

# Data Structures

## Binary Search Trees 2

CS 225

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# Extra Credit Project

→ New tabs!

→ PL

Propose, implement, and benchmark an algorithm or data structure not covered in CS 225

→ Discuss theory

Teams of two

Extra credit only if you complete the project.

← \*  
😊

Four key parts:

1. Proposal

S

2. Development Logs

S

3. Mid-project Checkin

S

4. Final Submission

AS

Do this or zero

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# Learning Objectives

Review binary search trees

Continue implementing BST ADT

Discuss pros and cons of BST (and possible improvements)

BT → BST → ????

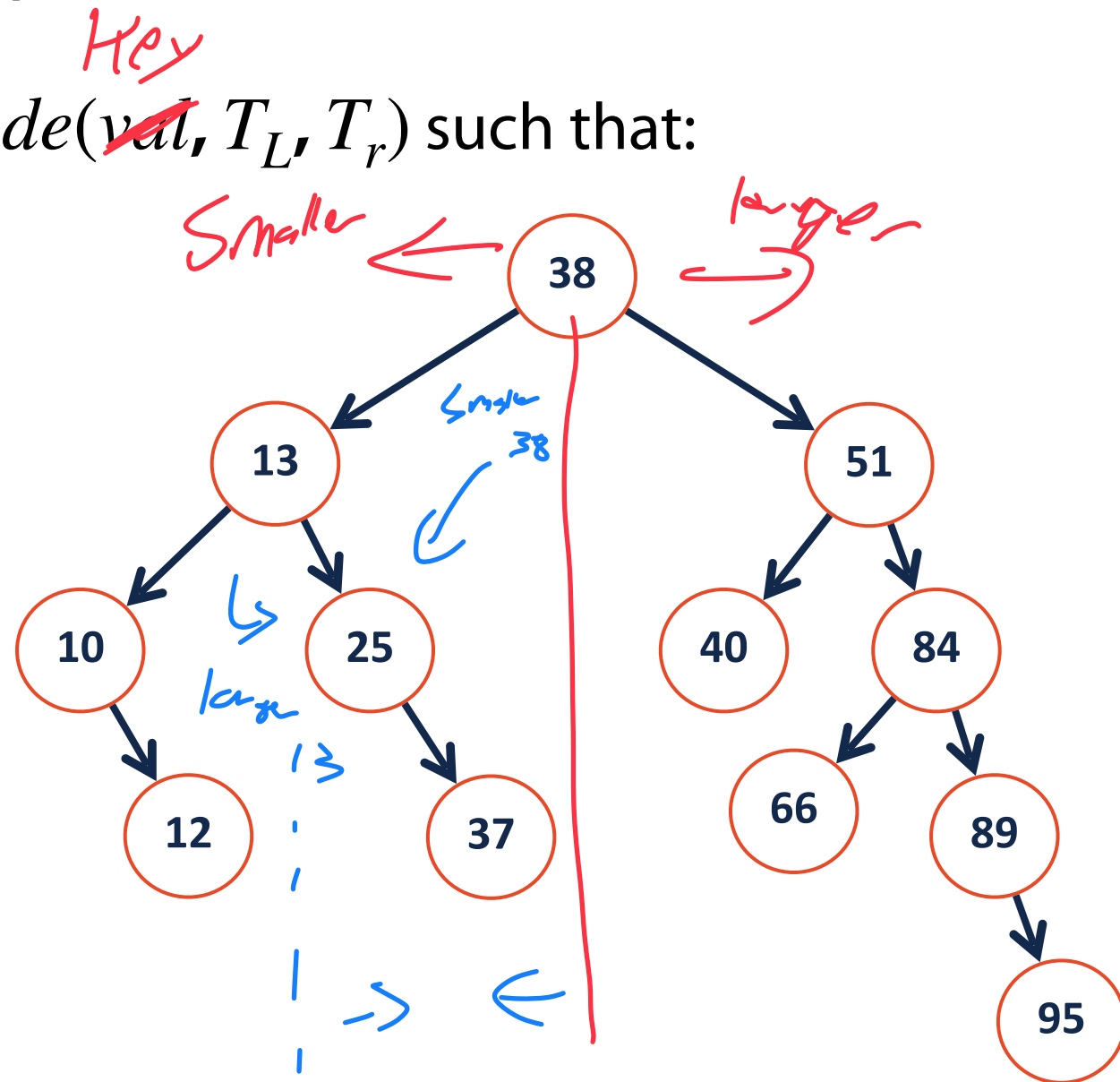
# Binary Search Tree (BST)

Flatt()

A **BST** is a binary tree  $T = \text{TreeNode}(\text{val}, T_L, T_R)$  such that:

