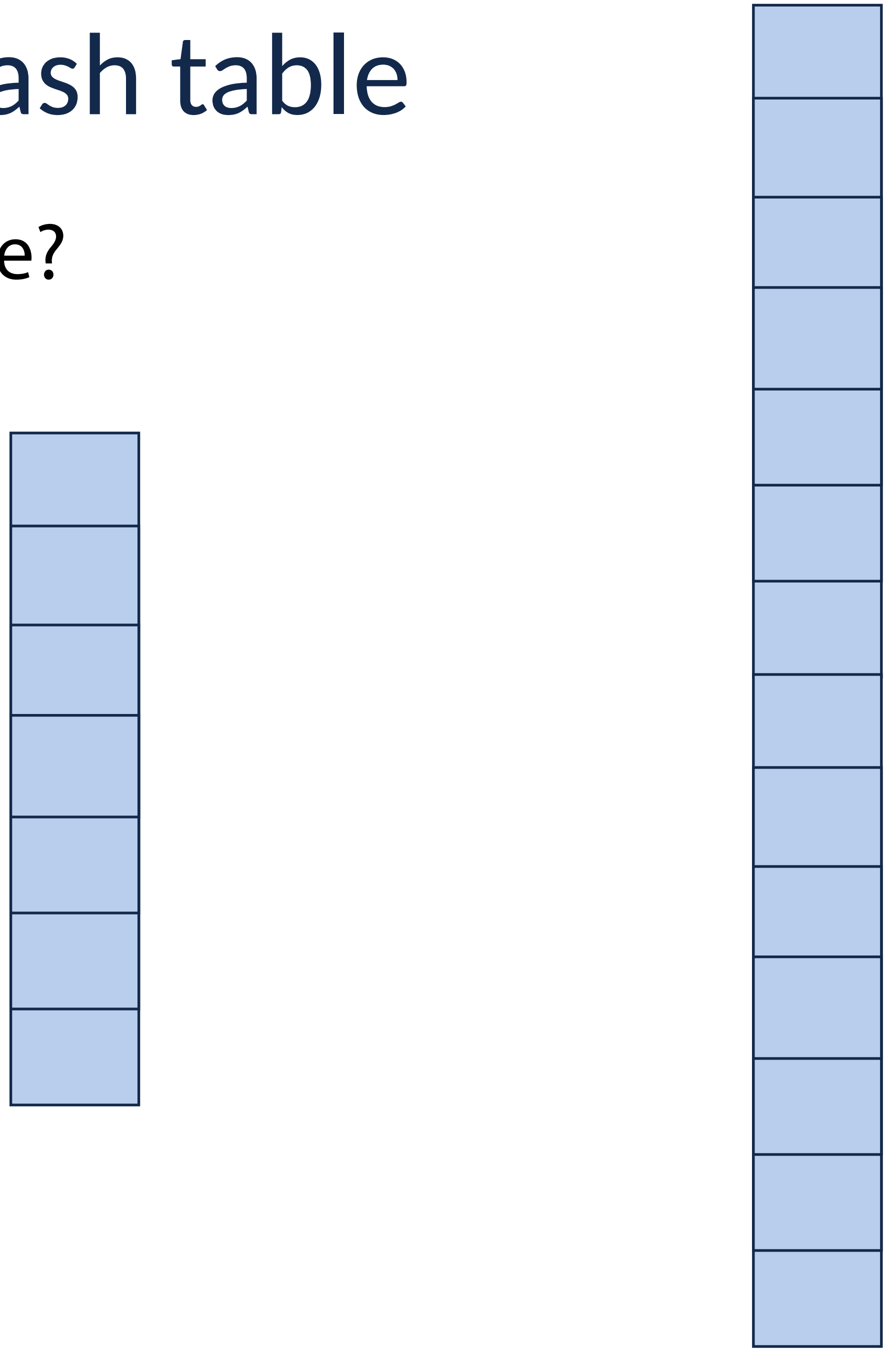
- Successful: **1/α * ln(1/(1-α))**
- Unsuccessful: **1/(1-α)**



When do we resize?

Resizing a hash table

How do you resize?



Which collision resolution strategy is better?

- Big Records:
- Structure Speed:

What structure do hash tables implement?

What constraint exists on hashing that doesn't exist with BSTs?

Why talk about BSTs at all?

Running Times

	Hash Table	AVL	Linked List
Find	Expectation*: Worst Case:		
Insert	Expectation*: Worst Case:		
Storage Space			