# String Algorithms and Data Structures
# Approximate Pattern Matching

CS 199-225
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# Approximate Pattern Matching

**Input:** A text *T*, a pattern *P*, and a distance *d*

**Output:** All positions in *T* where *P* has at most *d* mismatches or edits

*P*: `word`

*T*: `There would have been a time for such a word`

Alignment 1: `word`

Alignment 2: `word`

~~Not a match!~~

Distance 2 match!

Match!

Distance 0 match!

# Hamming Distance

The number of **substitutions** required to turn one string into another

```
T: G G A A A A A G A G G T A G C G G C G T T T A A C A G T A G
                       | | |   | | | | |
P:                     G T A A C G G C G
                             ↑
```

Mismatch
(Substitution)

# Edit Distance

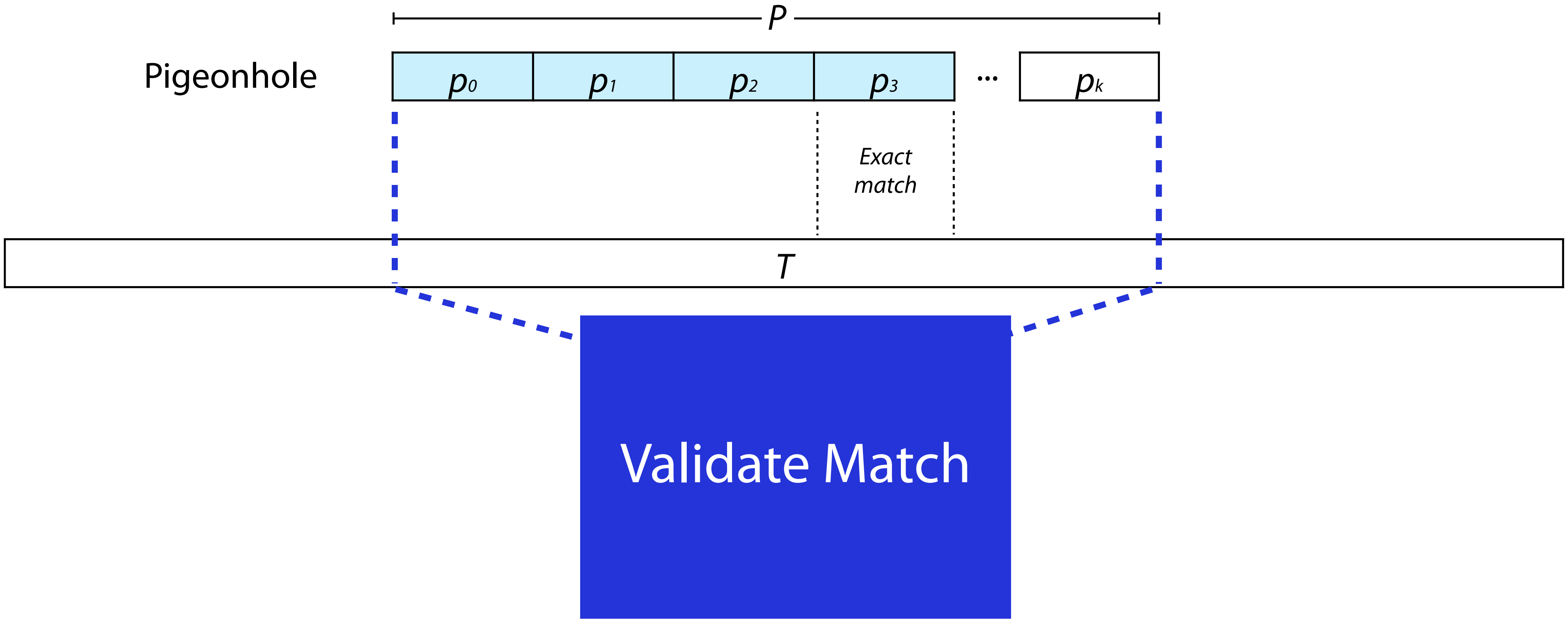
```
Score =  248 bits (129), Expect = 1e-63
Identities = 213/263 (80%), Gaps = 34/263 (12%)
Strand = Plus / Plus
                                                    Substitution
Query: 161 atatcaccacgtcaaaggtgactccaactcca---ccactccatttttgttcagataatgc 217
           |||||||||||||||||||||||||||||  |     | |   || |||||||||||||
Sbjct: 481 atatcaccacgtcaaaggtgactccaact-tattgatagtgttttatgttcagataatgc 539

Query: 218 ccgatgatcatgtcatgcagctccaccgattgtgagaacgacagcgacttccgtcccagc 277
           |||||||   |||||||||||||||||||| || |            ||||||||||||
Sbjct: 540 ccgatgactttgtcatgcagctccaccgattttg-g------------ttccgtcccagc 586
                                                     Deletion
Query: 278 c-gtgcc--aggtgctgcctcagattcaggttatgccgctcaattcgctgcgtatatcgc 334
           |  || |  | |||||||||||||||||||||||||||||||||||||| |||||||||
Sbjct: 587 caatgacgta-gtgctgcctcagattcaggttatgccgctcaattcgctgggtatatcgc 645

Query: 335 ttgctgattacgtgcagctttcccttcaggcggga------------ccagccatccgtc 382
           |||||||||||||||||||||||||||||||||||            |||||||||||||
Sbjct: 646 ttgctgattacgtgcagctttcccttcaggcgggattcatacagcggccagccatccgtc 705
                                                    Insertion
Query: 383 ctccatatc-accacgtcaaagg 404
            |||||||| |||||||||||||
Sbjct: 706 atccatatcaaccacgtcaaagg 728
```

