String Algorithms and Data Structures
Suffix Tries (and Trees)

CS 199-225
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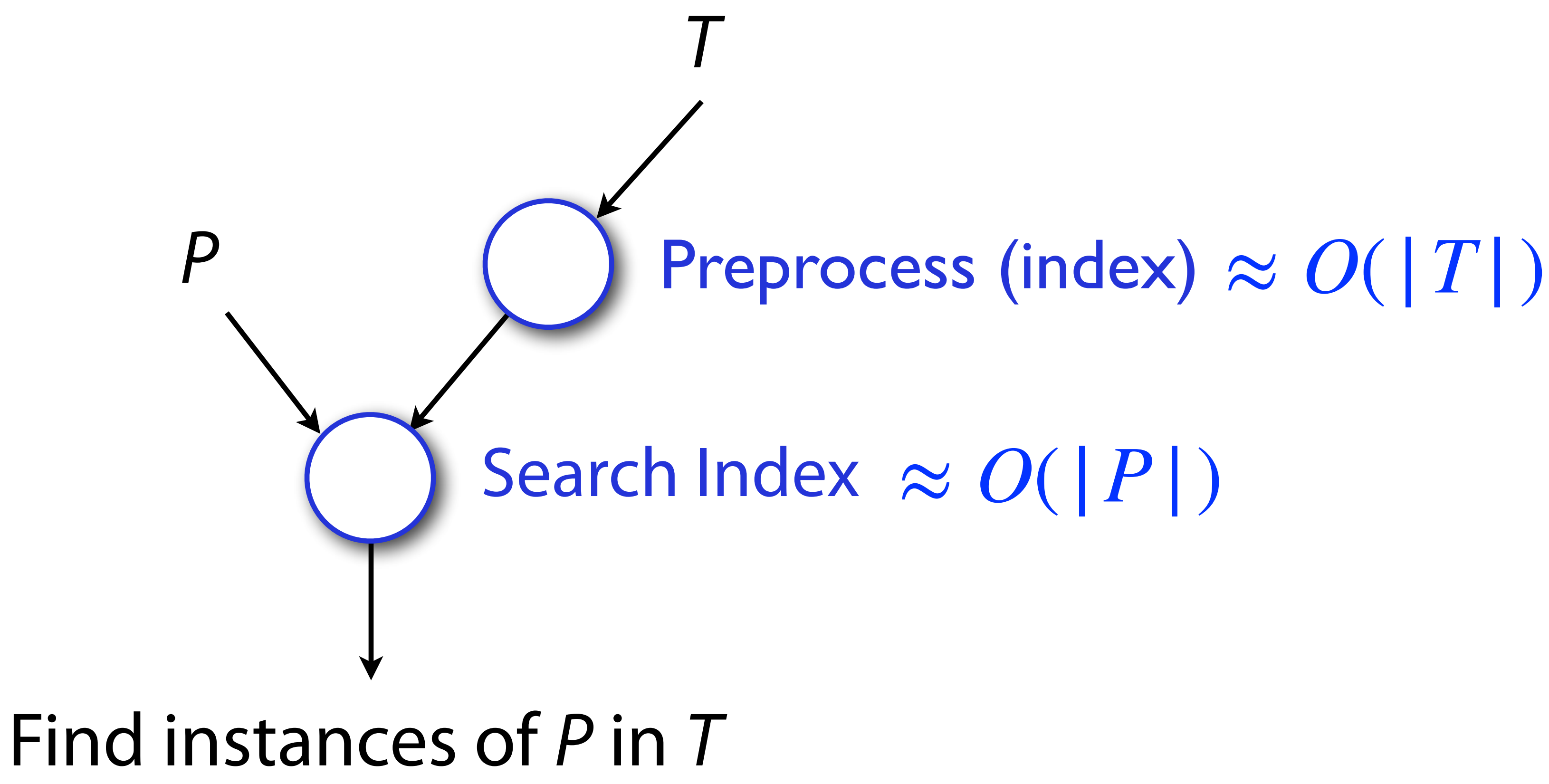
Department of Computer Science

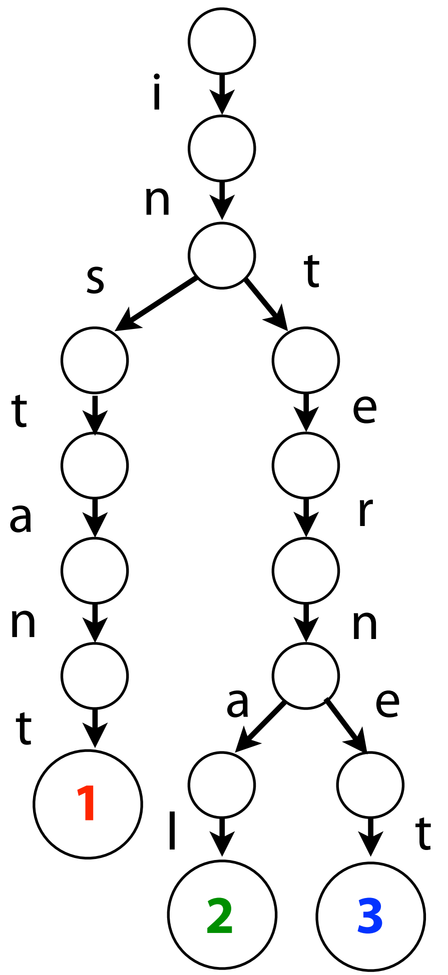
Exact pattern matching *w/ indexing*



String indexing with Tries

Trie: A rooted tree storing a collection of (key, value) pairs

Keys:	Values:
`instant`	1
`internal`	2
`internet`	3



Each edge is labeled with a character $c \in \Sigma$

For given node, at most one child edge has label c, for any $c \in \Sigma$

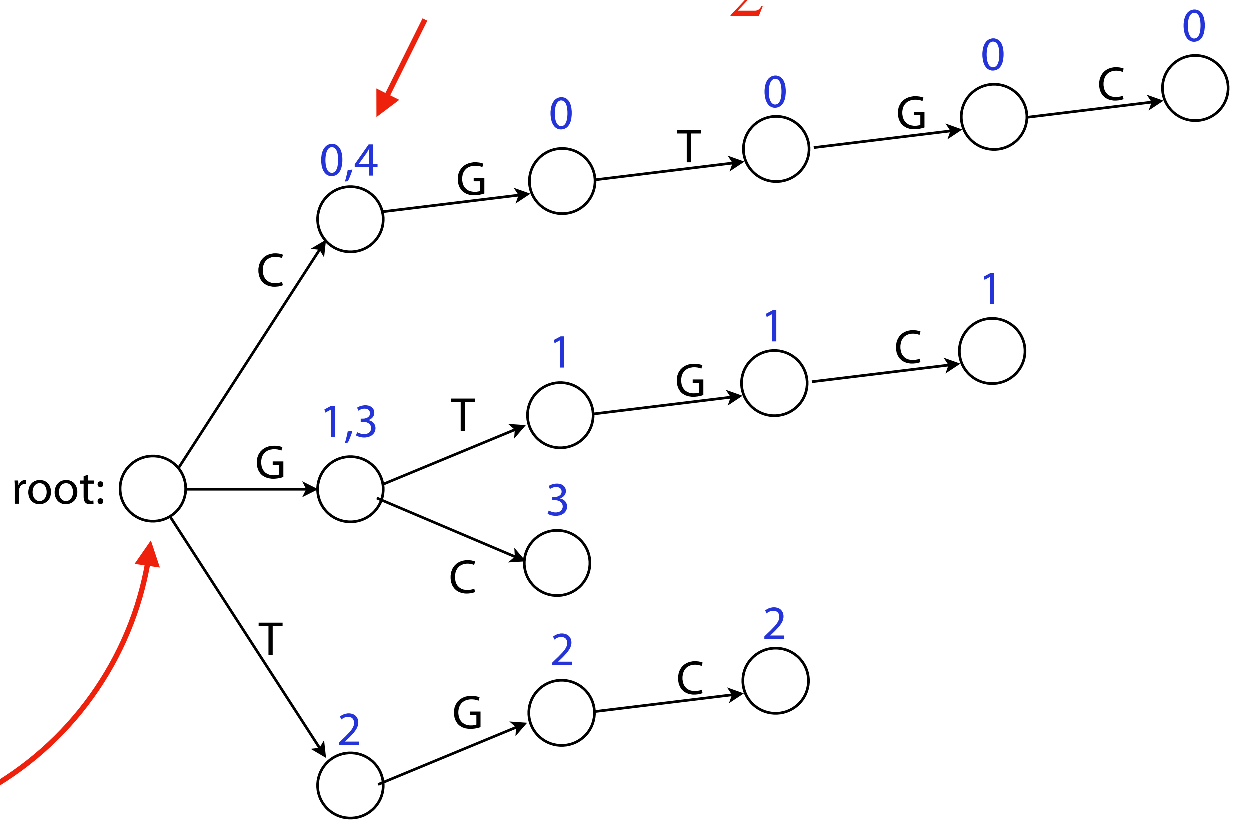
Each key is "spelled out" along some path starting at root and each value is stored at the leaf

NaryTree build_trie(std::string T)

T: **C G T G C**

Key	Value
C	0
G	1
T	2
G	3
C	4
CG	0
GT	1
TG	2
…	…

We are storing $\frac{|T|(|T|+1)}{2}$ values



We had to do $\frac{|T|(|T|+1)}{2}$ insertions

std::vector<int> searchPattern(P)

T: **C G T G C**

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G	1
T	2
G	3
C	4
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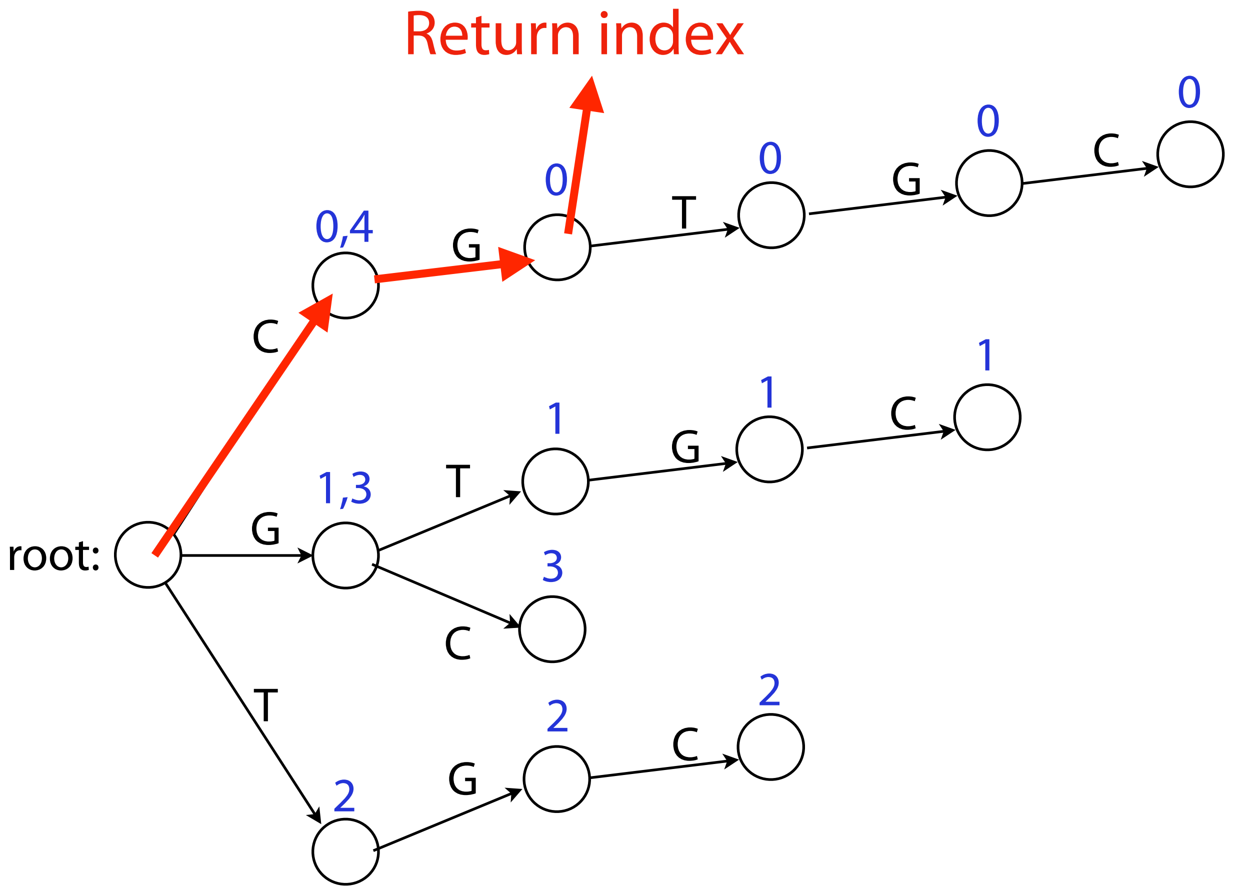
GT	1
TG	2
…	…



We can do exact pattern matching in $O(P)$ time!

Exact pattern matching *w/ indexing*

How can we be more efficient in our preprocessing?

1) Perform fewer insertions to store T

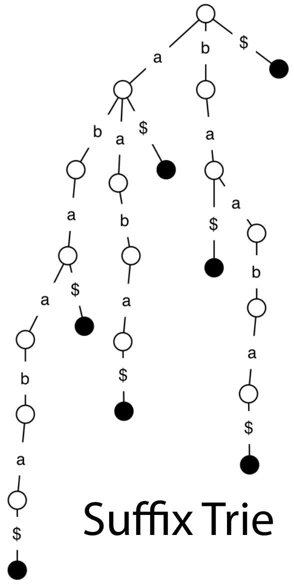
2) Store fewer values in index

What if we just stored the suffixes?

