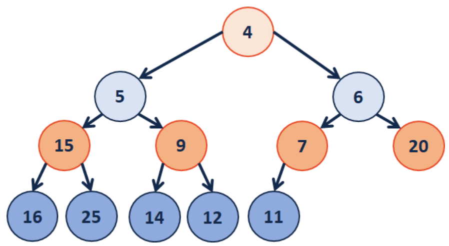
# #23: Heap Analysis

## A Heap Data Structure

***(specifically a minHeap in this example, as the minimum element is at the root)***

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| – | 4 | 5 | 6 | 15 | 9 | 7 | 20 | 16 | 25 | 14 | 12 | 11 | | |

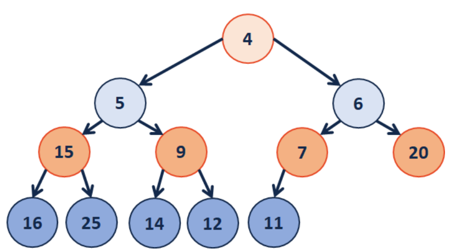
Given an index *i*, it's parent and children can be reached in O(1) time:

- leftChild := 2i
- rightChild := 2i + 1
- parent := floor( i / 2 )

**Formally, a complete binary tree T is a minHeap if:**

- **T = {} or**
- **T = {r, $T_L$, $T_R$} and r is less than the roots of $T_L$, $T_R$ and $T_L$, $T_R$ are minHeaps**

## Inserting into a Heap

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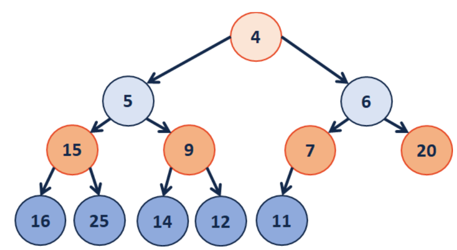
**Heap.hpp (partial)**

```
template <class T>
void Heap<T>::_insert(const T & key) {
  // Check to ensure there's space to insert an element
  // ...if not, grow the array
  if ( size_ == capacity_ ) { _growArray(); }

  // Insert the new element at the end of the array
  item_[++size] = key;

  // Restore the heap property
  _heapifyUp(size);
}
template <class T>
void Heap<T>::_heapifyUp( _______________ ) {
  if ( index > _________ ) {
    if ( item_[index] < item_[ parent(index) ] ) {
      std::swap( item_[index], item_[ parent(index) ]
);
      _heapifyUp( ________________ );
    }
  }
}
```

**How do we complete this code?**

**Running time of insert?**

## Heap Operation: removeMin / heapifyDown:

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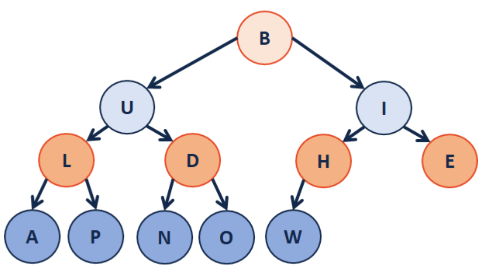
```
Heap.hpp (partial)
template <class T>
T Heap<T>::_removeMin() {
  // Swap with the last value
  T minValue = item_[1];
  item_[1] = item_[size_];
  size--;

  // Restore the heap property
  heapifyDown();

  // Return the minimum value
  return minValue;
}
template <class T>
void Heap<T>::_heapifyDown(int index) {
  if ( !_isLeaf(index) ) {
    int minChildIndex = _minChild(index);
    if ( item_[index] ___ item_[minChildIndex] ) {
       std::swap( item_[index], item_[minChildIndex] );
       _heapifyDown( ________________ );
    }
  }
}
```

**Q: How do we construct a heap given data?**

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```
Heap.cpp (partial)
template <class T>
void Heap<T>::buildHeap() {
  for (unsigned i = parent(size); i > 0; i--) {
    heapifyDown(i);
  }
}
```

**Running Time?**

**Theorem:** The running time of buildHeap on array of size n is: ____________.

**Strategy:**

**Define S(h):**
Let **S(h)** denote the sum of the heights of all nodes in a complete tree of height **h**.

**S(0)** =

**S(1)** =

**S(h)** =

**Proof of S(h) by Induction:**

**Finally, finding the running time:**