# Approximate Pattern Matching

Find an exact match partition and validate the overall alignment



# Learning Objectives

Review approximate pattern matching

Formalize edit distance storage as an ‘edit string’

Discuss strategies for efficient APM with edits

Introduce dynamic programming

# Edit Distance

Imagine edits are introduced by an *optimal editor* working left-to-right:

*Edit string* summarizes how editor turns *x* into *y:*

```
x:  G C G T A T G C G G C T A A C G C

y:  G C T A T G C G G C T A T A C G C
```

```
x:  G C G T A T G C G G C T A A C G C
    | |
y:  G C - T A T G C G G C T A T A C G C
```

```
x:  G C G T A T G C G G C T A - A C G C
    | |   | | | | | | | | | |
y:  G C - T A T G C G G C T A T A C G C
```

```
x:  G C G T A T G C G G C T A - A C G C
    | |   | | | | | | | | | |   | | | |
y:  G C - T A T G C G G C T A T A C G C
```

Operations:
**M** = match, **R** = replace (substitute),
**I** = insert into *x*, **D** = delete from *x*

**MMD**

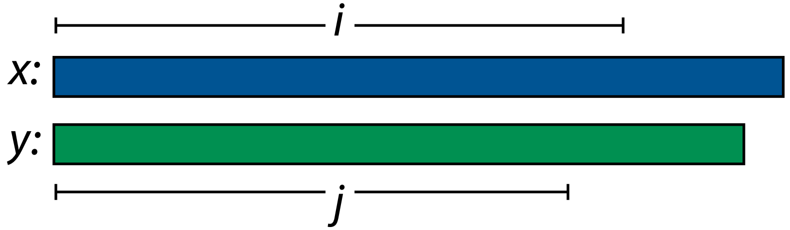
Reminder: this is **D**, not **I**, because we have to delete a character from *x* to make it more like *y*

**MMDMMMMMMMMMMI**

**MMDMMMMMMMMMMIMMMM**

# Edit Distance

D[$i$, $j$]: edit distance between length-$i$ prefix of $x$ and length-$j$ prefix of $y$




Optimal edit string for D[$i$, $j$] is built by ***extending a shorter optimal string by 1 operation***. 3 options:

Append **D** to transcript for D[$i$-1, $j$]

Append **I** to transcript for D[$i$, $j$-1]

Append **M** or **R** to transcript for D[$i$-1, $j$-1]

We choose based on whichever option has the fewest edits

# Edit Distance

X: GTTTAA Y: GGTTTA

D[*5*, *6*]
GTTTA
GGTTTA

D[*5*, *5*]
GTTTA
GGTTT

D[*6*, *5*]
GTTTAA
GGTTT

GTTTAA
GGTTTA
D[*6*, *6*]

# Edit Distance

X: GTTTAA        Y: GGTTTA

D[*5*, *6*]
```
GTTTA
GGTTTA
```

D[*5*, *5*]
```
GTTTA
GGTTT
```

D[*6*, *5*]
```
GTTTAA
GGTTT
```

```
A
-
```

```
A
A
```

```
-
A
```

```
GTTTAA
GGTTTA
```
D[*6*, *6*]

# Edit Distance

X: GTTTAA Y: GGTTTA

MIMMMM

| | | | | | | |
|---|---|---|---|---|---|---|
| D[5, 6] | G | - | T | T | T | A |
| | G | G | T | T | T | A |

MRMMR

| | | | | | |
|---|---|---|---|---|---|
| D[5, 5] | G | T | T | T | A |
| | G | G | T | T | T |

MRMMRD

| | | | | | | |
|---|---|---|---|---|---|---|
| D[6, 5] | G | T | T | T | A | A |
| | G | G | T | T | T | - |

# Edit Distance

X: GTTTAA Y: GGTTTA

MIMMMM

D[5, 6]

| | | | | | |
|---|---|---|---|---|---|
| G | - | T | T | T | A |
| G | G | T | T | T | A |

| | | | | | | |
|---|---|---|---|---|---|---|
| G | - | T | T | T | A | |
| G | G | T | T | T | A | |

$D[6, 6]^1$

MRMMR

D[5, 5]