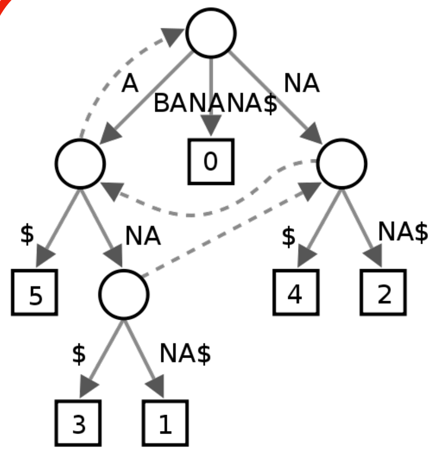
Exact pattern matching *w/ indexing*

There are many data structures built on ***suffixes***

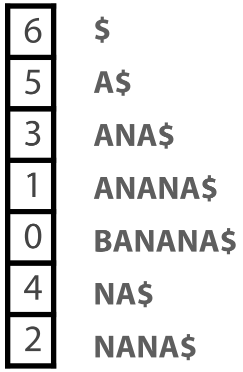
Modern methods still use these today



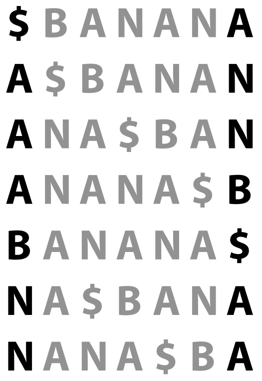
Suffix Trie



Suffix Tree



Suffix Array



FM Index

Suffix Trie

Build a **trie** containing all **suffixes** of a text *T*

T: **C G T G C**

C G T G C
G T G C
T G C
G C
C

T suffixes

Suffix Trie

Build a **trie** containing all **suffixes** of a text *T*

T: **C G T G C**

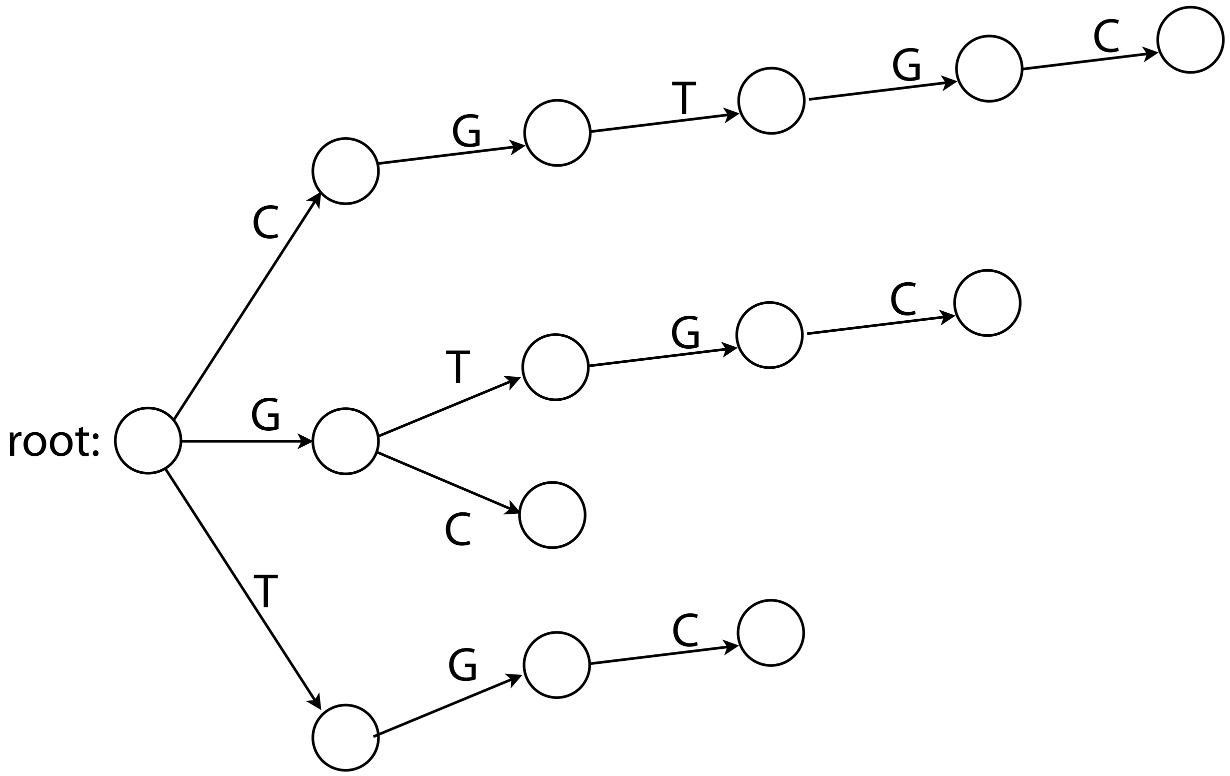
C G T G C

G T G C

T G C

G C

C



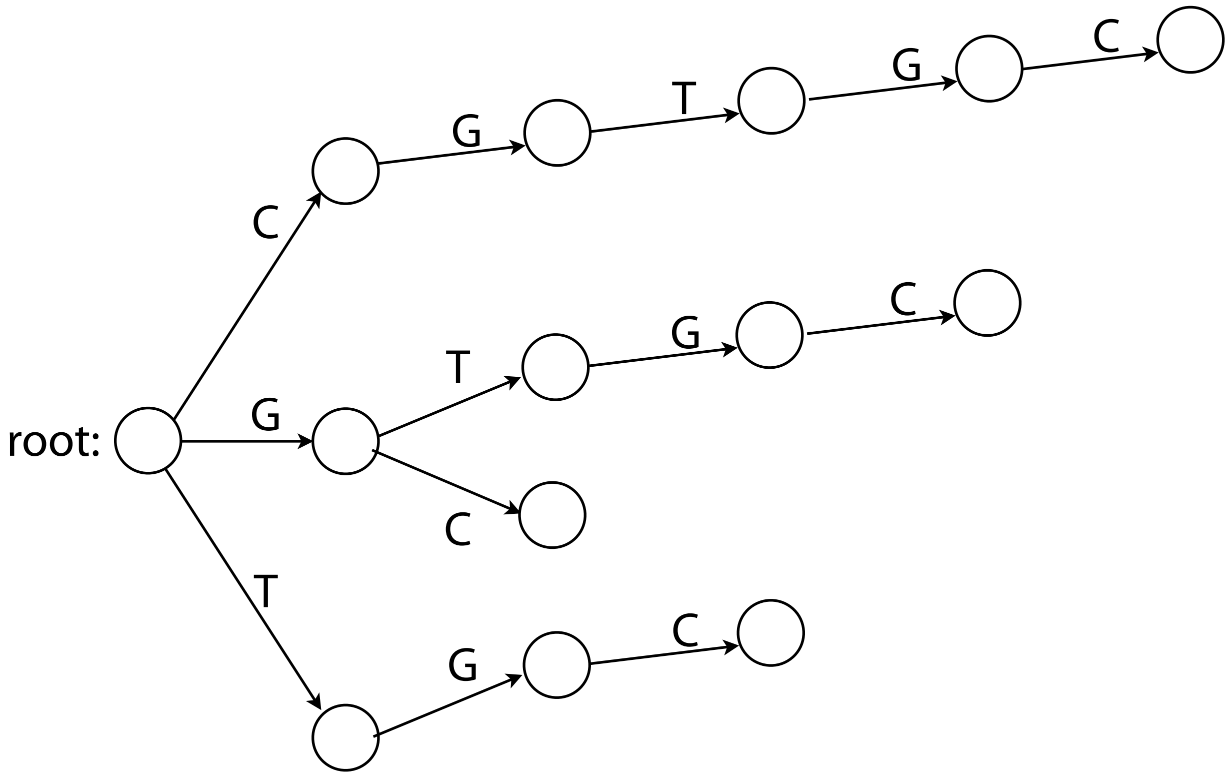
Inserting just T suffixes gets us the same tree*!

Suffix Trie

To prevent loss of information, add a **terminal character**

T: **C G T G C $**

Key	Value
CGTGC$	0
GTGC$	1
TGC$	2
GC$	3
C$	4
$	5



A terminal character cannot be a part of the alphabet!

Suffix Trie

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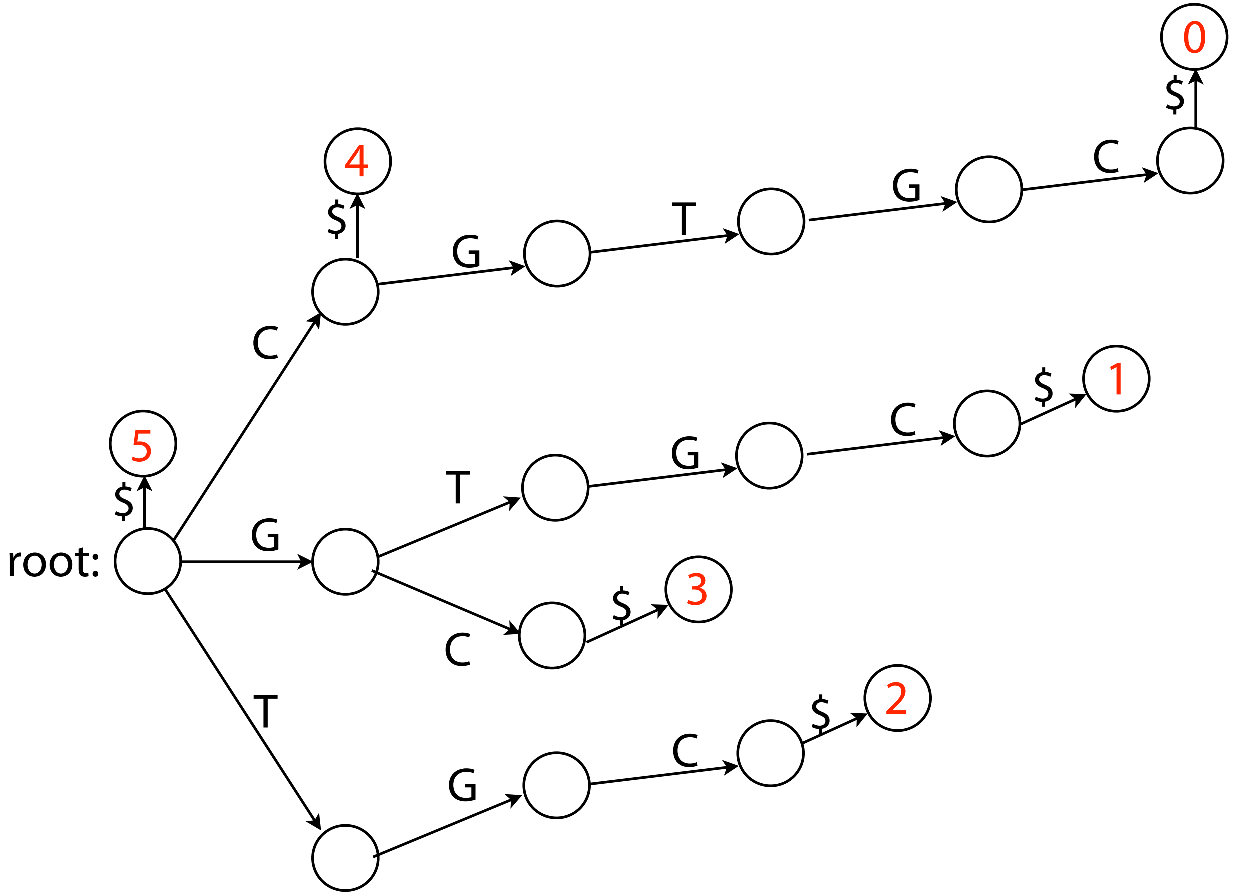
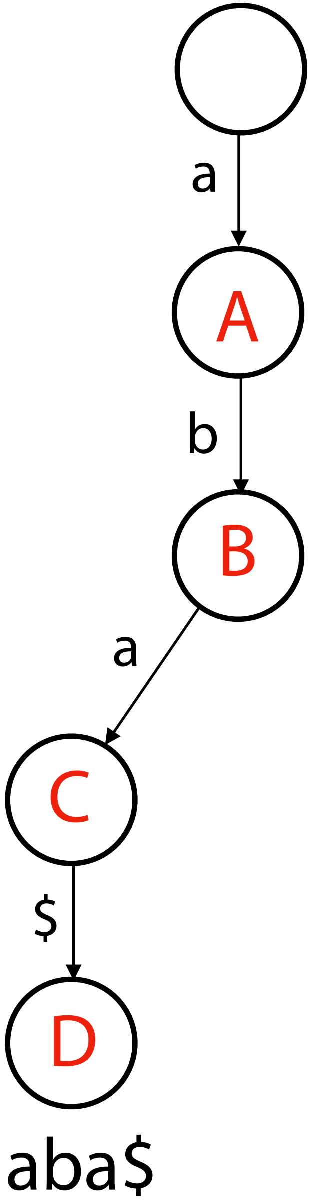


Suffix Trie Construction

T: **a b a $**

Key	Value
aba$	0
	1
	2

Where do I put value?



Suffix Trie Construction

T: **a b a $**

Key	Value
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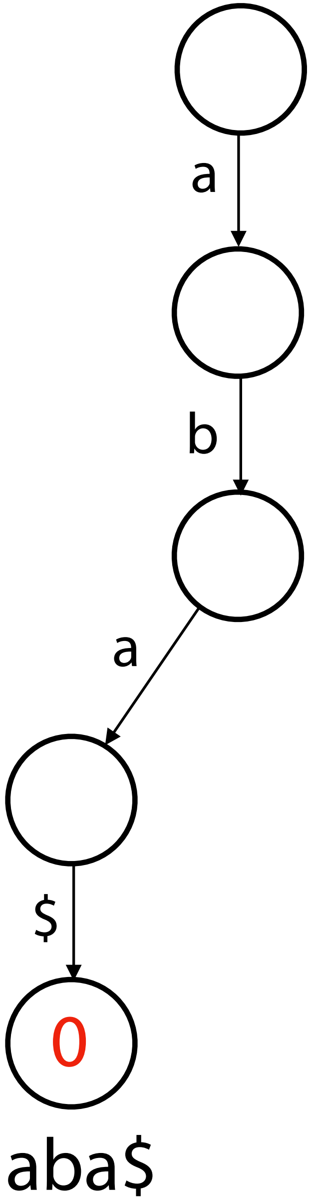
	1
	2

What are my other keys?