$\forall n \in T_L, n.\text{val} < T.\text{val}$

$\forall n \in T_R, n.\text{val} > T.\text{val}$



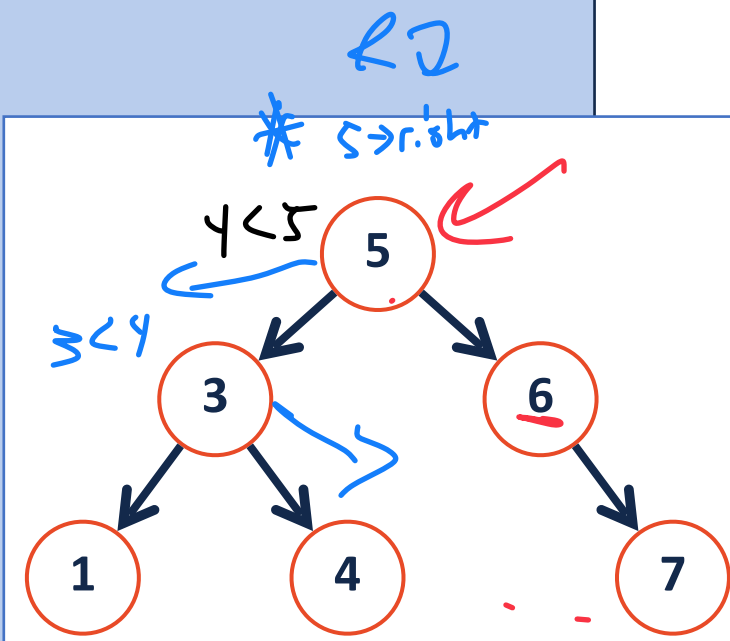
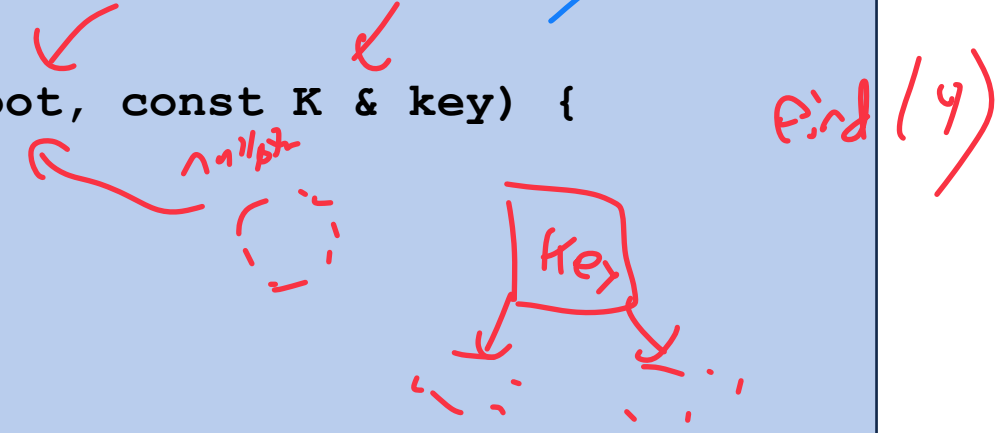
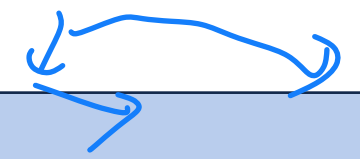
1  
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23

```

template<typename K, typename V>
Tree Node * & __find(TreeNode *& root, const K & key) {
// Base case
1) root == nullptr -> return root;
2) root->key == key -> return root;
// Recursive Step
Look at root->key
-> ==
-> root->key < key -> go right
-> root->key > key -> go left
return
}

```

~~us~~ us \*\*\*  
"us" (with arrow pointing to 'us')