| | | | | |
|---|---|---|---|---|
| G | T | T | T | A |
| G | G | T | T | T |

| | | | | | |
|---|---|---|---|---|---|
| G | T | T | T | A | |
| G | G | T | T | T | |

$D[6, 6]^2$

MRMMRD

D[6, 5]

| | | | | | |
|---|---|---|---|---|---|
| G | T | T | T | A | A |
| G | G | T | T | T | - |

| | | | | | | |
|---|---|---|---|---|---|---|
| G | T | T | T | A | A | |
| G | G | T | T | T | - | |

$D[6, 6]^3$

# Edit Distance

X: GTTTAA Y: GGTTTA

MIMMMM

D[5, 6]

| | | | | | |
|---|---|---|---|---|---|
| G | - | T | T | T | A |
| G | G | T | T | T | A |

MIMMMMD

| | | | | | | |
|---|---|---|---|---|---|---|
| G | - | T | T | T | A | A |
| G | G | T | T | T | A | - |

D[6, 6][1]

MRMMR

D[5, 5]

| | | | | |
|---|---|---|---|---|
| G | T | T | T | A |
| G | G | T | T | T |

MRMMRM

| | | | | | |
|---|---|---|---|---|---|
| G | T | T | T | A | A |
| G | G | T | T | T | A |

D[6, 6][2]

MRMMRD

D[6, 5]

| | | | | | |
|---|---|---|---|---|---|
| G | T | T | T | A | A |
| G | G | T | T | T | - |

MRMMRDI

| | | | | | | |
|---|---|---|---|---|---|---|
| G | T | T | T | A | A | - |
| G | G | T | T | T | - | A |

D[6, 6][3]

# Edit Distance

X: ACCT

Y: AGCC

D[*3*, *4*]

| | | | | |
|---|---|---|---|---|
| A | - | C | C | |
| A | G | C | C | |

MIMM

D[*3*, *3*]

| | | | |
|---|---|---|---|
| A | C | C | |
| A | G | C | |

MRM

D[*4*, *3*]

| | | | | |
|---|---|---|---|---|
| A | C | C | T | |
| A | G | C | - | |

MRMD

D[4,4] extends one of these options

D[*4*, *4*]

# Edit Distance

X: ACCT Y: AGCC

D[*3*, *4*]

| | | | | |
|---|---|---|---|---|
| A | - | C | C | T |
| A | G | C | C | - |

MIMMD

D[*3*, *3*]

| | | | |
|---|---|---|---|
| A | C | C | T |
| A | G | C | C |

MRMR

D[*4*, *3*]

| | | | | |
|---|---|---|---|---|
| A | C | C | T | - |
| A | G | C | - | C |

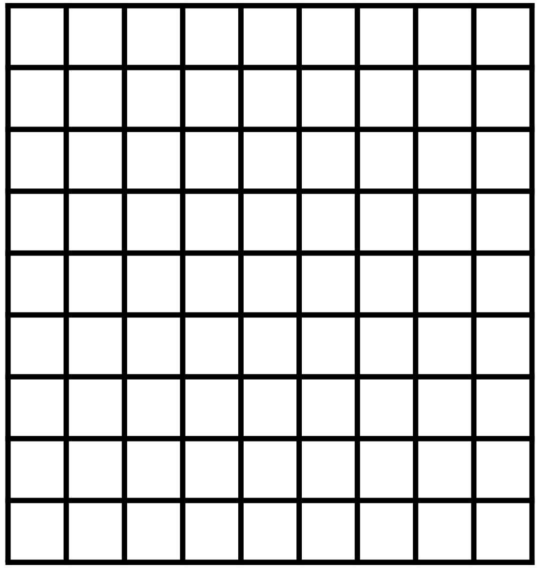
MRMDI

D[4,4] extends one of these options

D[*4*, *4*]

# Edit Distance

We can store $D$ as a 2D matrix:



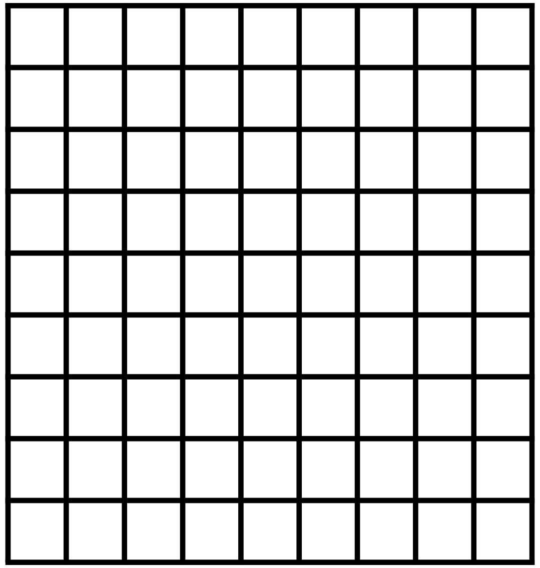
Let $\overbrace{D[0,j] = j}^{\text{Is at beginning}}$, and let $\overbrace{D[i,0] = i}^{\text{Ds at beginning}}$

```
----BBBBBBB
    |||||||
AAAABBBBBBB
```

```
the longest----
    |||||||
----longest day
```