Suffix Trie Construction

T: **a b a $**

Key	Value
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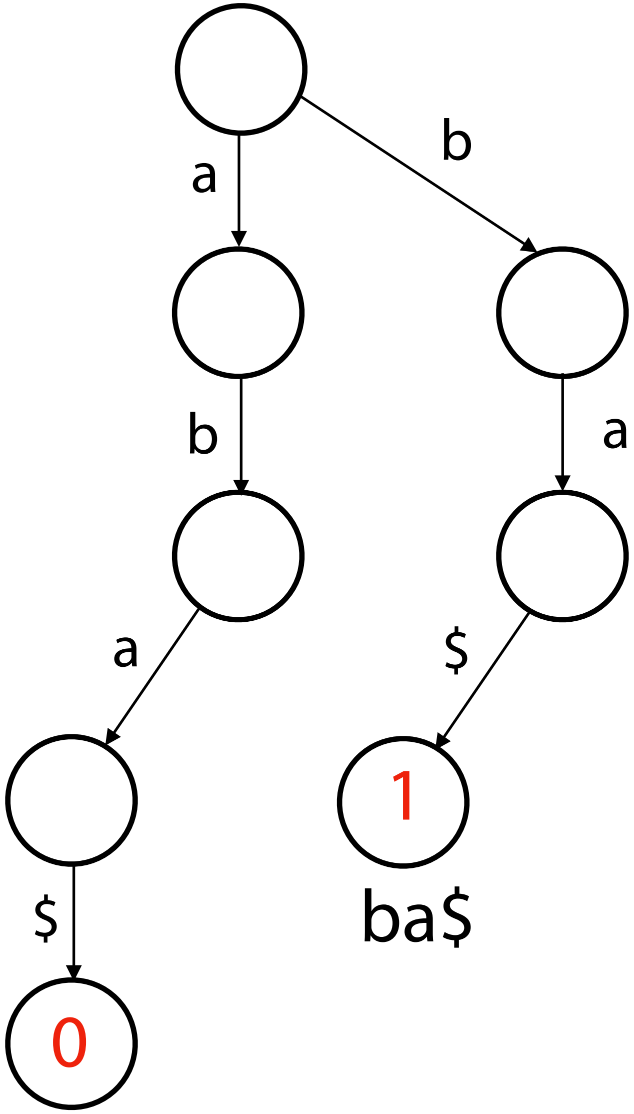
ba$	1
a$	2

What edges do I add?



Suffix Trie Construction

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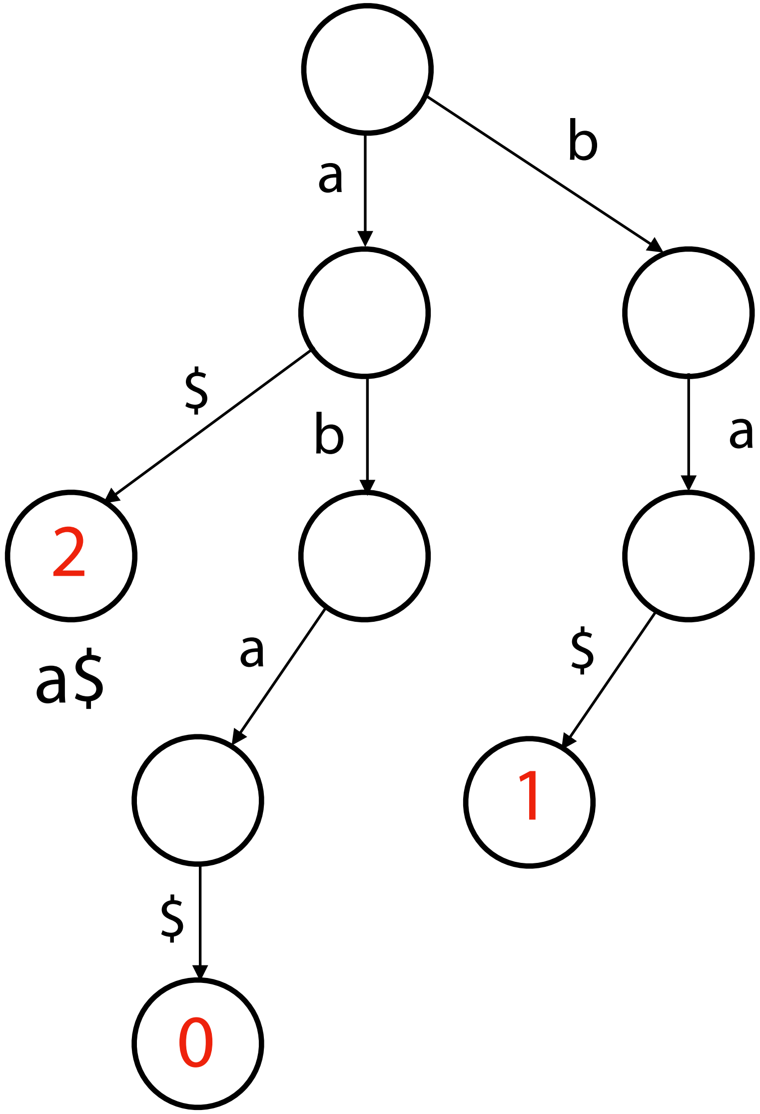
Are we done?



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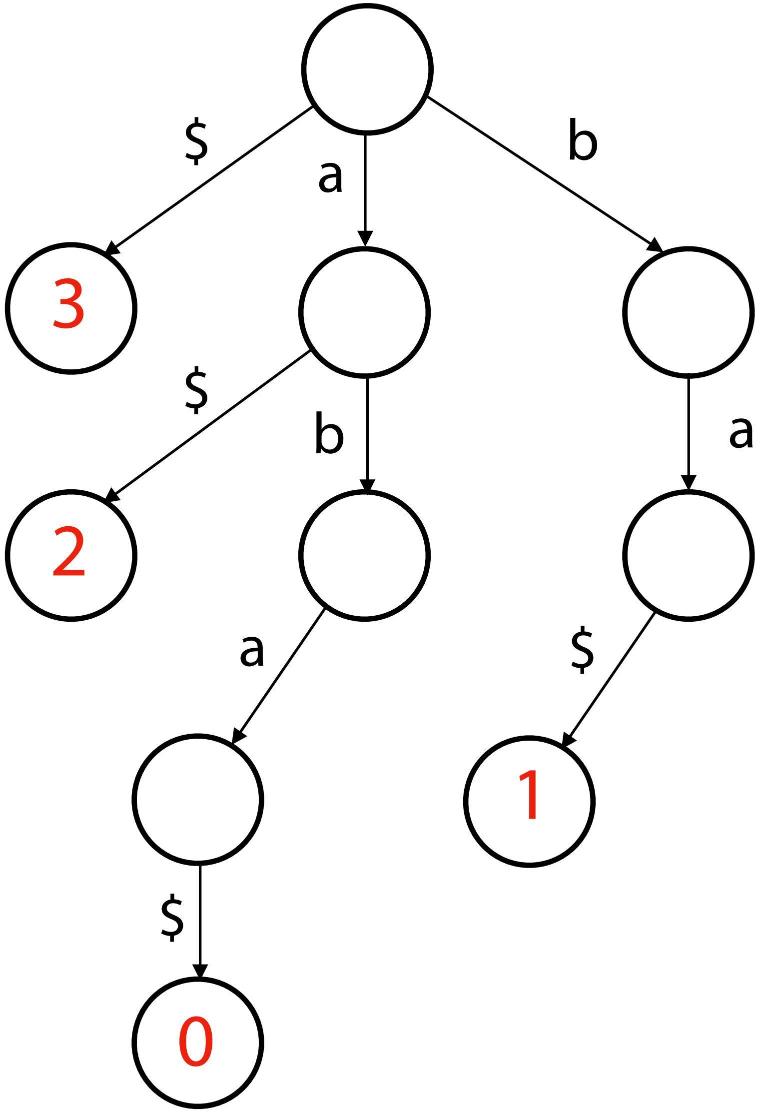

NaryTree Modification I and II

stree.h

```
class NaryTree
{
    public:
      struct Node {
        int index;
        std::map<std::string, Node*> children;

        Node(std::string s, int i)
        {
          if(s.length() > 0 ){
            std::string f = s.substr(0,1);
            children[f] = new Node(s.substr(1), i );
          } else {
            index = i;
          }
        }
      };

    protected:
        Node* root;
…
```

NaryTree build_strie(std::string T)

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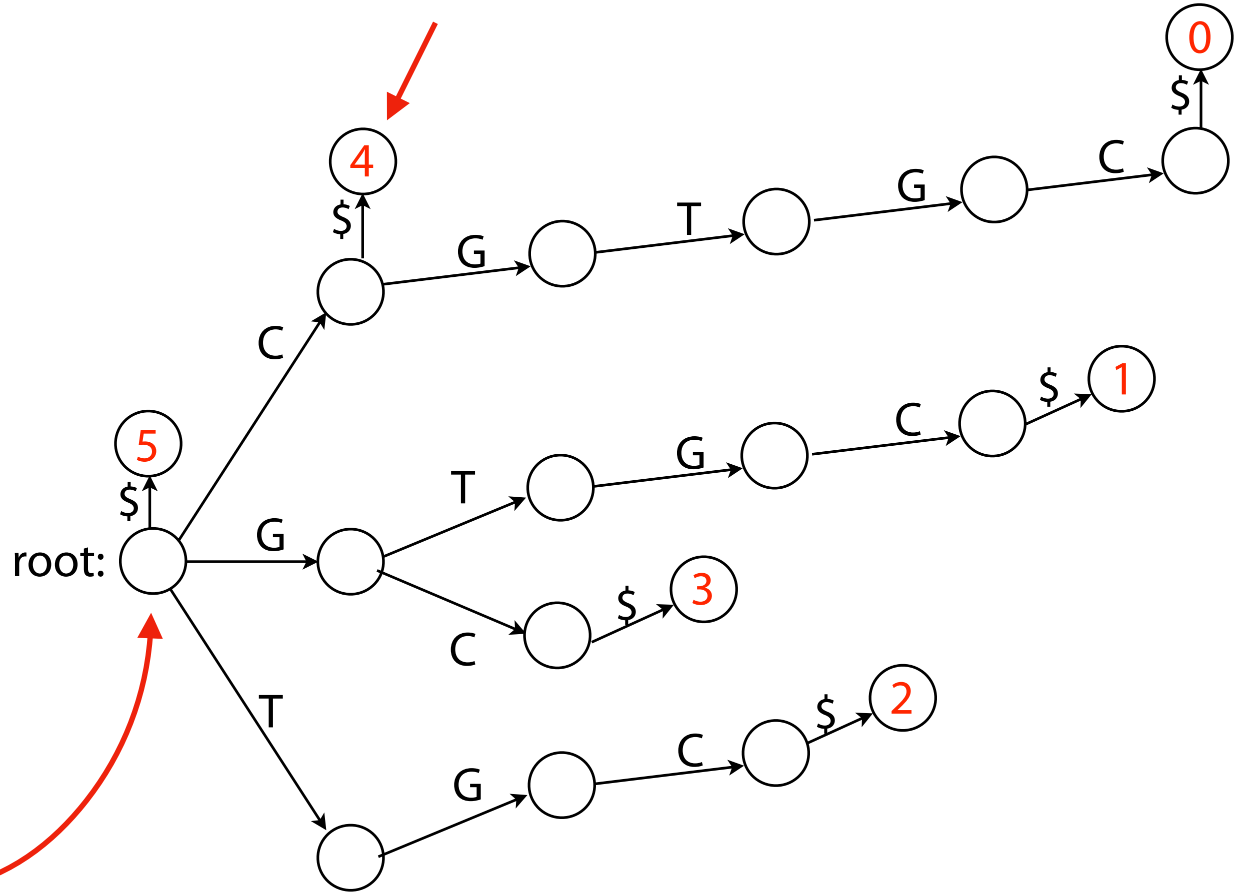
Store $|T| + 1$ values



Perform $|T| + 1$ insertions

Assignment 6: a_stree

Learning Objective:

Use an existing implementation of a suffix trie as a N-ary Tree

Implement exact pattern matching using a suffix trie

Construct a suffix tree from a suffix trie

Suffix Trie Search

Each of *T*'s substrings is spelled out along a path from the root.

Every ***substring*** is a ***prefix*** of some ***suffix***

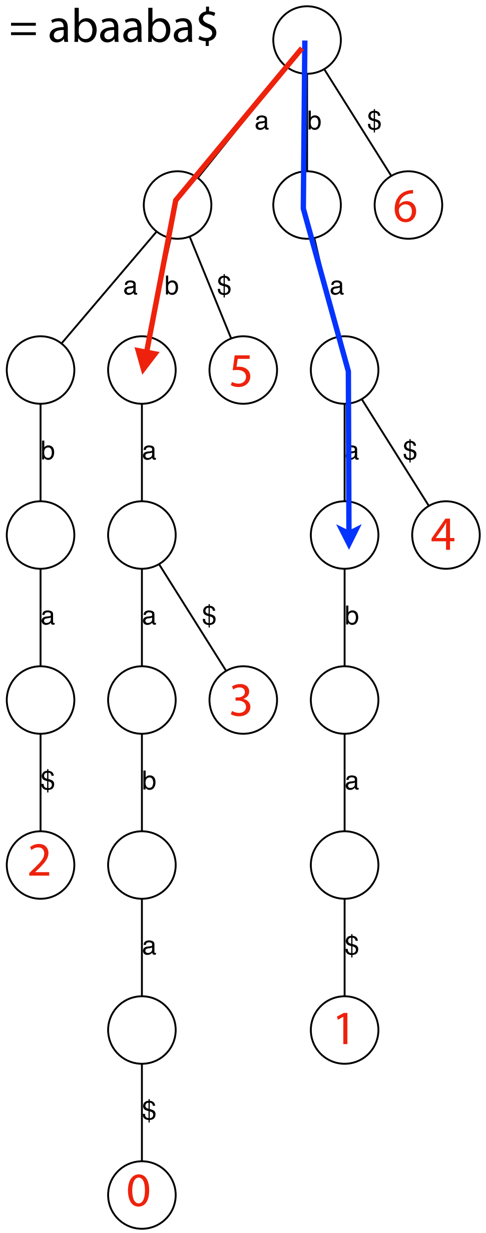
P = baa

Search(P) =

P = ab

Search(P) =

T = abaaba$



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Starting at root:

(0) If *P* empty, return values of all leaves.