# BST Insert

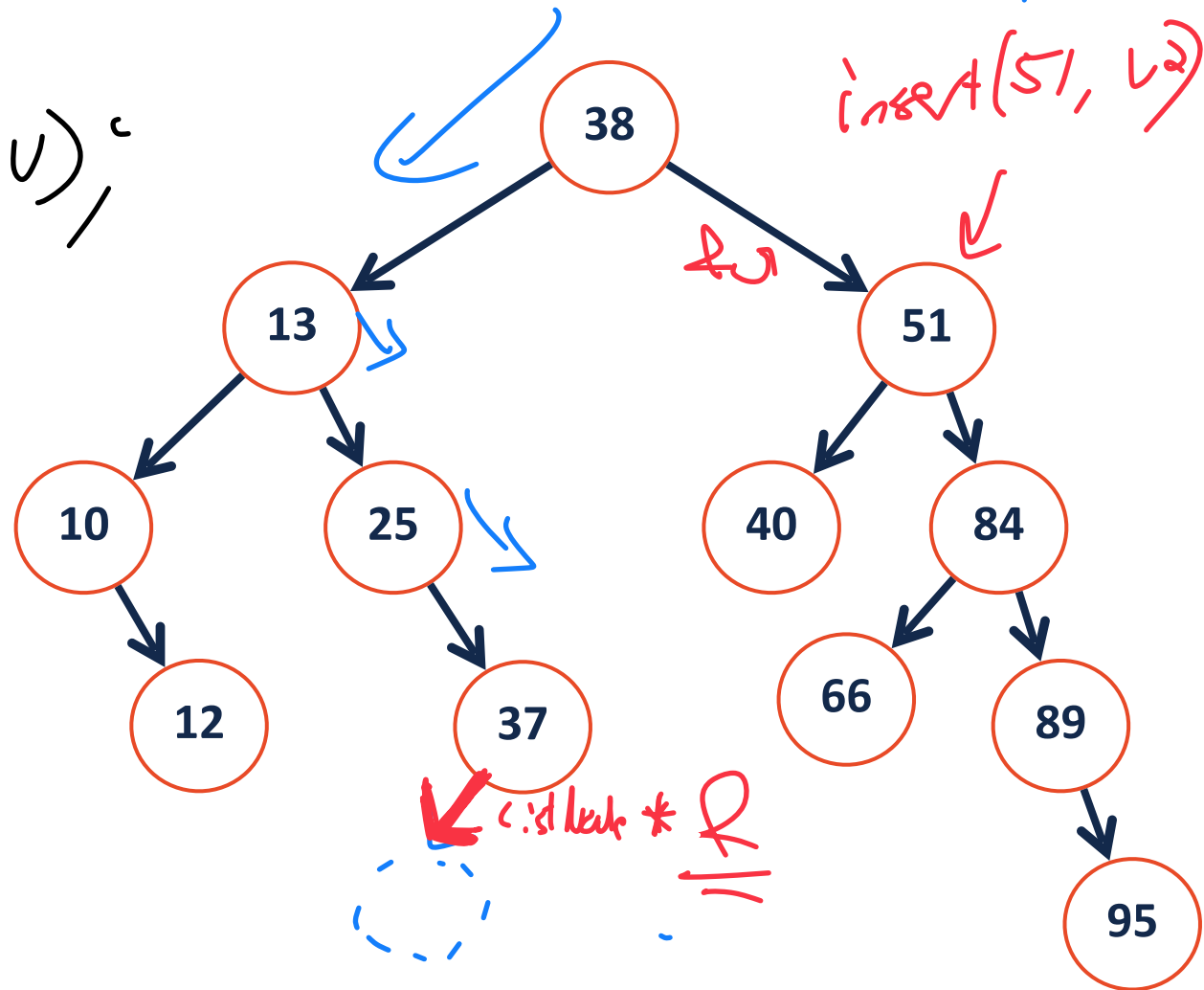
1) use find(33) Tree Node \* R  
 tmp =

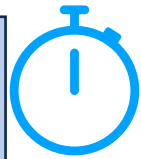
2) tmp = new TreeNode(33, v)  
 Main Point →

De duplicate  
 key = 'Bob\_1', 'Bob\_2'  
 value []

TreeNode  
 ↳ key K  
 value V

**insert(33, v)**





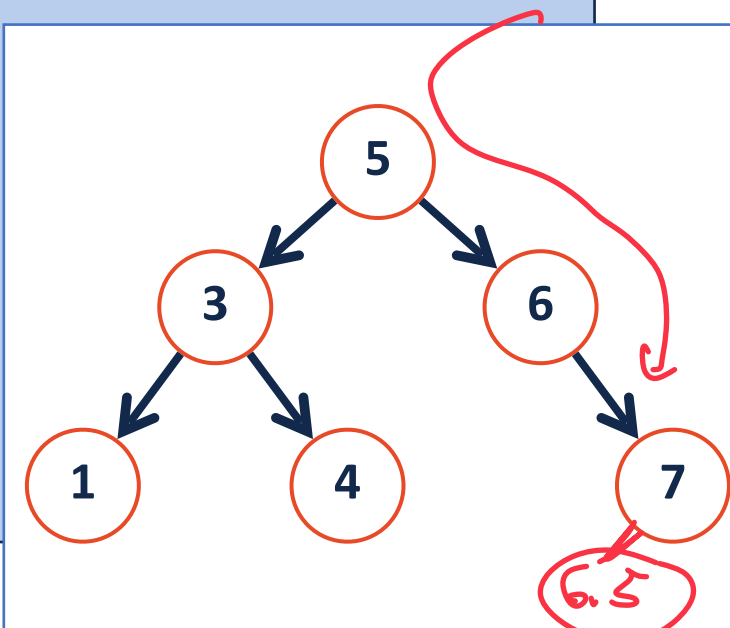
```
1 template<typename K, typename V>
2
3 void _insert(const K & key, const V & val) {
4
5     return _insert(root, key, val);
6 }
7
```

*Handwritten notes:*  $T^E$  data (next to line 3), helper function (next to line 5), with a red arrow pointing from the helper function back to the main function.

```
1 template<typename K, typename V>
2
3 void _insert(TreeNode *& root, const K & key, const V & val) {
4
5
6
7
8
9
10
11
12
13
14
15
16 }
```

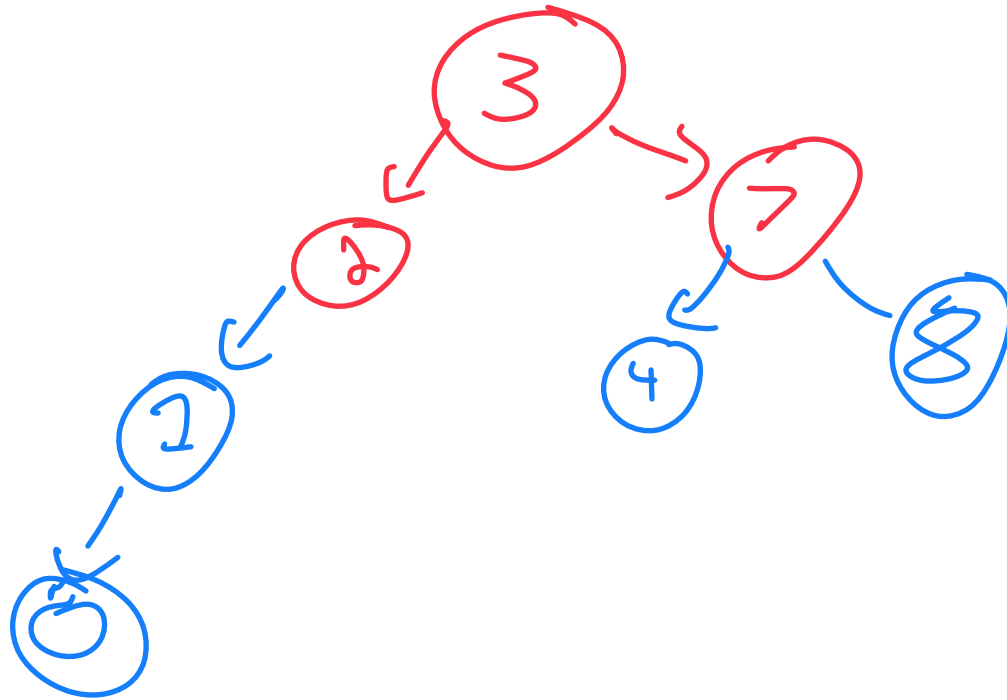
*Handwritten notes:* 1) Do a find (with a red arrow pointing to the root parameter), 2) Insert new Node.