# Edit Distance

We can store $D$ as a 2D matrix:



Let $\overbrace{D[0,j] = j}^{\texttt{I}\text{s at beginning}}$, and let $\overbrace{D[i,0] = i}^{\texttt{D}\text{s at beginning}}$

Otherwise, let $D[i,j] = \min \begin{cases} D[i-1,j] + 1 & \text{vertical } (\texttt{D}) \\ D[i,j-1] + 1 & \text{horizontal } (\texttt{I}) \\ D[i-1,j-1] + \delta(x[i-1], y[j-1]) & \end{cases}$

diagonal (**M** or **R**)

$\delta(a,b)$ is 0 if $a = b$, 1 otherwise

# Edit Distance

**editRecursive**(“ABC”, “BBC”)

(“ABC”, “BB”) (“AB”, “BB”) (“AB”, “BBC”)

(“ABC”, “B”) (“AB”, “B”) (“AB”, “BB”)

*(only part of recursion tree shown)*

# Edit Distance

**editRecursive**(“ABC”, “BBC”)

(“ABC”, “BB”) (“AB”, “BB”) (“AB”, “BBC”)

(“ABC”, “B”) (“AB”, “B”) (“AB”, “BB”)

*(only part of recursion tree shown)*

**editRecursive**(“Shakespeare”, “shake spear”)

**Calculate (“Shake”, “shake”) 8989 times**

How can we address this problem?

# Edit Distance

```
editRecursive("ABC", "BBC")

("ABC", "BB")   ("AB", "BB")   ("AB", "BBC")

("ABC", "B")   ("AB", "B")   ("AB", "BB")
```

**Memoization:** Top-down

**Dynamic Programming:** Bottom-up

**Both:** Solve individual sub-problems once

# Edit Distance: dynamic programming

ϵ is empty string

$y$

D: $x$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | | | | | | | | | | | | | |
| G | | | | | | | | | | | | | |
| C | | | | | | | | | | | | | |
| G | | | | | | | | | | | | | |
| T | | | | | | | | | | | | | |
| A | | | | | | | | | | | | | |
| T | | | | | | | | | | | | | |
| G | | | | | | | | | | | | | |
| C | | | | | | | | | | | | | |
| A | | | | | | | | | | | | | |
| C | | | | | | | | | | | | | |
| G | | | | | | | | | | | | | |
| C | | | | | | | | | | | | | |

Let $n = |x|$, $m = |y|$

D: $(n+1)$ x $(m+1)$ matrix

D[$i$, $j$] = edit distance b/t length-$i$ prefix of $x$ and length-$j$ prefix of $y$

# Edit Distance: dynamic programming

Let $D[0, j] = j$, and let $D[i, 0] = i$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | | | A | | | | | | | | | | |
| G | | | | | | | | | | | | | |
| C | | | | | | | | | | | | | |
| G | | | | | | | | | | | | | |
| T | | | | | | | | | | | | | |
| A | | | | | | | | | | | | | |
| T | | | | | | | | | | | | | |
| G | | | | | | | | | | | | | |
| C | | | | | | | | | | | | | |
| A | | | | | | | | | | | | | |
| C | | | | | | | | | | | | | |
| G | | | | | | | | | | | | | |
| C | | | | | | | | | | | | | |

What is A?

D[$i$, $j$] = edit distance b/t length-$i$ prefix of $x$ and length-$j$ prefix of $y$

# Edit Distance: dynamic programming

Let $D[0, j] = j$, and let $D[i, 0] = i$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | | | 2 | | | | | | | | | | |
| G | | | | | | | | | | | | | |
| C | | | | | | | | | | | | | |
| G | | | | | | | | | | | | | |
| T | | | | | | | | | | | | | |
| A | | | | | | | | | | | | | |
| T | | | | | | | | | | | | | |
| G | | | | | | | | | | | | | |
| C | | | | | | | | | | | | | |
| A | | | | | | | | | | | | | |
| C | | | | | | | | | | | | | |
| G | | | | | | | | | | | | | |
| C | | | | | | | | | | | | | |

What is A?

# Edit Distance: dynamic programming

Let $D[0, j] = j$, and let $D[i, 0] = i$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | | | | | | | | | | | | |
| C | 2 | | | | | | | | | | | | |
| G | 3 | | | | | | | | | | | | |
| T | 4 | | | | | | | | | | | | |
| A | 5 | | | | | | | | | | | | |
| T | 6 | | | | | | | | | | | | |
| G | 7 | | | | | | | | | | | | |
| C | 8 | | | | | | | | | | | | |
| A | 9 | | | | | | | | | | | | |
| C | 10 | | | | | | | | | | | | |
| G | 11 | | | | | | | | | | | | |
| C | 12 | | | | | | | | | | | | |