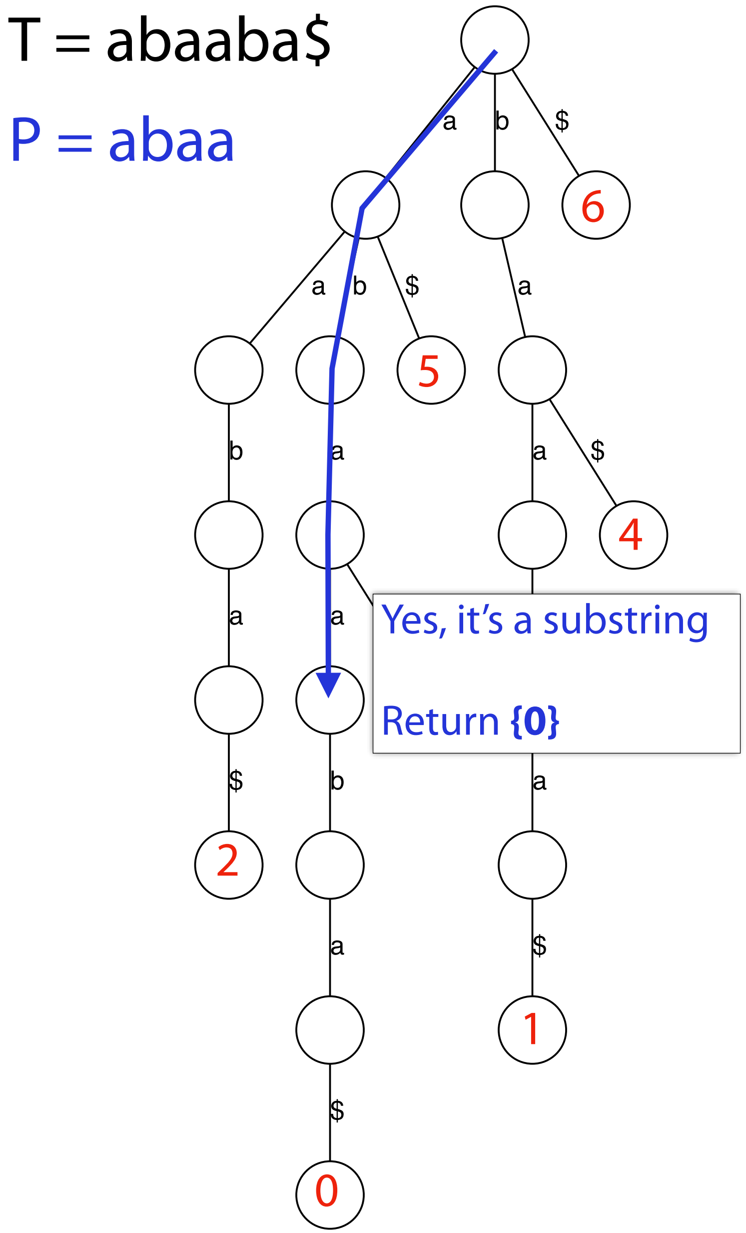
(1) Try to match front character

(2) If match, move to appropriate child

(2.5) Set pattern equal to remainder

(2.5) Go back to (0)

(3) If mismatch, *P* is not a substring!



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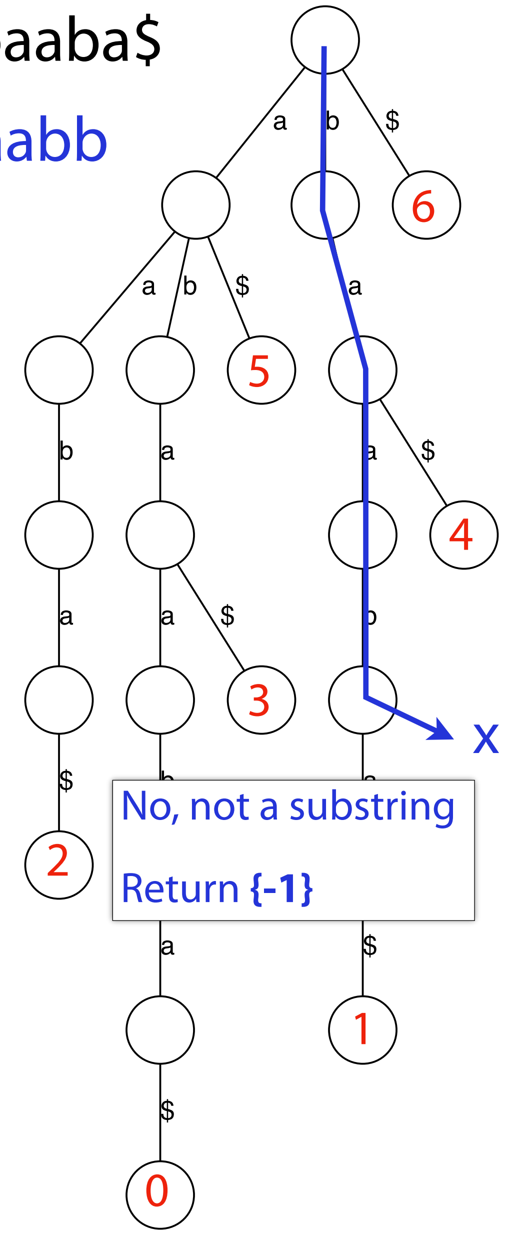
T = abaaba$

P = baabb



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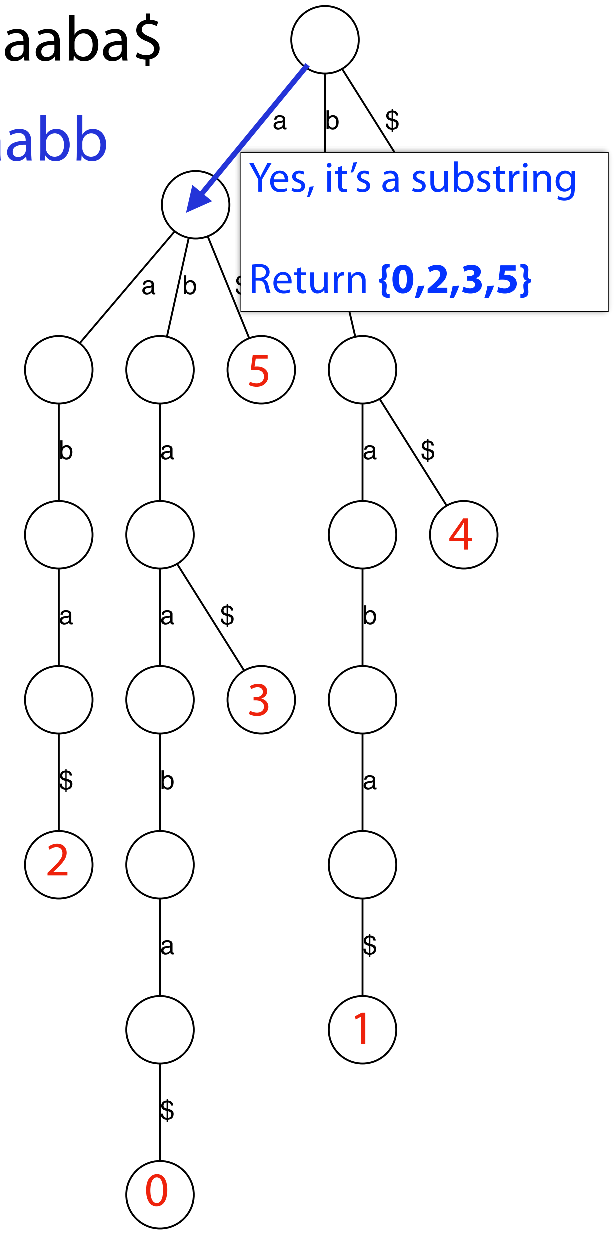

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Trie Search Big O:

Suffix Trie Search Big O:

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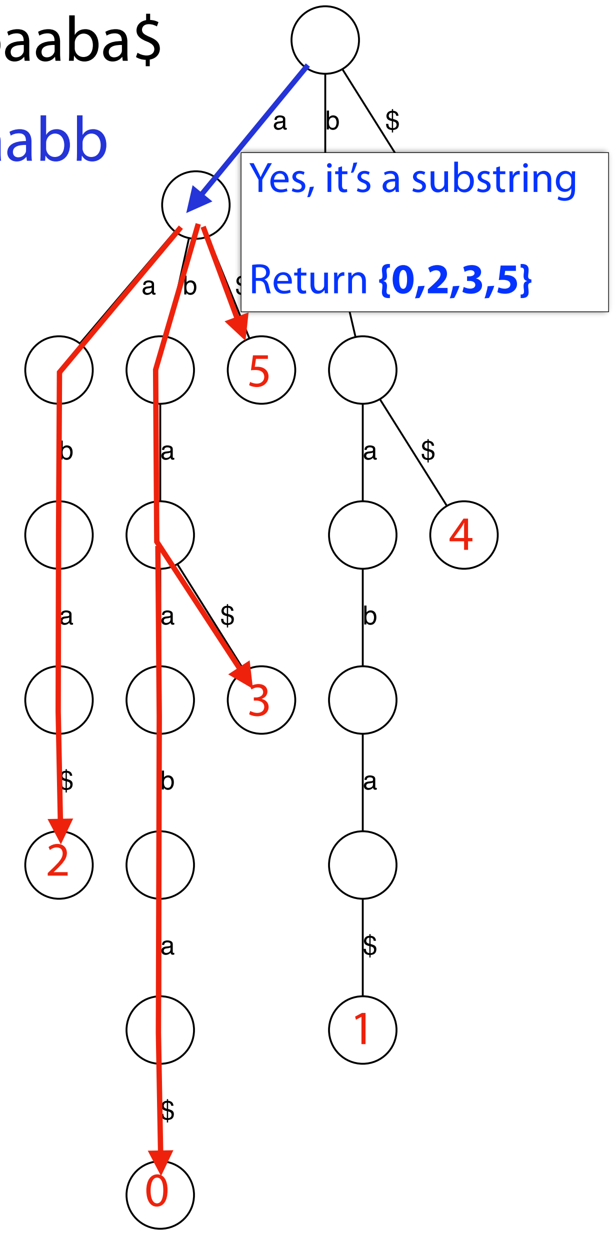


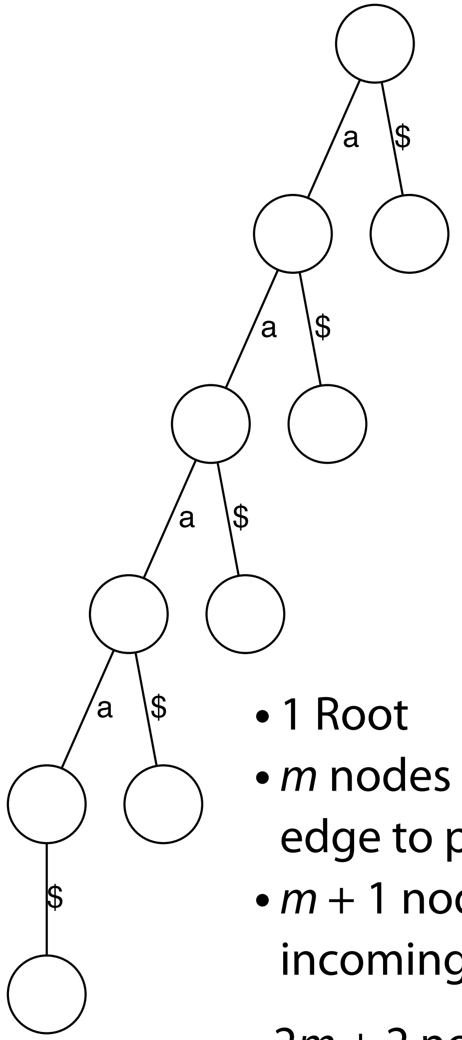
Suffix Trie

How does the suffix trie grow with | *T* | = *m* ?

Is there a class of string where the number of suffix trie nodes grows linearly with m?

Yes: a string of *m* a's in a row (a^m)

T = aaaa$



- 1 Root
- *m* nodes with "a" edge to parent
- *m* + 1 nodes with incoming **$** edge

2*m* + 2 nodes

Suffix Trie

How does the suffix trie grow with $|T| = m$?

Is there a class of string where the number of suffix trie nodes grows with m^2?

Yes: $a^n b^n$ where $2n = m$

- 1 root
- n nodes along "b chain," right
- n nodes along "a chain," middle
- n chains of n "b" nodes hanging off "a chain" (n^2 total)
- $2n + 1$ **\$** leaves (not shown)