6.5



# BST Insert *The order of insert matters!*

What binary tree would be formed by inserting the following sequence of integers: [3, 7, 2, 1, 4, 8, 0]





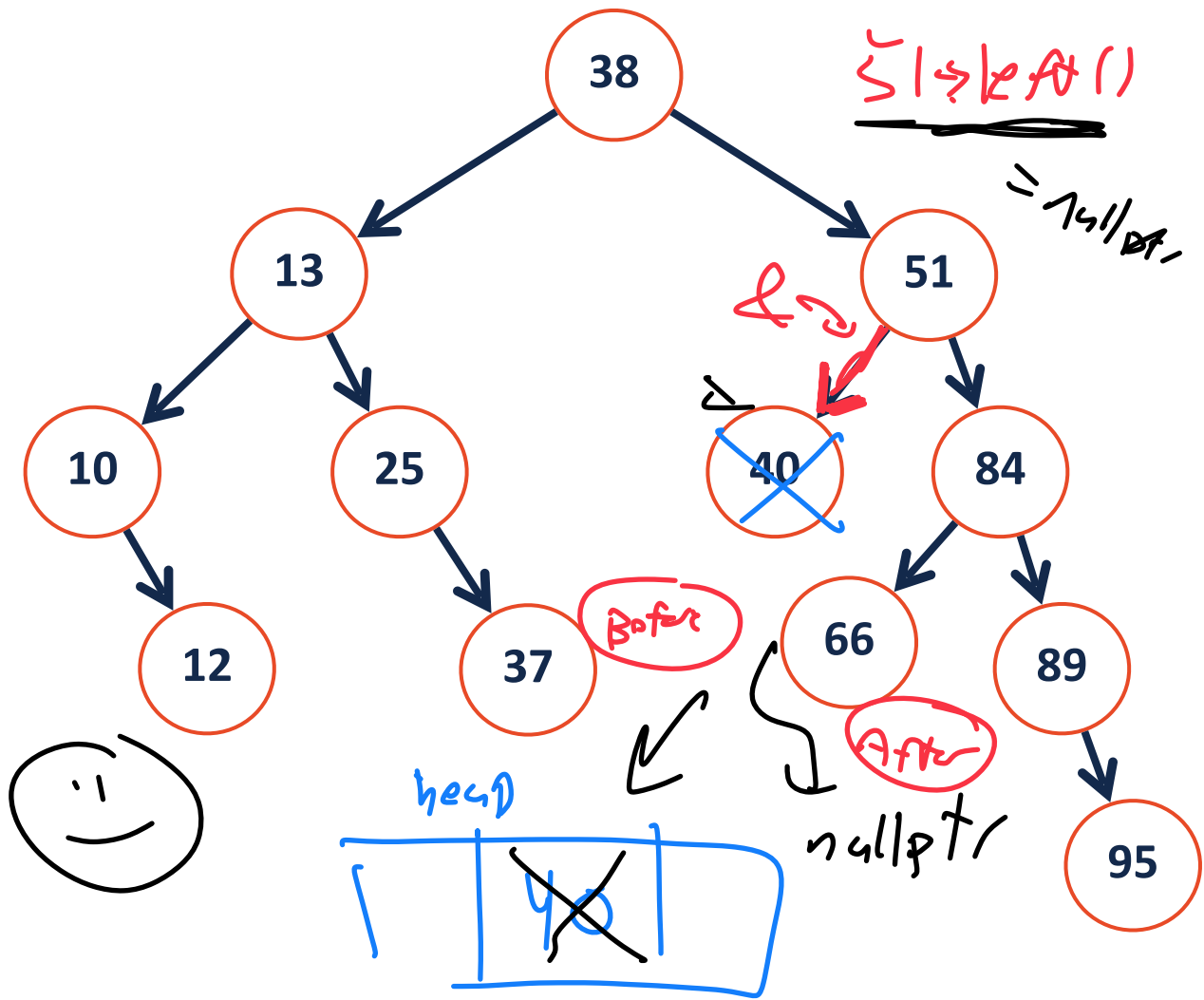
# BST Remove

**remove (40)**

- 1) tmp = find (40)
- 2) delete tmp;
- 3) tmp = null ptr;

