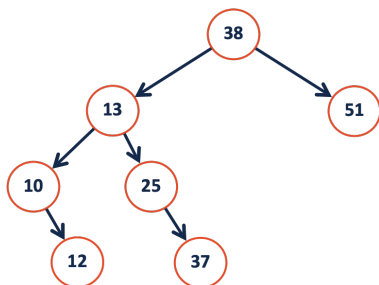
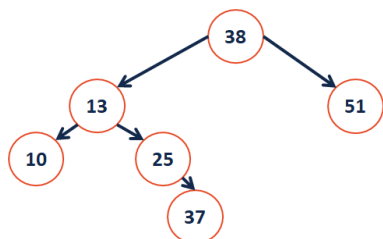
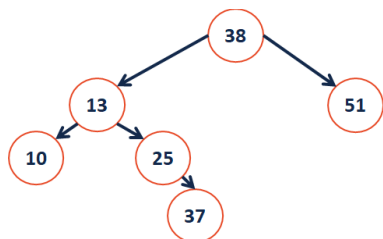


**Example 1: Right Rotation**



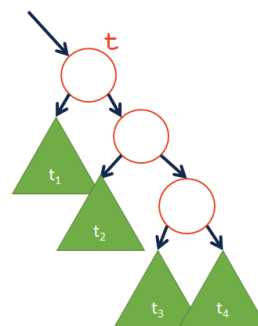
**Example 2: A Complex Rotation**



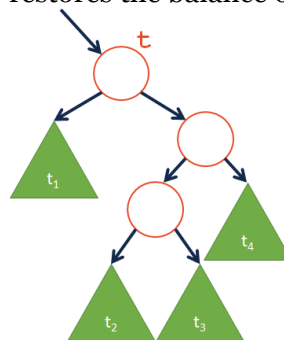
**BST Rotation Summary:**

1. Four kinds of rotations (L, R, LR, and RL)
2. All rotations are local
3. All rotations run in constant time,  $O(1)$
4. BST property is maintained!

**AVL Theorem #1:** If an insertion occurred in subtrees  $t_3$  or  $t_4$  and a subtree was detected at  $t$ , then a \_\_\_\_\_ rotation about  $t$  restores the balance of the tree.



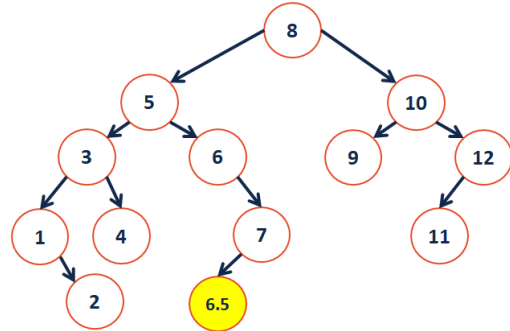
**AVL Theorem #2:** If an insertion occurred in subtrees  $t_2$  or  $t_3$  and a subtree was detected at  $t$ , then a \_\_\_\_\_ rotation about  $t$  restores the balance of the tree.



**Do you understand the mirrored versions of these theorems?**

## AVL Insertion

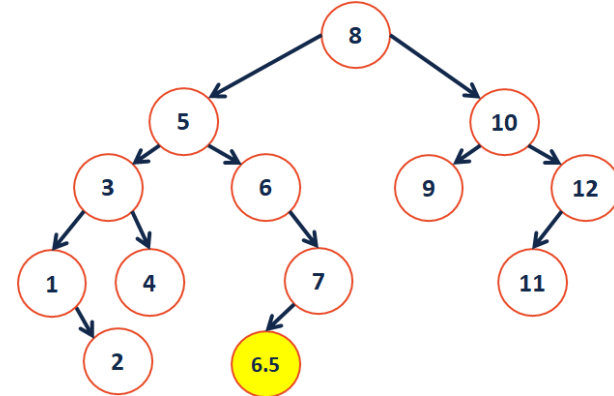
Pseudocode:



AVL.h (snippet)	
23	class TreeNode {
24	public:
25	T key;
26	unsigned height;
27	TreeNode *left;
28	TreeNode *right;
...	

134	
135	_updateHeight (cur) ;
136	};

## AVL Insertion



## AVL Insertion

AVL .hpp	
151	template <typename K, typename V>
152	void AVL<K, D>::_insert(const K & key, const V & data, TreeNode
	*& cur) {
153	if (cur == NULL)           { cur = new TreeNode(key, data); } }
157	else if (key < cur->key) { _insert( key, data, cur->left ); }
160	else if (key > cur->key) { _insert( key, data, cur->right ); }
166	_ensureBalance(cur);
167	}
---	
119	template <typename K, typename V>
120	void AVL<K, D>::_ensureBalance(TreeNode *& cur) {
121	// Calculate the balance factor:
122	int balance = height(cur->right) - height(cur->left);
123	
124	// Check if the node is current not in balance:
125	if ( balance == -2 ) {
126	int l_balance =
	height(cur->left->right) - height(cur->left->left);
127	if ( l_balance == -1 ) { _____; }
128	else           { _____; }
129	} else if ( balance == 2 ) {
130	int r_balance =
	height(cur->right->right) - height(cur->right->left);
131	if( r_balance == 1 ) { _____; }
132	else           { _____; }
133	}

## AVL Removal