$n^2 + 4n + 2$ nodes, where $m = 2n$

T = aaabbb\$

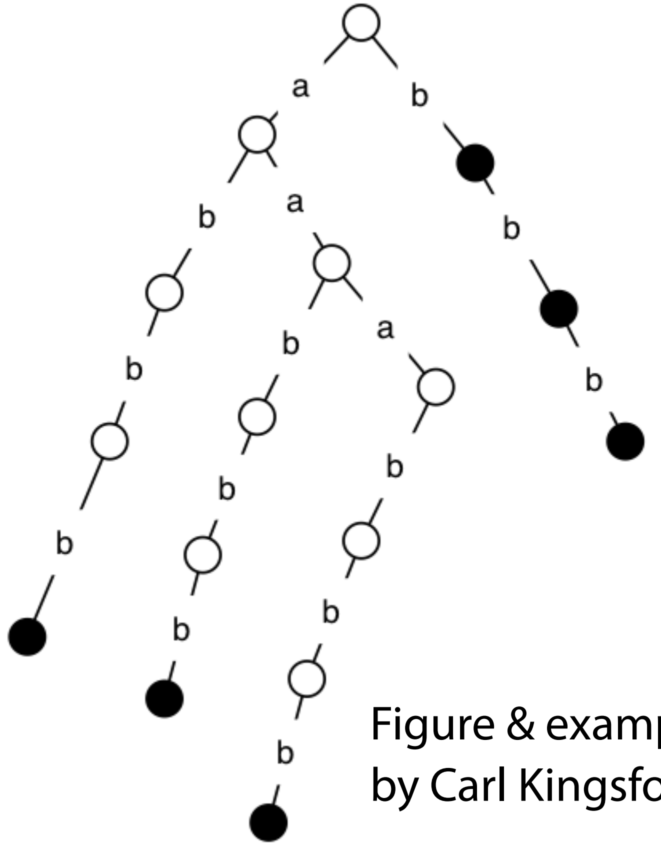


Figure & example by Carl Kingsford

Suffix trie: actual growth

Built suffix tries for the first 500 prefixes of a virus genome

Black curve shows how # nodes increases with prefix length

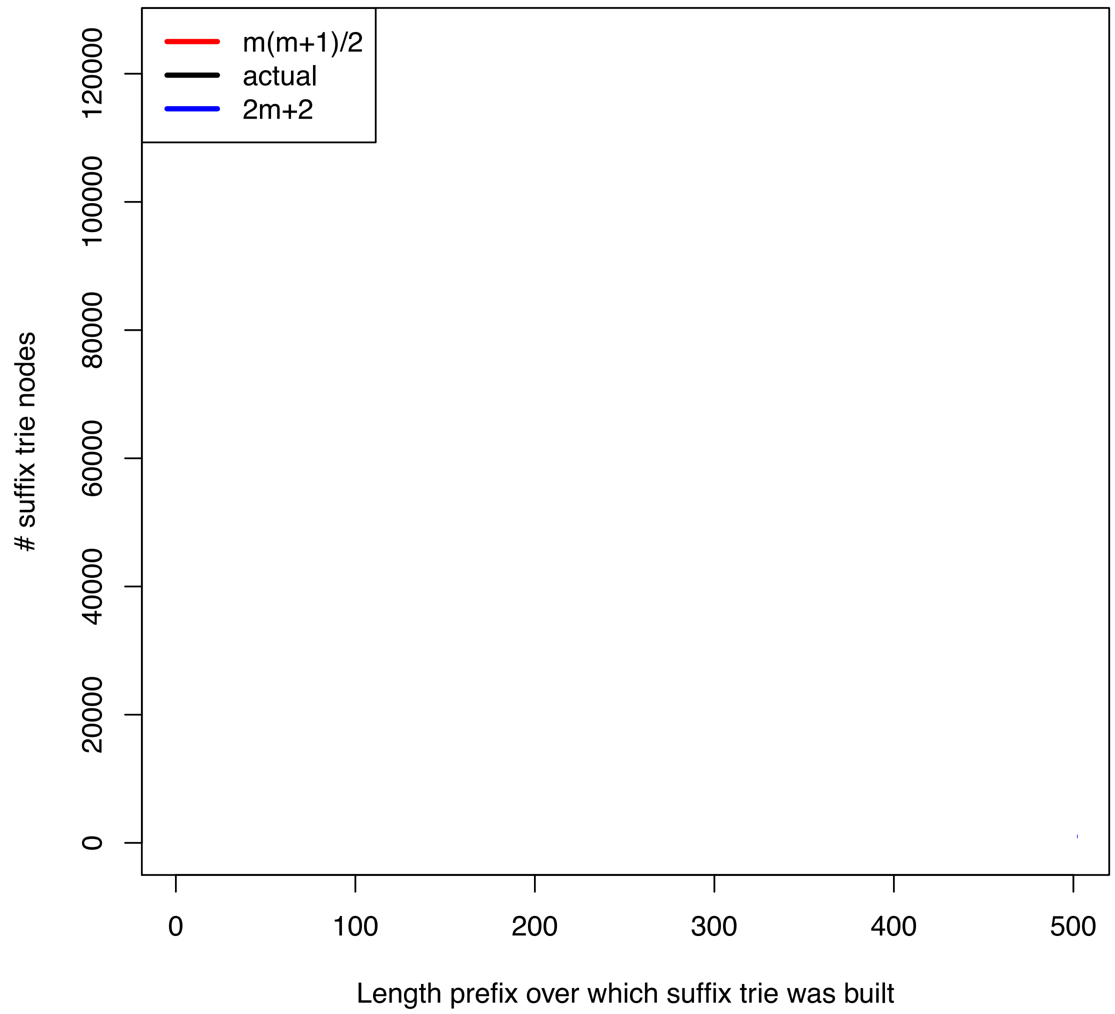


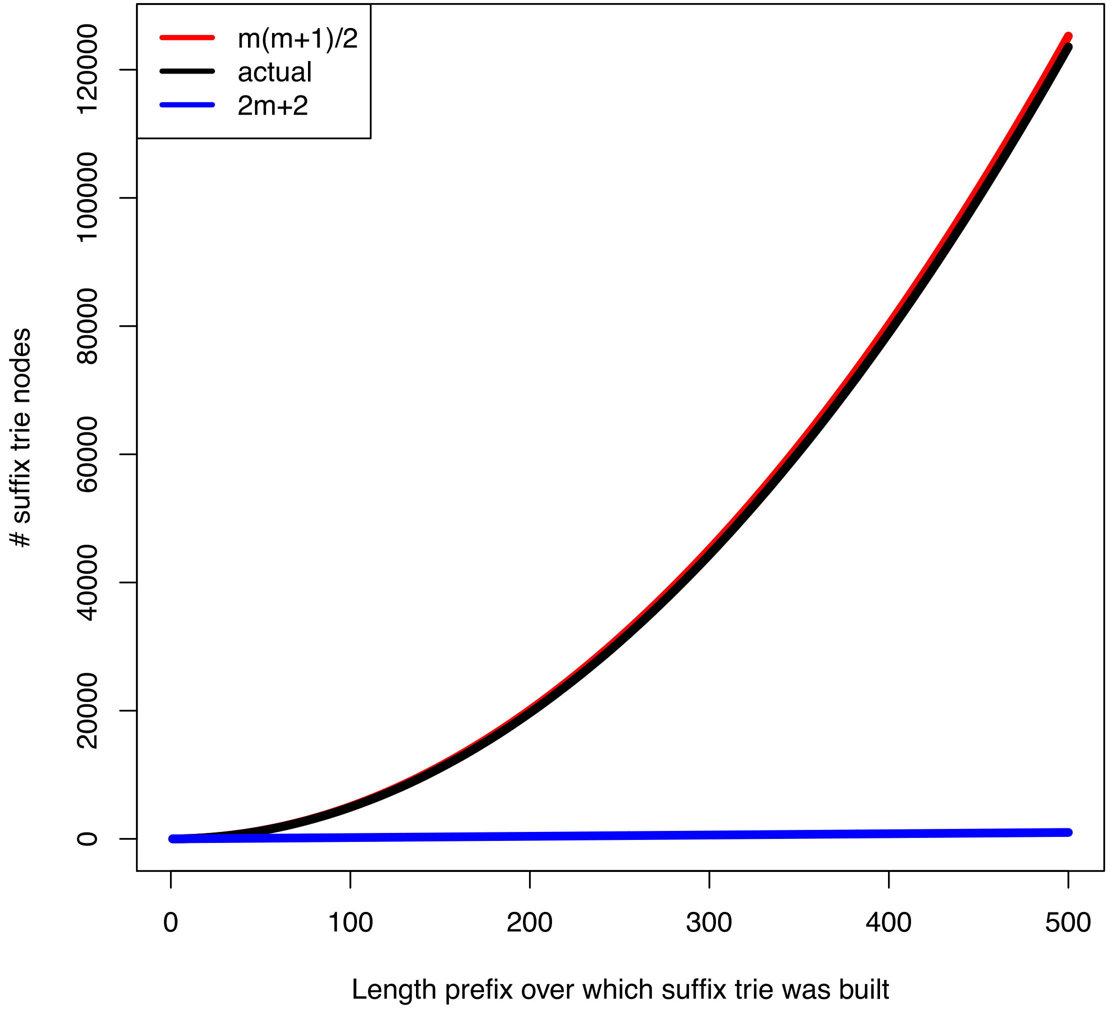
Figure & example by Ben Langmead

Suffix trie: actual growth

Built suffix tries for the first 500 prefixes of a virus genome

Black curve shows how # nodes increases with prefix length

Actual growth *much* closer to worst case than to best!



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Assignment 6: a_stree

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Use an existing implementation of a suffix trie as a N-ary Tree

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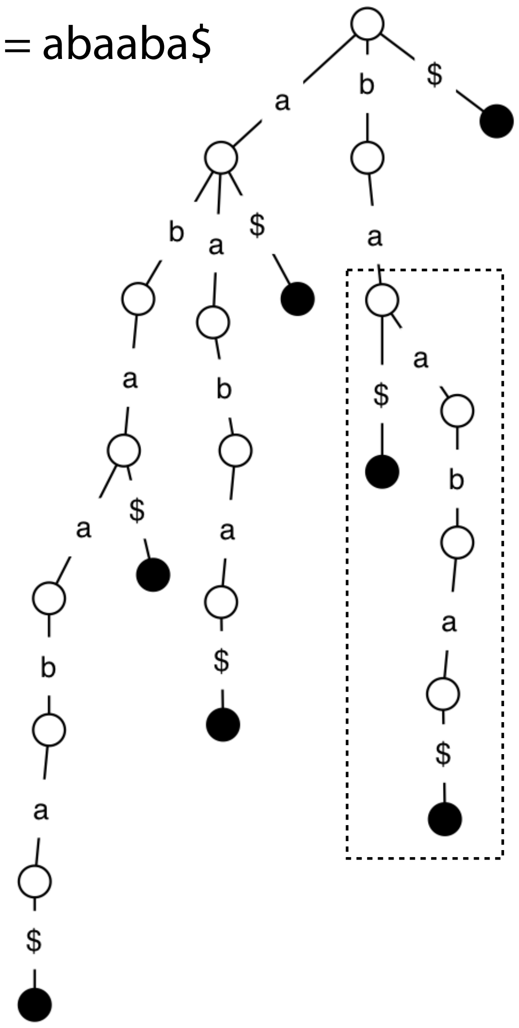
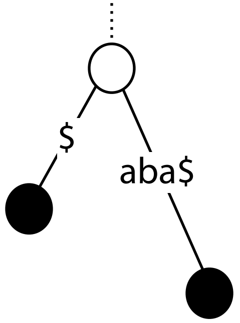
Construct a suffix tree from a suffix trie

Suffix Trie: Making it smaller

T = abaaba$



Idea 1: Coalesce non-branching paths into a *single edge* with a *string* label



Reduces # nodes, edges,
guarantees non-leaf nodes have >1 child

Suffix Trie: Making it smaller

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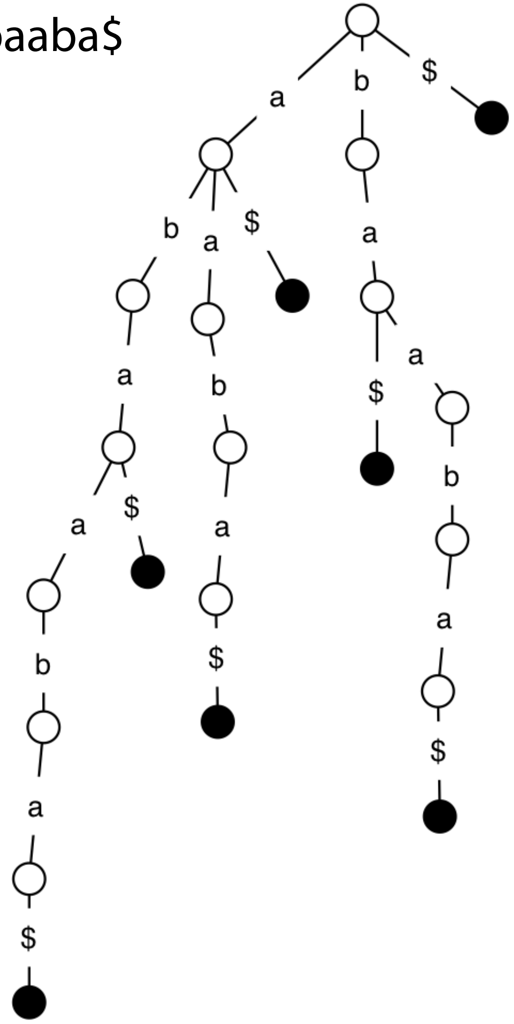

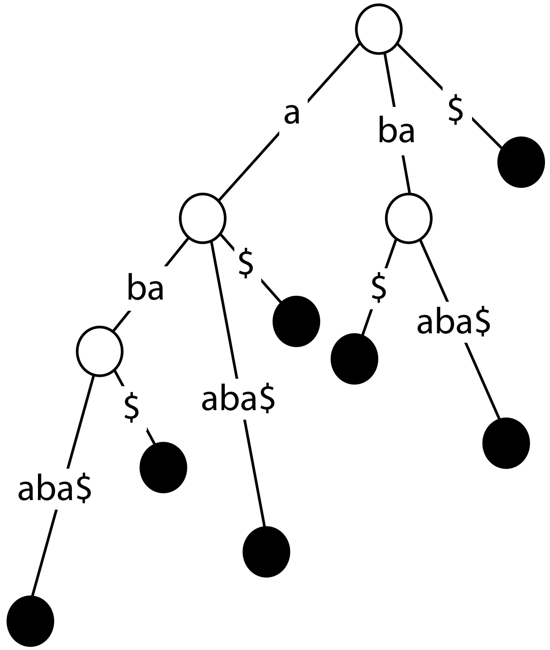

Idea 1: Coalesce non-branching paths into a *single edge* with a *string* label

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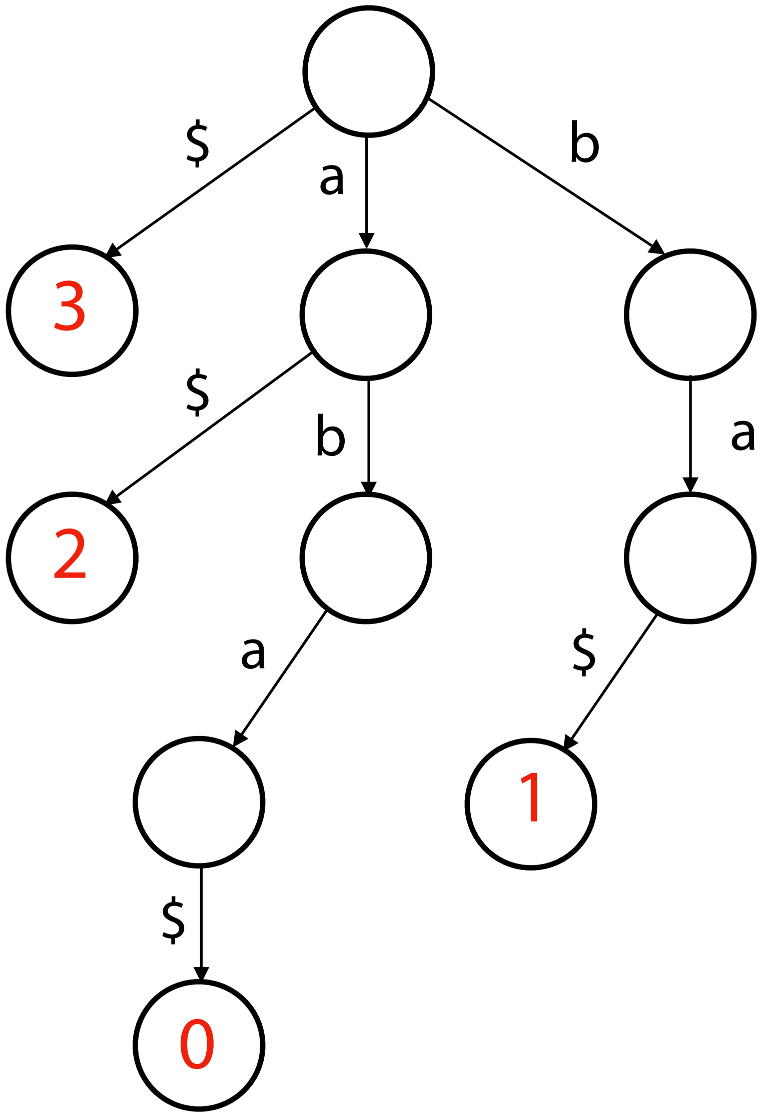
Coalescing edges

We want to coalesce paths that don’t branch.