Because we are removing a leaf

↳ 0-child remove 😊

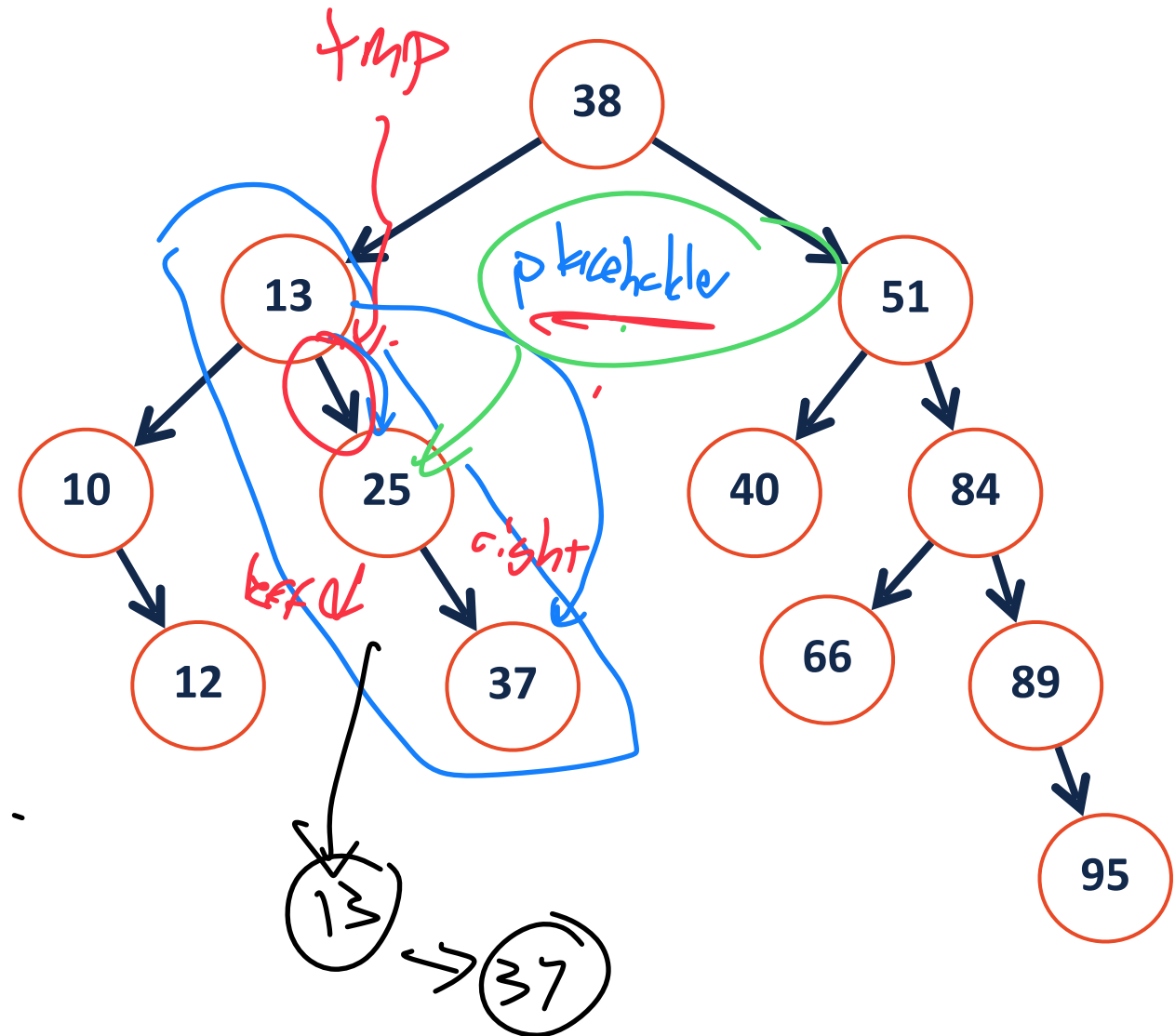


# BST Remove

- 1)  $tmp = \text{find}(25)$
- 2) Tree Node \* placeholder = tmp;
- 3)  $tmp = tmp \rightarrow \text{right}$
- 4) Delete placeholder

Linked list remove 😊

remove (25)



# BST Remove

1) find (13)  
 find <sup>tmp</sup>

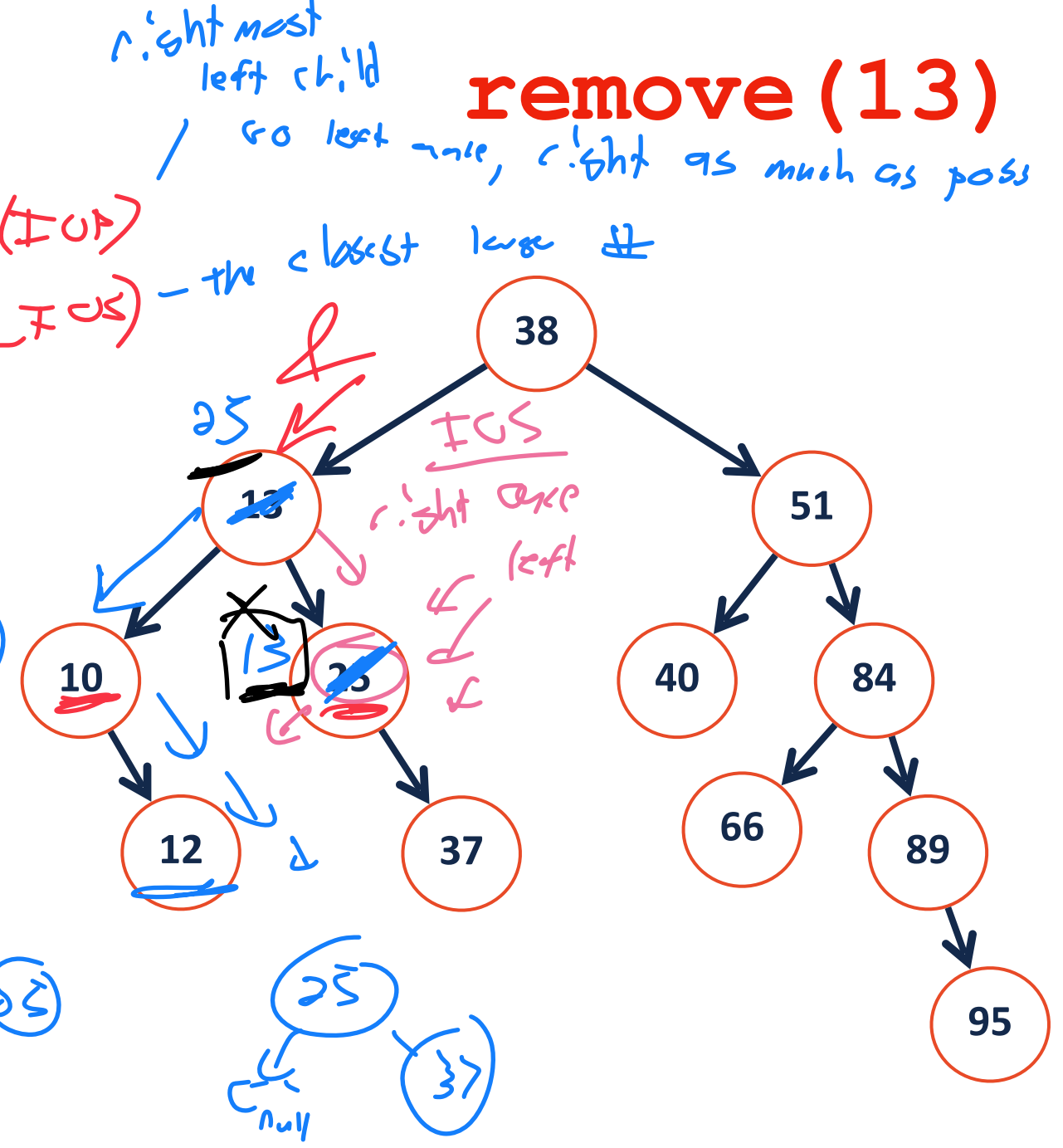
2) find In-order predecessor (IOP) / In-order Successor (IOS) - the closest larger &

