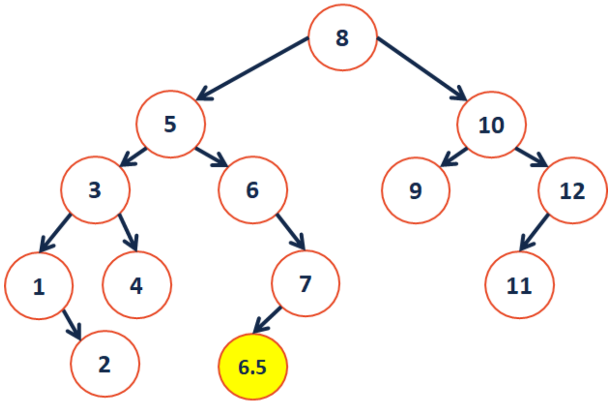
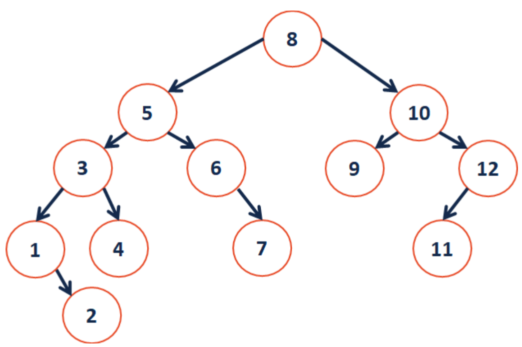
#16: AVL Analysis

AVL Insertion



AVL Removal



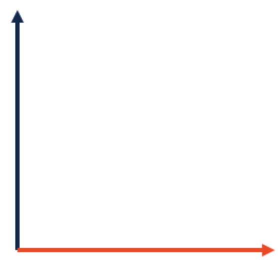
Running Times:

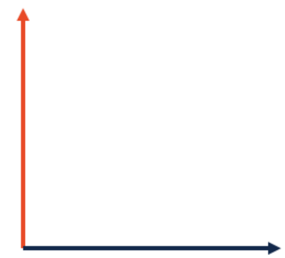
	AVL Tree
find	
insert	
remove	

Motivation:

Big-O is defined as:

Let **f(n)** describe the height of an AVL tree in terms of the number of nodes in the tree (**n**). Visually, we can represent the big-O relation:





$\mathbf{f(n) \le c \times g(n)}$: Provides an <u>upper bound</u>:

The height of the tree, **f(n)**, will always be <u>less than</u> $\mathbf{c \times g(n)}$ for all values where **n** > **k**.

$\mathbf{f^{-1}(h) \ge c \times g^{-1}(h)}$: Provides a <u>lower bound</u>:

The number of nodes in the tree, $\mathbf{f^{-1}(h)}$, will always be <u>greater than</u> **c** × $\mathbf{g^{-1}(h)}$ for all values where **n** > **k**.

Plan of Action:
Goal: Find a function that defines the lower bound on **n** given **h**.

Given the goal, we begin by defining a function that describes the smallest number of nodes in an AVL of height **h**:

Theorem:
An AVL tree of height **h** has at least ________________________.

I. Consider an AVL tree and let **h** denote its height.

II. Case: __________________

III. Case: ___________________

IV. Case: __________________

Inductive hypothesis (IH):

Proving our IH:

V. Using a proof by induction, we have shown that:

...and by inverting our finding:

Summary of Balanced BSTs:

Advantages	Disadvantages