# Edit Distance: dynamic programming

Let $D[0, j] = j$, and let $D[i, 0] = i$

| | ε | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ε | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | | | | | | | | | | | | |
| C | 2 | | | | | | | | | | | | |
| G | 3 | | | | | | | | | | | | |
| T | 4 | | | | | | | | | | | | |
| A | 5 | | | | | | | | | | | | |
| T | 6 | | | | | | | | | | | | |
| G | 7 | | | | | | | | | | | | |
| C | 8 | | | | | | | | | | | | |
| A | 9 | | | | | | | | | | | | |
| C | 10 | | | | | | | | | | | | |
| G | 11 | | | | | | | | | | | | |
| C | 12 | | | | | | | | | | | | |

```
- - -
G C T
```

# Edit Distance: dynamic programming

Let $D[0, j] = j$, and let $D[i, 0] = i$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | | | | | | | | | | | | |
| C | 2 | | | | | | | | | | | | |
| G | 3 | | | | | | | | | | | | |
| T | 4 | | | | | | | | | | | | |
| A | 5 | | | | | | | | | | | | |
| T | 6 | | | | | | | | | | | | |
| G | 7 | | | | | | | | | | | | |
| C | 8 | | | | | | | | | | | | |
| A | 9 | | | | | | | | | | | | |
| C | 10 | | | | | | | | | | | | |
| G | 11 | | | | | | | | | | | | |
| C | 12 | | | | | | | | | | | | |