3) Swap tmp w/ IOP / IOS  
 ↳ In btree are swapped  
 ↳ Swap either value (if easy) or swap pointers

4) delete (13)  
 ↳ This will always be 0 or 1 child remove

right most left child / go left until, right as much as poss

## remove (13)



# BST Remove

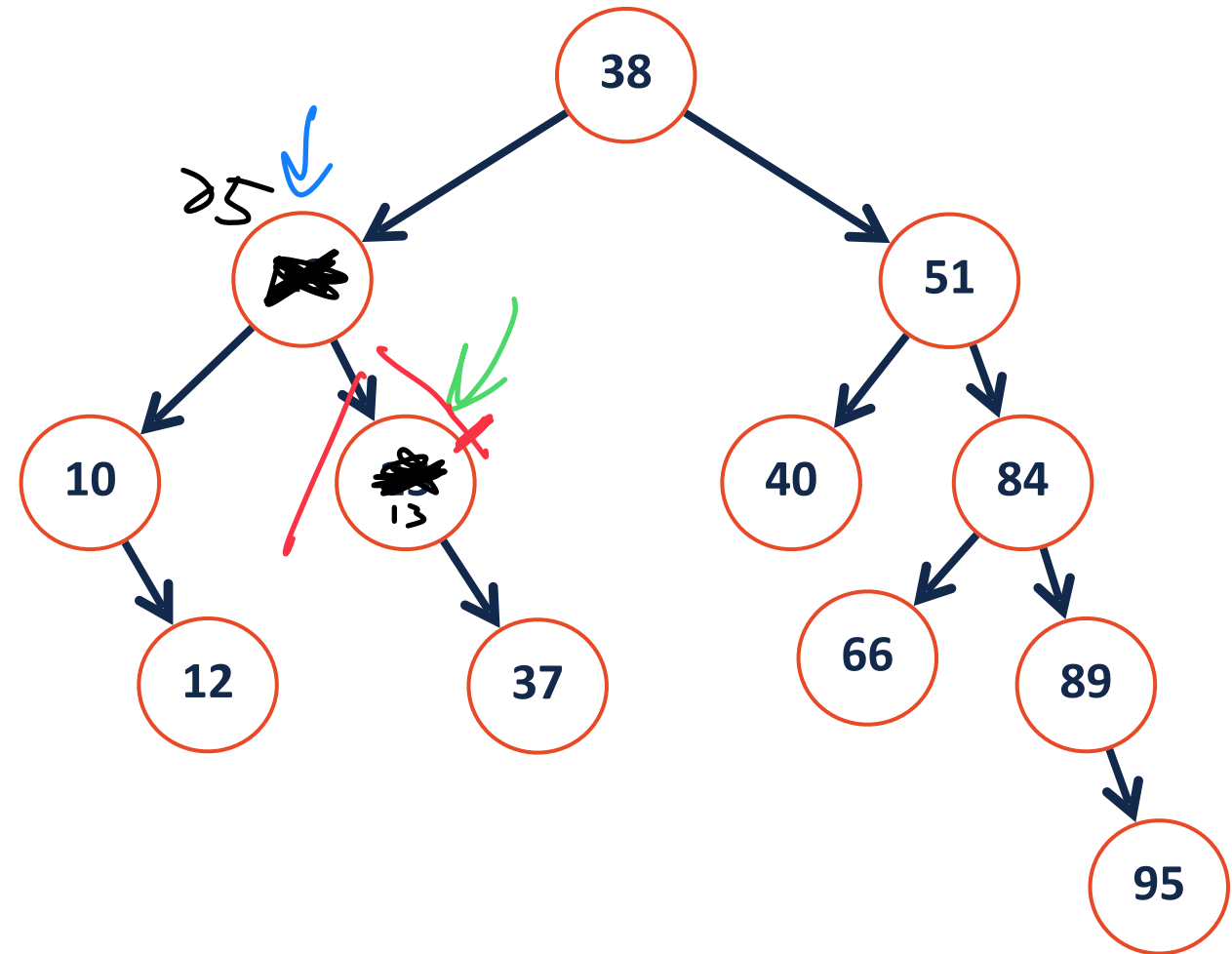
1) find B

2) find IOS

3) swap