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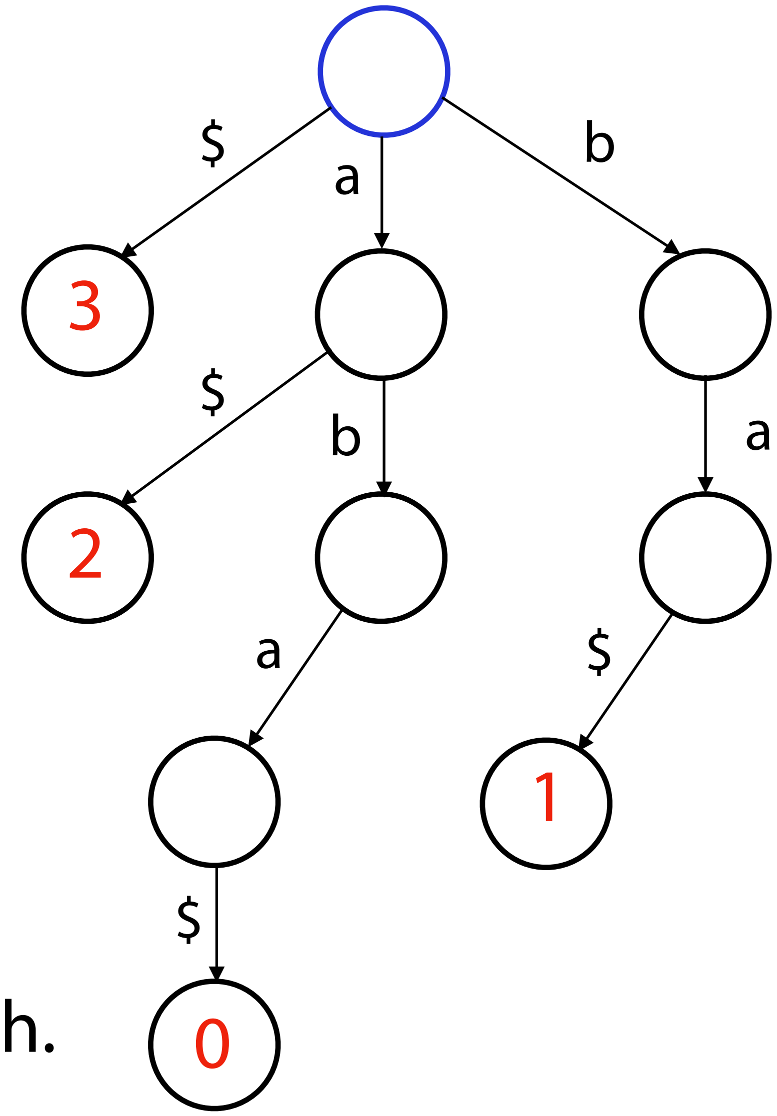
‘Current root’ in blue

Coalesce ‘$’? **No**, nothing to merge

Coalesce ‘a’? **No**, child has a branch

Coalesce ‘b’? **Yes**, b->a is the only path.



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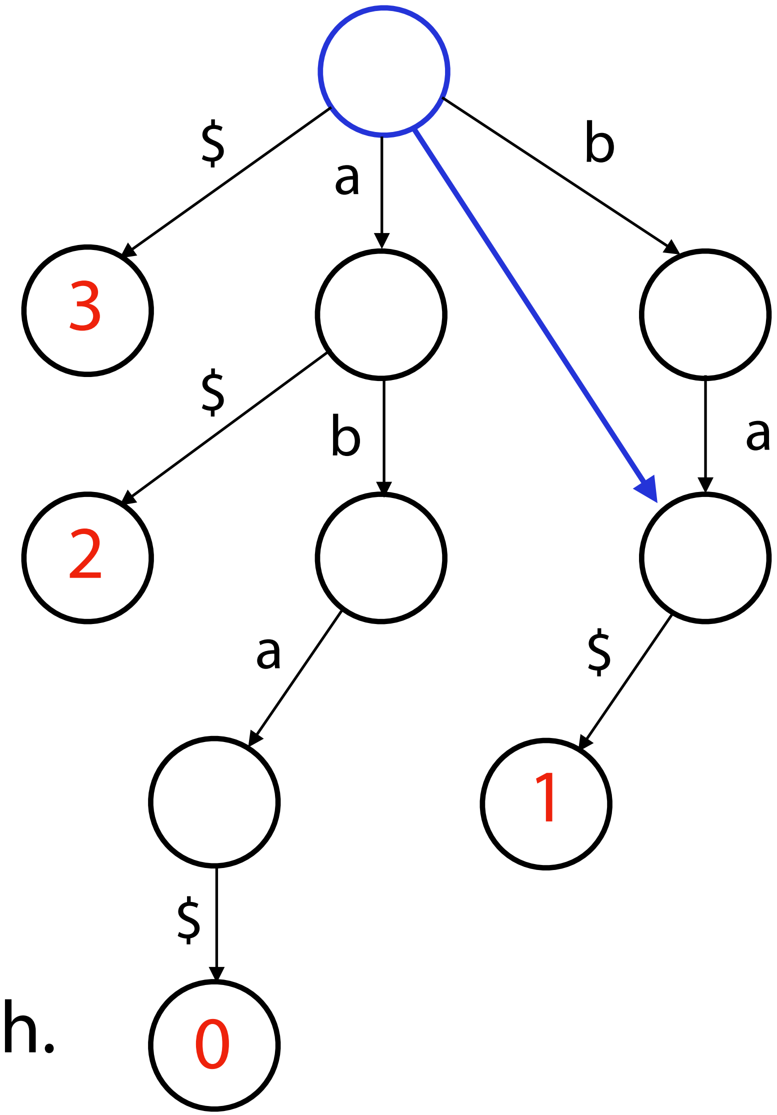

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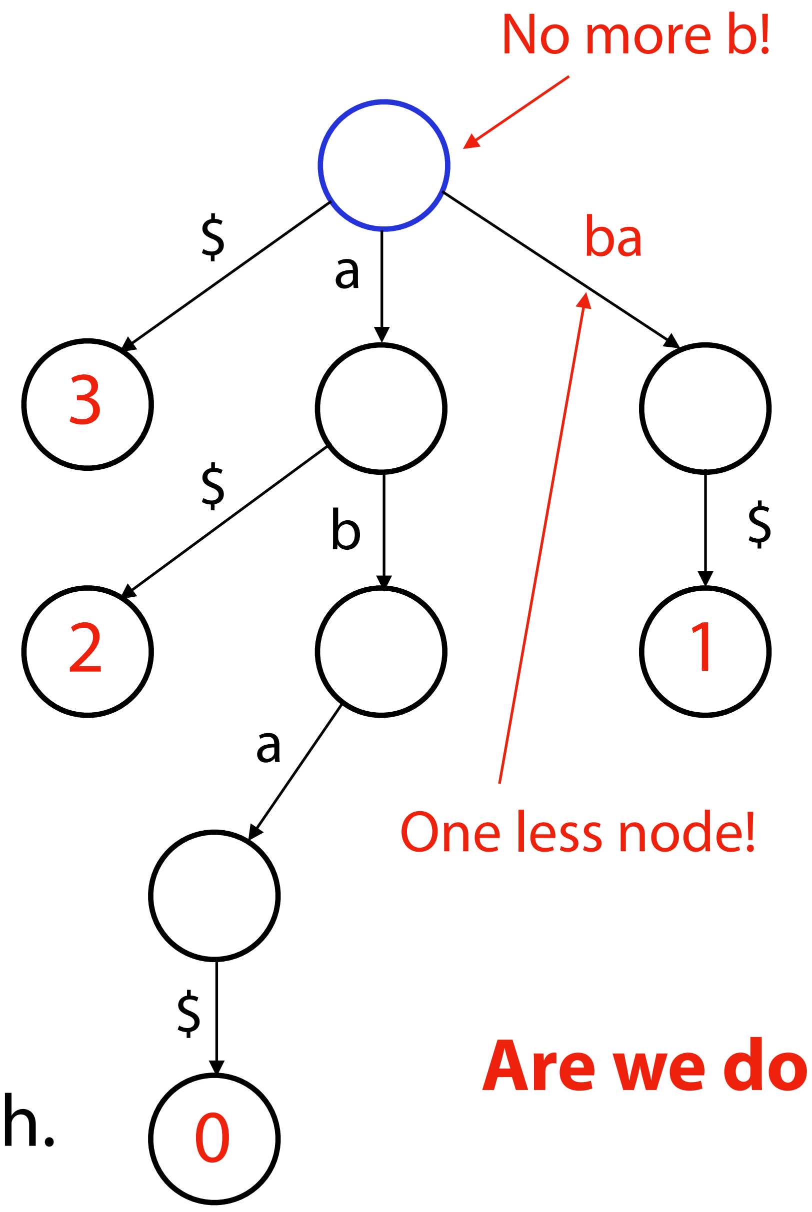

Are we done?

Coalescing edges

We want to coalesce paths that don't branch.

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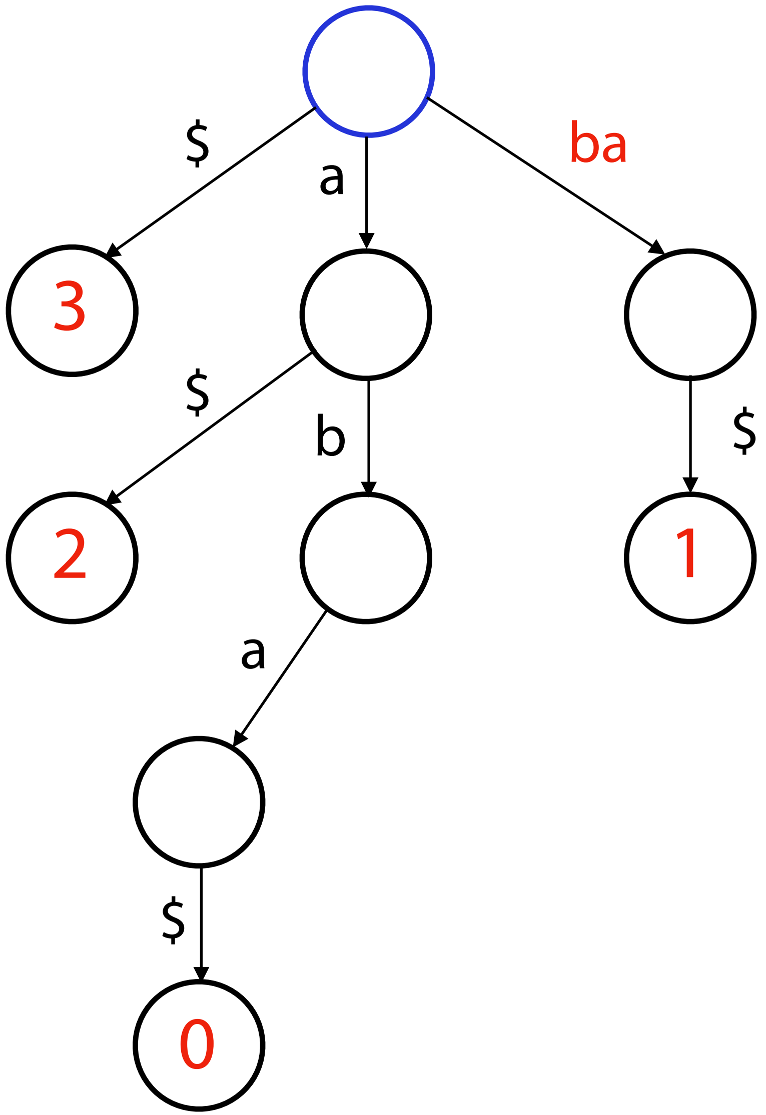
We added a new edge 'ba'!

We might need to coalesce again!



Coalescing edges

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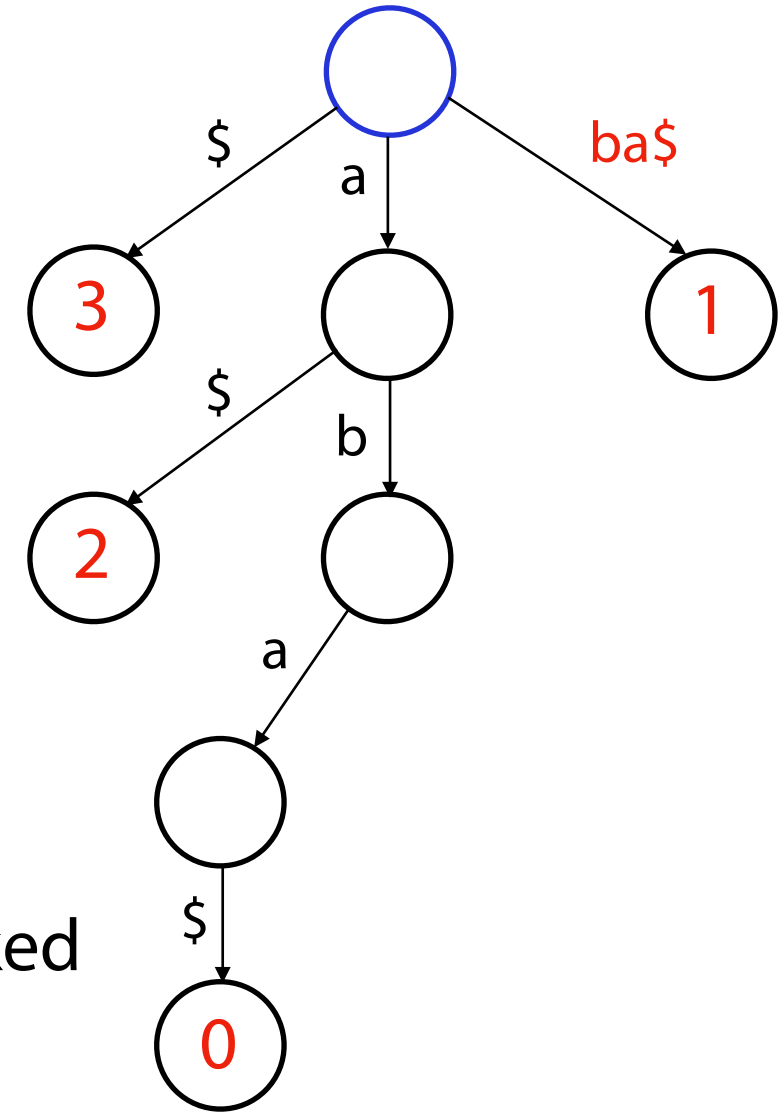
Repeat until ***all*** edges have been checked



Coalescing Edges

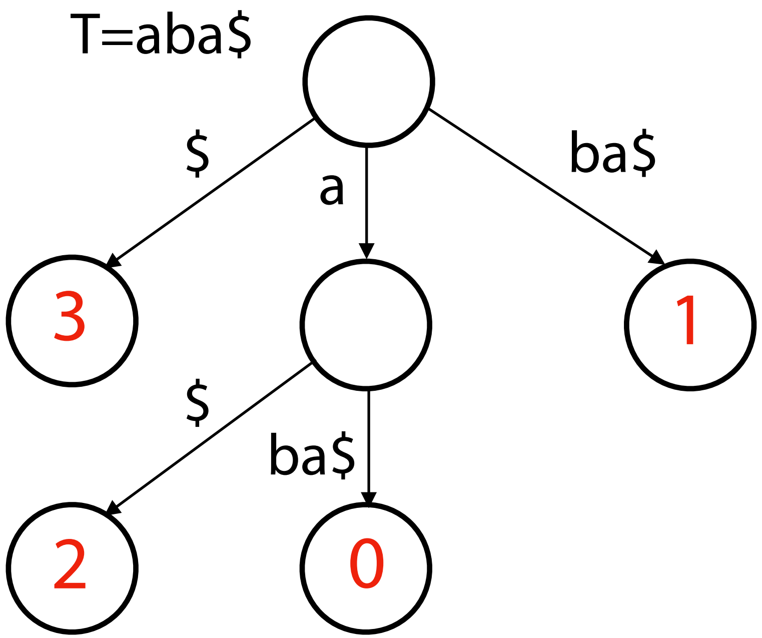
Coalesce all paths that don't branch.