GCGTAT

------

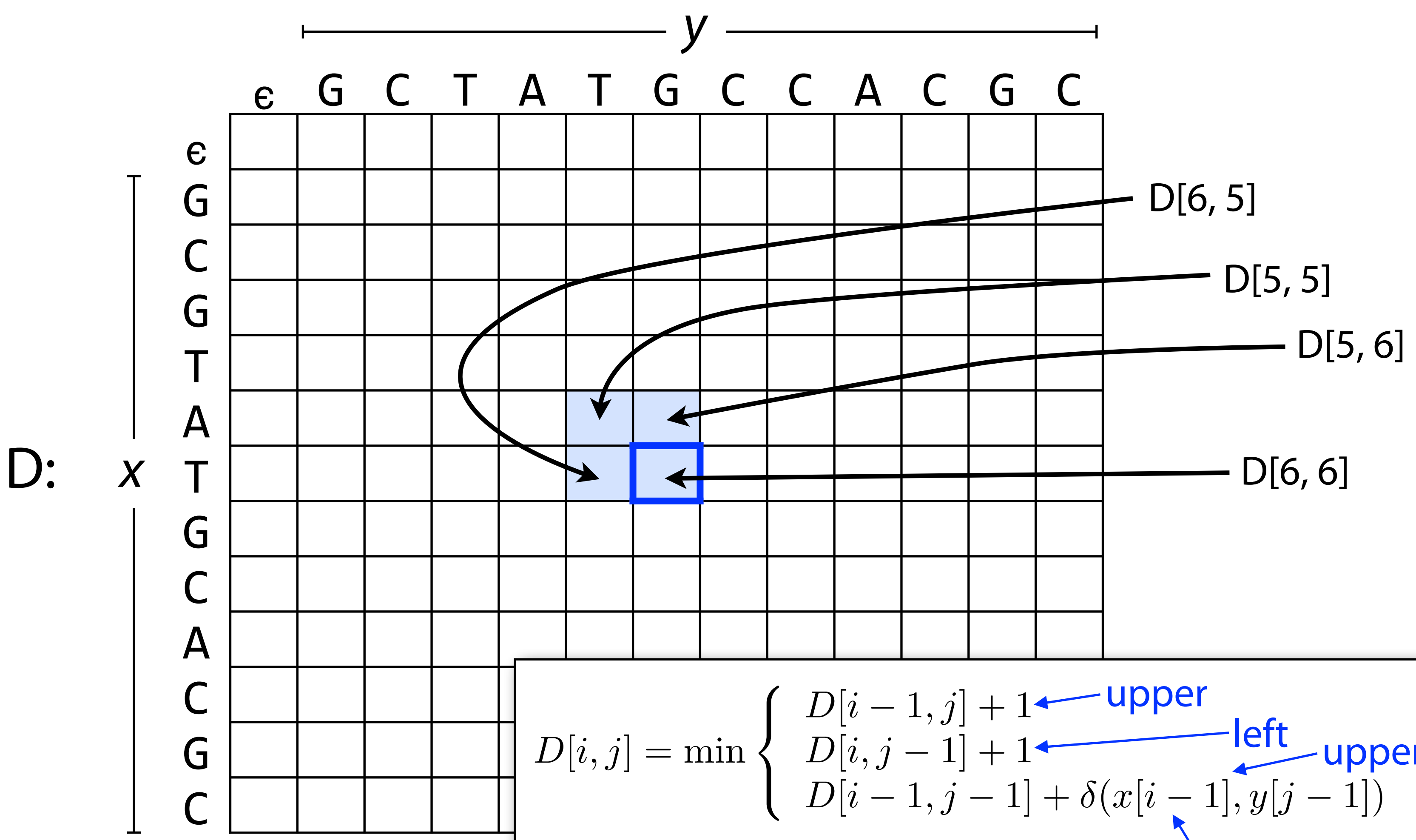
# Edit Distance: dynamic programming

Let $D[0, j] = j$, and let $D[i, 0] = i$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | | | | | | | | | | | | |
| C | 2 | | | | | | | | | | | | |
| G | 3 | | | | | | | | | | | | |
| T | 4 | | | | | | | | | | | | |
| A | 5 | | | | | | | | | | | | |
| T | 6 | | | | | | | | | | | | |
| G | 7 | | | | | | | | | | | | |
| C | 8 | | | | | | | | | | | | |
| A | 9 | | | | | | | | | | | | |
| C | 10 | | | | | | | | | | | | |
| G | 11 | | | | | | | | | | | | |
| C | 12 | | | | | | | | | | | | |

*D* is one row / column larger than x / y!

# Edit Distance: dynamic programming




$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

upper

left

upper-left

Cell depends upon its upper, left, and upper-left neighbors

*i*, *j* are D-based

D[0] is 'empty string'

# Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | | | | | | | | | | | | |
| C | 2 | | | | | | | | | | | | |
| G | 3 | | | | | | | | | | | | |
| T | 4 | | | | | | | | | | | | |
| A | 5 | | | | | | **etc** | | | | | | |
| T | 6 | | | | | | | | | | | | |
| G | 7 | | | | | | | | | | | | |
| C | 8 | | | | | | | | | | | | |
| A | 9 | | | | | | | | | | | | |
| C | 10 | | | | | | | | | | | | |
| G | 11 | | | | | | | | | | | | |
| C | 12 | | | | | | | | | | | | |

Fill remaining cells from top row to bottom and from left to right

# Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | ? | | | | | | | | | | | |
| C | 2 | | | | | | | | | | | | |
| G | 3 | | | | | | | | | | | | |
| T | 4 | | | | | | | | | | | | |
| A | 5 | | | | | | | | | | | | |
| T | 6 | | | | | | | | | | | | |
| G | 7 | | | | | | | | | | | | |
| C | 8 | | | | | | | | | | | | |
| A | 9 | | | | | | | | | | | | |
| C | 10 | | | | | | | | | | | | |
| G | 11 | | | | | | | | | | | | |
| C | 12 | | | | | | | | | | | | |

What goes here in **i=1,j=1**?

| | | |
|---|---|---|
| - G | G - | G |
| | | \| |
| G - | - G | G |
| **ID** | **DI** | **M** |

**x[i-1]** = '**G**',
**y[j-1]** = '**G**',
so **delt** = **0**

```
D[i, j] = min(D[i-1, j]+1,
              D[i, j-1]+1,
              D[i-1, j-1]+delt)
        = min(1 + 1, 1 + 1, 0 + 0)
        = 0
```

# Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

| | | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| ϵ | 0 | | | | | | | | | | | | | |
| G | 1 | 0 | ? | | | | | | | | | | | |
| C | 2 | | | | | | | | | | | | | |
| G | 3 | | | | | | | | | | | | | |
| T | 4 | | | | | | | | | | | | | |
| A | 5 | | | | | | | | | | | | | |
| T | 6 | | | | | | | | | | | | | |
| G | 7 | | | | | | | | | | | | | |
| C | 8 | | | | | | | | | | | | | |
| A | 9 | | | | | | | | | | | | | |
| C | 10 | | | | | | | | | | | | | |
| G | 11 | | | | | | | | | | | | | |
| C | 12 | | | | | | | | | | | | | |

What goes here in **i=1,j=2**?

```
- - G      G -      - G
           |
G C -      G C      G C

IID        MI       IR
```

**x[i-1]** = '**G**',
**y[j-1]** = '**C**',
so **delt** = **1**

```
D[i, j] = min(D[i-1, j]+1,
              D[i, j-1]+1,
              D[i-1, j-1]+delt)
        = min(2 + 1, 0 + 1, 1 + 1)
        = 1
```

# Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

| | ε | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ε | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | | | | | |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | | | | | |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | | | | | |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | **?** | | | | | |
| A | 5 | | | | | | | | | | | | |
| T | 6 | | | | | | | | | | | | |
| G | 7 | | | | | | | | | | | | |
| C | 8 | | | | | | | | | | | | |
| A | 9 | | | | | | | | | | | | |
| C | 10 | | | | | | | | | | | | |
| G | 11 | | | | | | | | | | | | |
| C | 12 | | | | | | | | | | | | |

What goes here in **`i=4,j=7`**?

# Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | | | | | |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | | | | | |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | | | | | |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | | | | | |
| A | 5 | | | | | | | | | | | | |
| T | 6 | | | | | | | | | | | | |
| G | 7 | | | | | | | | | | | | |
| C | 8 | | | | | | | | | | | | |
| A | 9 | | | | | | | | | | | | |
| C | 10 | | | | | | | | | | | | |
| G | 11 | | | | | | | | | | | | |
| C | 12 | | | | | | | | | | | | |

What goes here in **i=4,j=7**?