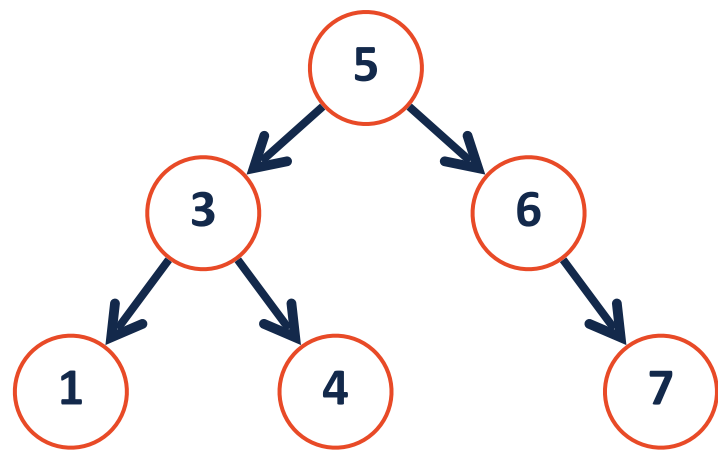
4) remove (13)  
↑ on my subtree

<sup>13</sup>  
**remove (51)**





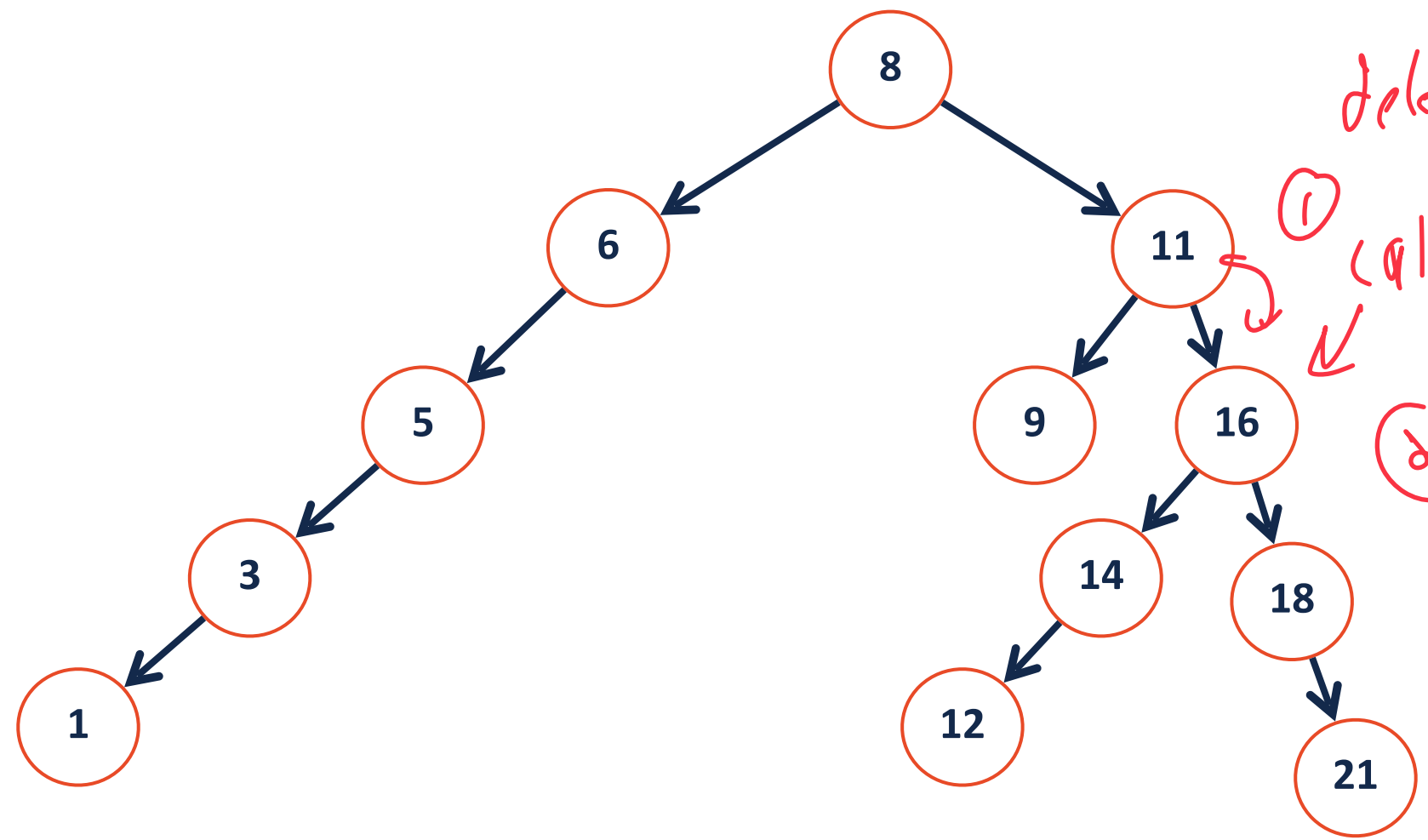
```
1 template<typename K, typename V>
2
3 void _remove(TreeNode *& root, const K & key) {
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23 }
```



# BST Remove

SKipped  $\rightarrow$  put on Friday!