A suffix tree of $|T| = m$ should have:

m leaves* (including '$' in T)

$\leq m - 1$ internal nodes

Each internal node ≥ 2 children

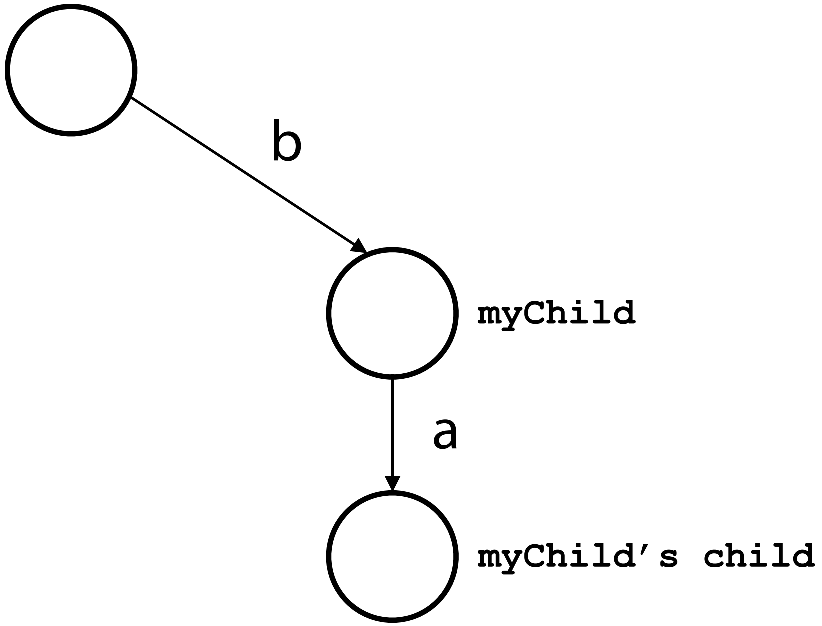


How to do this is up to you (and for you to work out)!

Iterator on Dynamic Data

stree.cpp

```
map<string,Node*>::iterator it = myMap.begin();

while(it != myMap.end()){

        Node* myChild = it->second;

        if < LOGIC STATEMENT >{
            Node* temp = < myChild's child >;

            myMap["NewEdge"] = temp;

            delete myChild;

            myMap.erase(it++);

        }
}
```



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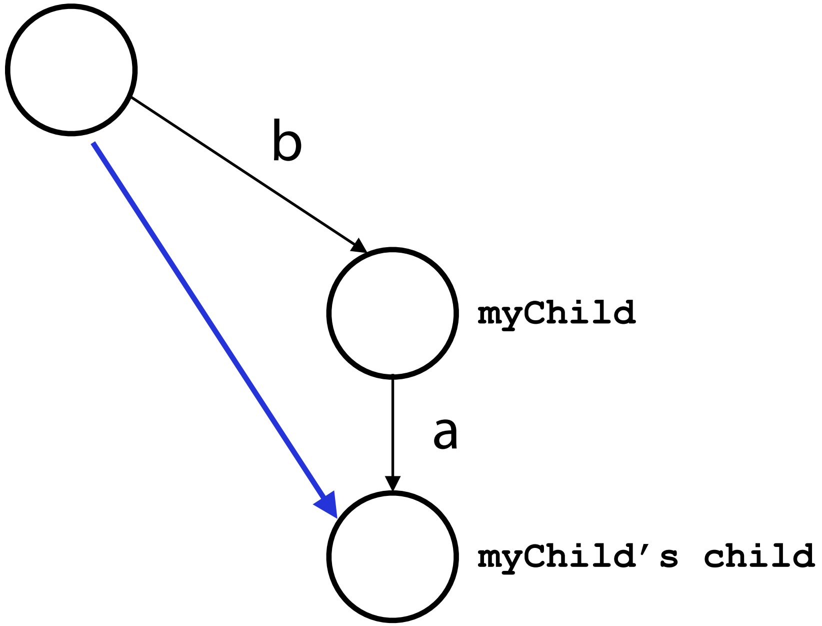

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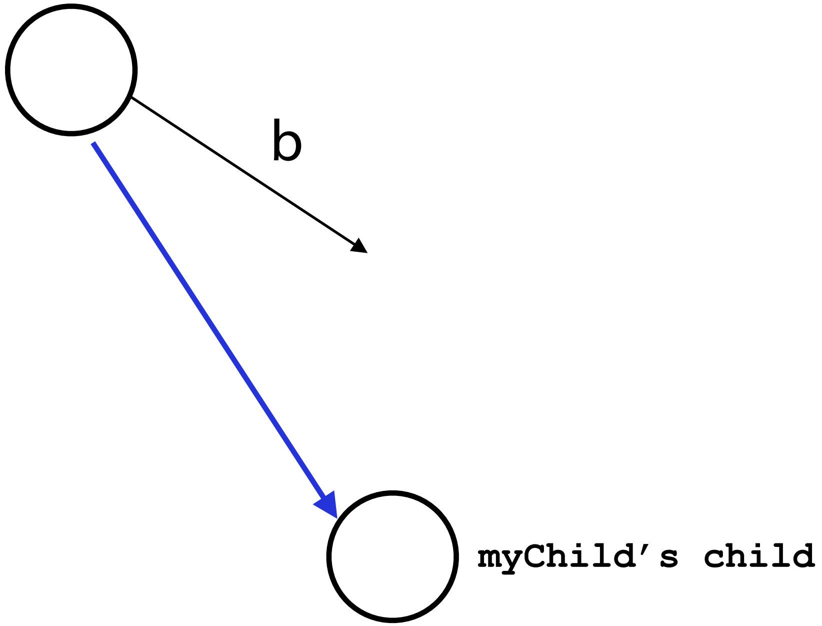

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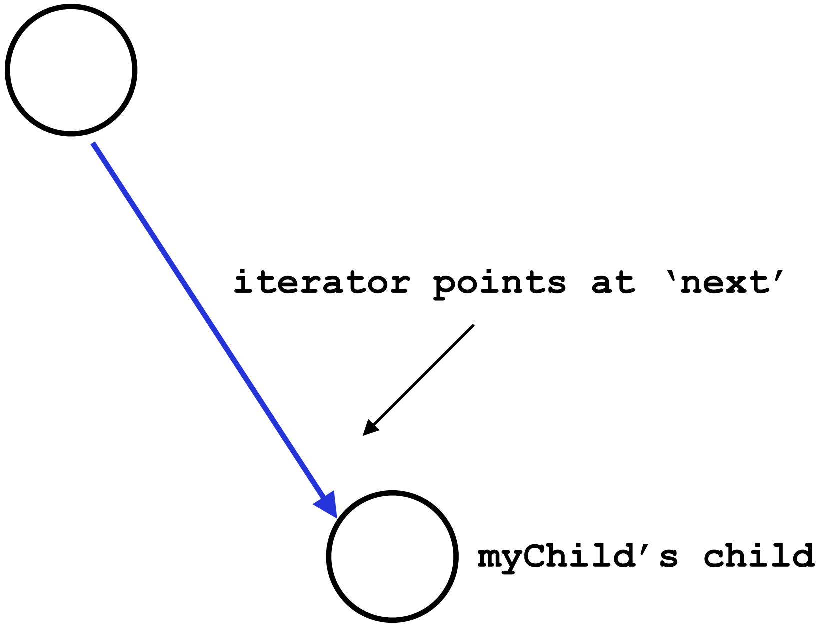

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            delete myChild;

            it = myMap.erase(it);

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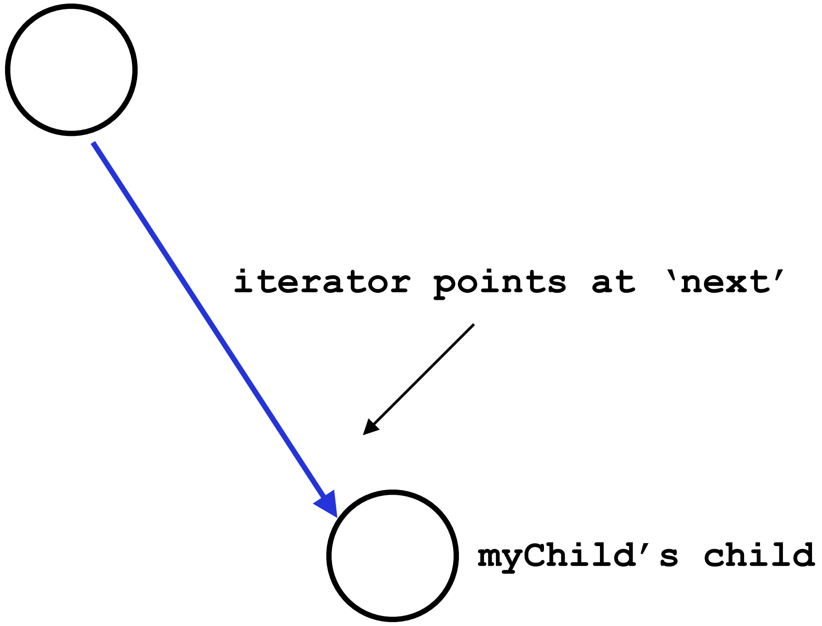

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Implement exact pattern matching using a suffix trie

Construct a suffix tree from a suffix trie

Consider: The modified NaryTree code works for both suffix tries and trees. Can you write a search that works for both trees and tries?

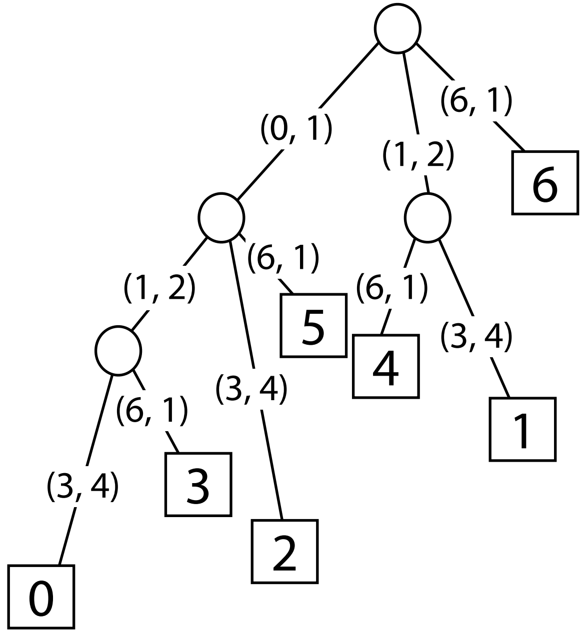
Suffix tree: building

Method 1: build suffix trie, coalesce non-branching paths, relabel edges

$O(m^2)$ time, $O(m^2)$ space

Method 2: build single-edge tree representing longest suffix, augment to include the 2nd-longest, augment to include 3rd-longest, etc (Gusfield 5.4)

$O(m^2)$ time, $O(m)$ space



Algorithmica (1995) 14: 249–260

Algorithmica

On-Line Construction of Suffix Trees[1]

E. Ukkonen[2]

Abstract. An on-line algorithm is presented for constructing the suffix tree for a given string in time linear in the length of the string. The new algorithm has the desirable property of processing the string symbol by symbol from left to right. It always has the suffix tree for the scanned part of the string ready. The method is developed as a linear-time version of a very simple algorithm for (quadratic size) suffix *tries*. Regardless of its quadratic worst case this latter algorithm can be a good practical method when the string is not too long. Another variation of this method is shown to give, in a natural way, the well-known algorithms for constructing suffix automata (DAWGs).

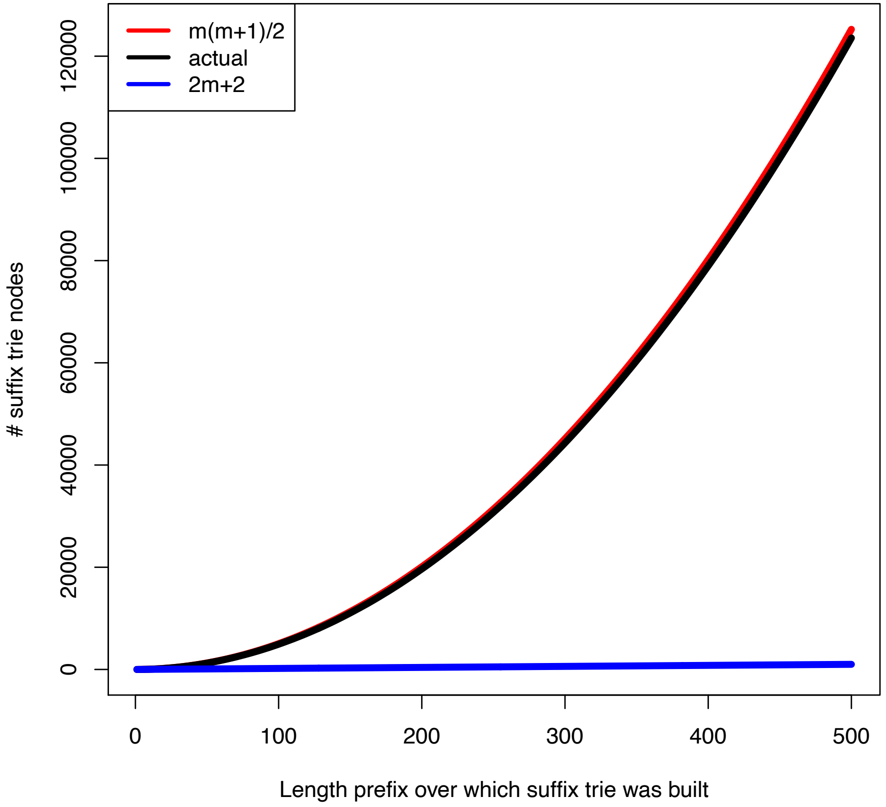
Key Words. Linear-time algorithm, Suffix tree, Suffix trie, Suffix automaton, DAWG.