**x[i-1]** = '**T**',
**y[j-1]** = '**C**',
so **delt** = **1**

```
GC---GT          GC-GT--

GCTATGC          GCTATGC

MMIIIMR          MMIRMII
```

```
D[i, j] = min(D[i-1, j]+1,
              D[i, j-1]+1,
              D[i-1, j-1]+delt)
        = min(4 + 1, 3 + 1, 3 + 1)
        = 4
```

# Edit Distance: dynamic programming

$$D[i,j] = \min \begin{cases} D[i-1,j]+1 \\ D[i,j-1]+1 \\ D[i-1,j-1]+\delta(x[i-1],y[j-1]) \end{cases}$$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| T | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| G | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| C | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 |
| A | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | 2 | 3 | 4 | 5 |
| C | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 2 | 3 | 4 |
| G | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 | 3 | 2 | 3 |
| C | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 3 | 3 | 2 |

Fill remaining cells from top row to bottom and from left to right

Edit distance for x, y

# Edit Distance

| | | Y | | | | |
|---|---|---|---|---|---|---|
| | | ϵ | C | A | A | T |
| | ϵ | 0 | 1 | 2 | 3 | 4 |
| X | C | 1 | 0 | A | 2 | 3 |
| | A | 2 | 1 | 0 | B | |
| | T | 3 | 2 | C | | |

# Edit Distance

$Y$

$X$

| | ϵ | C | A | A | T |
|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 |
| C | 1 | 0 | **1** | 2 | 3 |
| A | 2 | 1 | 0 | **B** | |
| T | 3 | 2 | **C** | | |

# Edit Distance

|  |  | *Y* |  |  |  |
|---|---|---|---|---|---|
|  | ϵ | C | A | A | T |
| ϵ | 0 | 1 | 2 | 3 | 4 |
| C | 1 | 0 | 1 | 2 | 3 |
| A | 2 | 1 | 0 | 1 |  |
| T | 3 | 2 | c |  |  |

*X*

# Edit Distance

|  |  | Y |  |  |  |  |
|---|---|---|---|---|---|---|
|  |  | ϵ | C | A | A | T |
|  | ϵ | 0 | 1 | 2 | 3 | 4 |
|  | C | 1 | 0 | 1 | 2 | 3 |
| X | A | 2 | 1 | 0 | 1 |  |
|  | T | 3 | 2 | 1 |  |  |

# Edit Distance

| | | ***Y*** | | | | |
|---|---|---|---|---|---|---|
| | | ϵ | C | A | A | T |
| | ϵ | 0 | 1 | 2 | 3 | 4 |
| | C | 1 | 0 | 1 | 2 | 3 |
| ***X*** | A | 2 | 1 | 0 | 1 | 2 |
| | T | 3 | 2 | 1 | 1 | 1 |

# `eMatrix buildEditMatrix(X, Y)`

Input:

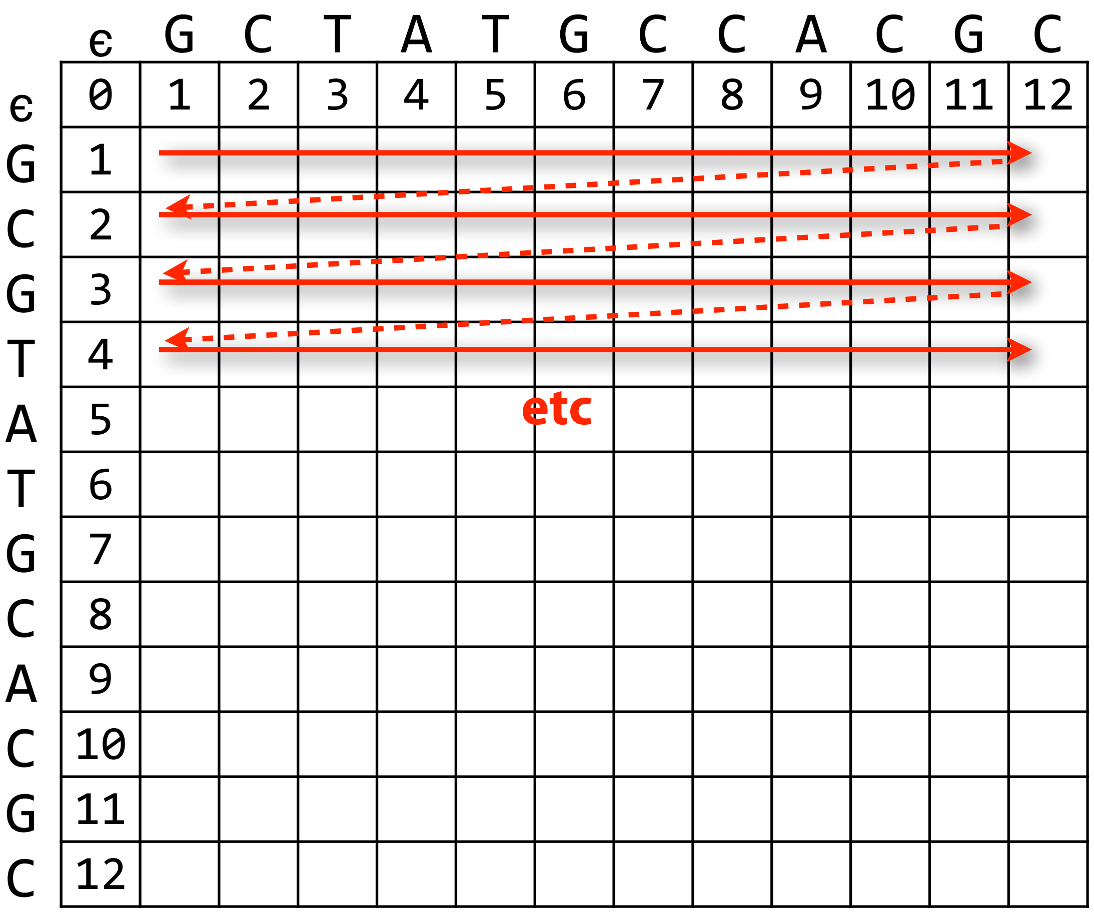
- **`string X`**: Input string X (edits with respect to X)
- **`string Y`**: Input string Y (edits turn X into Y)