What will the tree structure look like if we remove node 16 using IOS?

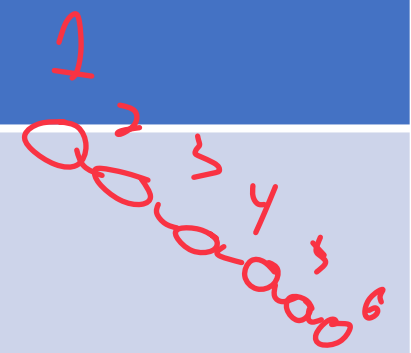


delete whole subtree  
(1) call tree destruct on 16  
(2) set 11  $\rightarrow$  r.child = null

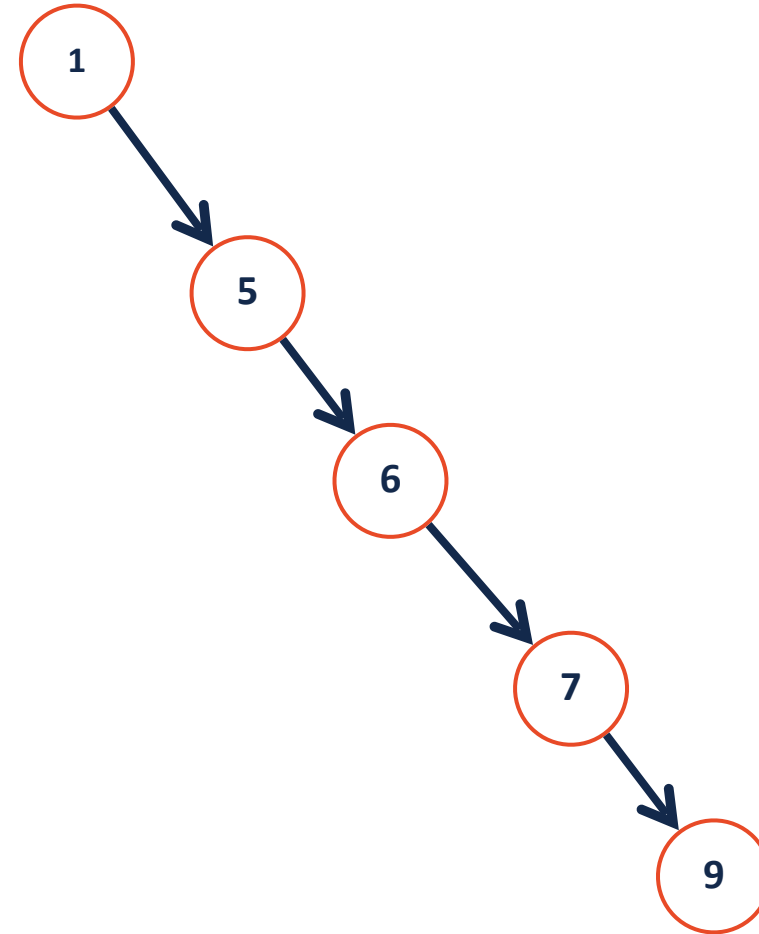
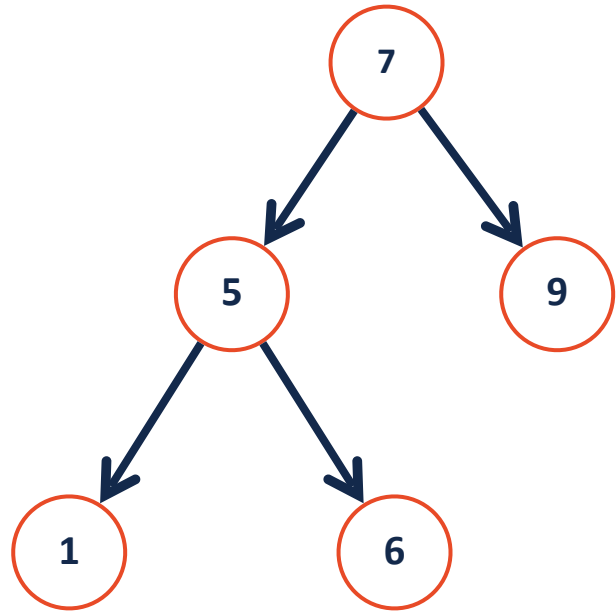
# BST Analysis – Running Time



Operation	BST Worst Case
<b>find</b> 	$O(h)$ → where $h$ is height $h$ is $O(n)$
<b>insert</b> 	find $O(h)$ insert $O(1)$ → $O(h)$
<b>remove</b> 	$\text{find}(O(h))$ + $\text{find}(O(h)) + 1$ <small>↳ remove obj</small> <small>↳ FUP/FOS</small> $O(h)$
<b>traverse</b> 	$O(n)$



# Limiting the height of a tree





# Option A: Correcting bad insert order

The height of a BST depends on the order in which the data was inserted

**Insert Order:** [1, 3, 2, 4, 5, 6, 7]

**Insert Order:** [4, 2, 3, 6, 7, 1, 5]

# AVL-Tree: A self-balancing binary search tree

Rather than fixing an insertion order, just correct the tree as needed!