Canonical algorithm for $O(m)$ time & space suffix tree construction

Won't cover it in class; see Gusfield Ch. 6 for details

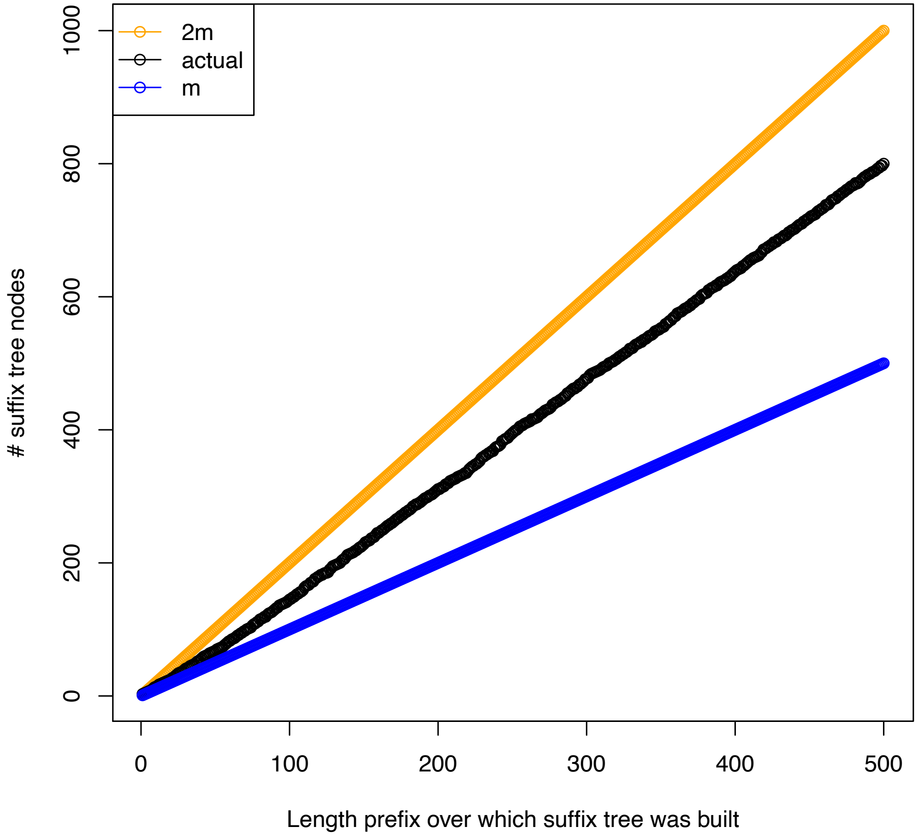
Suffix trie

>100K nodes



Suffix tree

<1K nodes



Suffix Tree

A rooted tree storing a collection of suffixes as (key, value) pairs

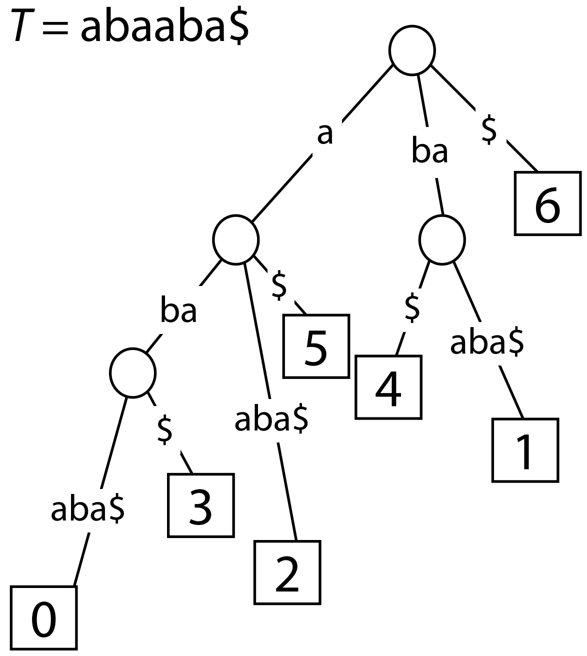
Each key is "spelled out" along some path starting at root

Each edge is labeled with a string *s*

For given node, at most one child edge *starts with character c*, for any $c \in \Sigma$

Each internal node contains >1 children

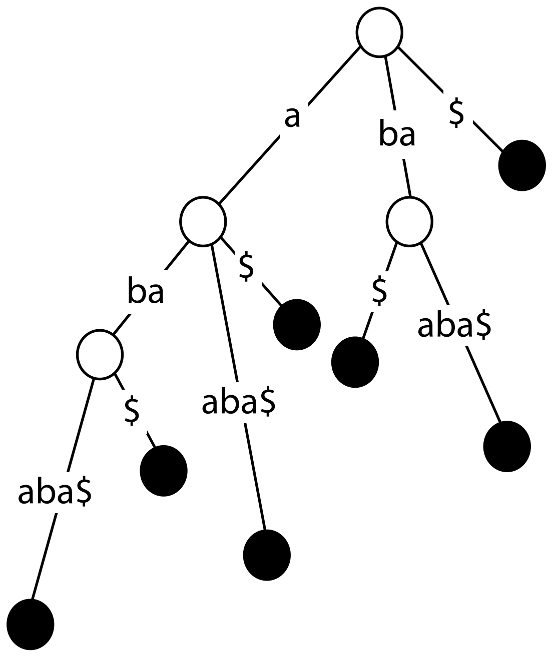
Each key's value is stored at a leaf



Bonus Slides

Suffix Tree: Size Redux

T = abaaba$ |T| = m



leaves?

non-leaf nodes (upper-bound)?

Suffix Tree: Size Redux

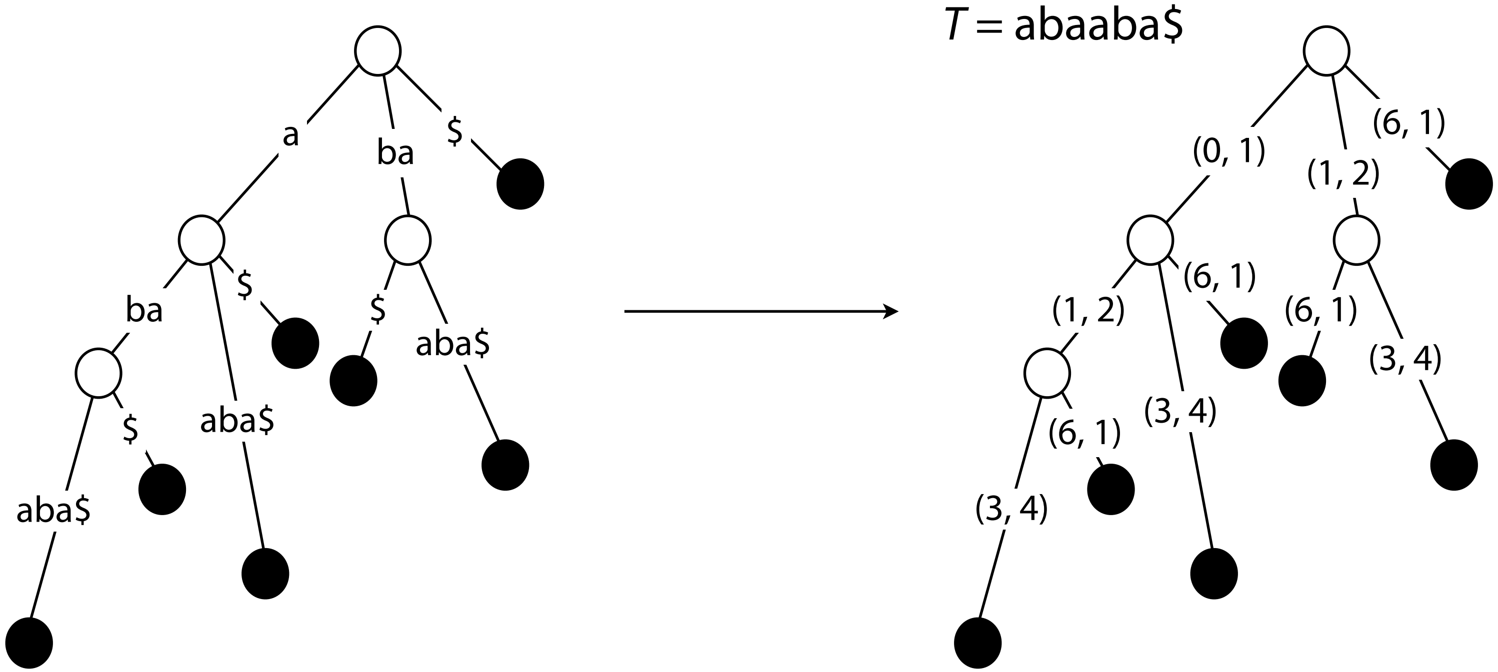
Trie or **tree:** we contain all **suffixes** of a text $|T| = m$

```
T:
GTTATAGCTGATCGCGGCGTAGCGG$
GTTATAGCTGATCGCGGCGTAGCGG$
 TTATAGCTGATCGCGGCGTAGCGG$
  TATAGCTGATCGCGGCGTAGCGG$
   ATAGCTGATCGCGGCGTAGCGG$
    TAGCTGATCGCGGCGTAGCGG$
     AGCTGATCGCGGCGTAGCGG$
      GCTGATCGCGGCGTAGCGG$
       CTGATCGCGGCGTAGCGG$
        TGATCGCGGCGTAGCGG$
         GATCGCGGCGTAGCGG$
          ATCGCGGCGTAGCGG$
           TCGCGGCGTAGCGG$
            CGCGGCGTAGCGG$
             GCGGCGTAGCGG$
              CGGCGTAGCGG$
               GGCGTAGCGG$
                GCGTAGCGG$
                 CGTAGCGG$
                  GTAGCGG$
                   TAGCGG$
                    AGCGG$
                     GCGG$
                      CGG$
                       GG$
                        G$
                         $
```

m(m+1)/2 chars

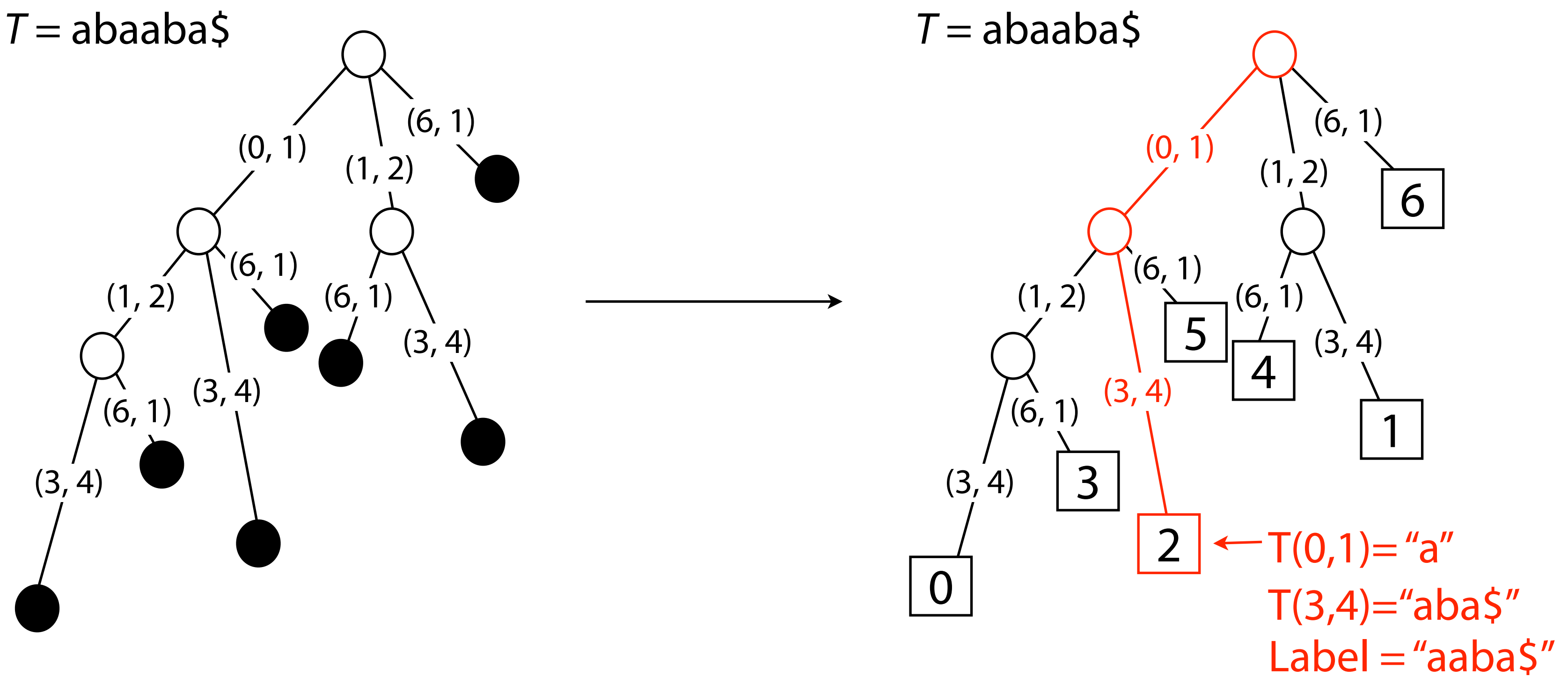
Suffix Tree: Size Redux

Store *T* itself in addition to the tree. Convert tree's edge labels to (index, length) pairs with respect to *T*.



Space is now $O(m)$ Suffix trie was $O(m^2)$!

Suffix Tree: Size Redux



Suffix Tree: Size Redux