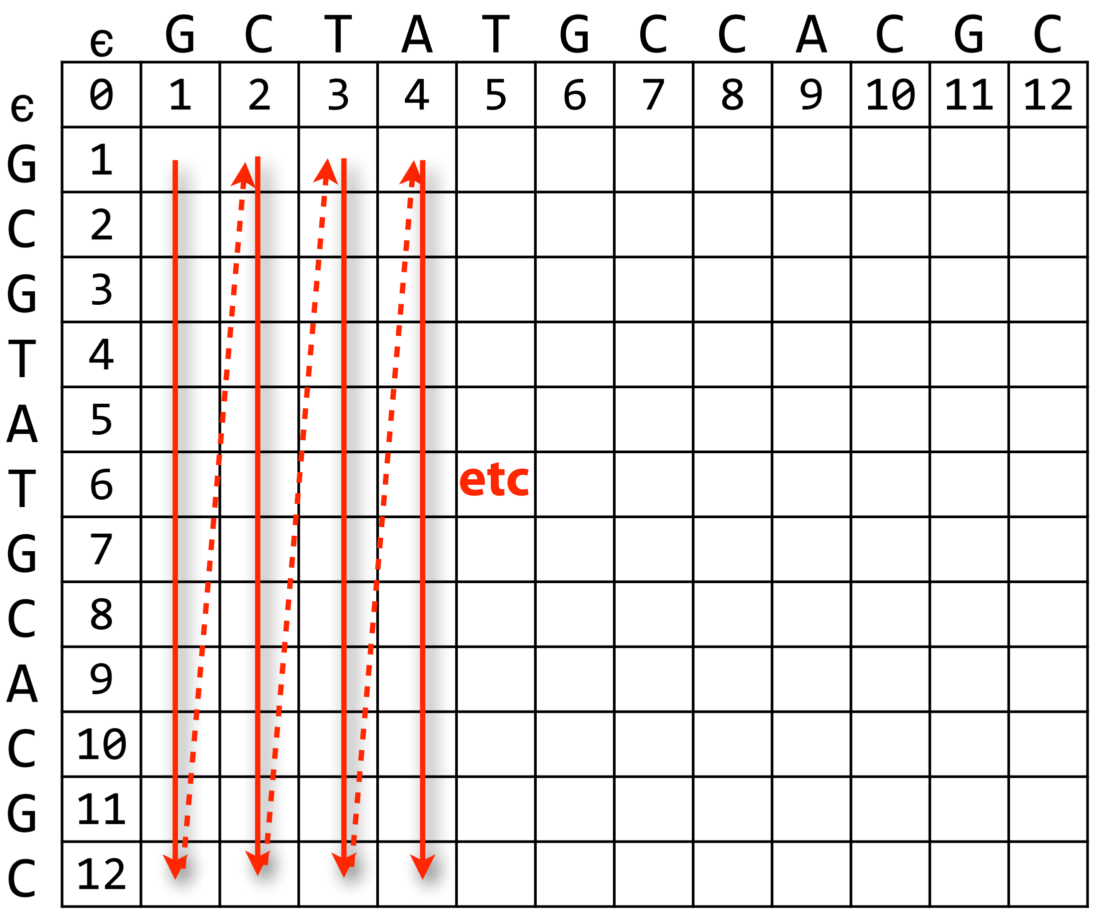
Output:

- `eMatrix` : vector<vector<int>> storing all optimal edit distances

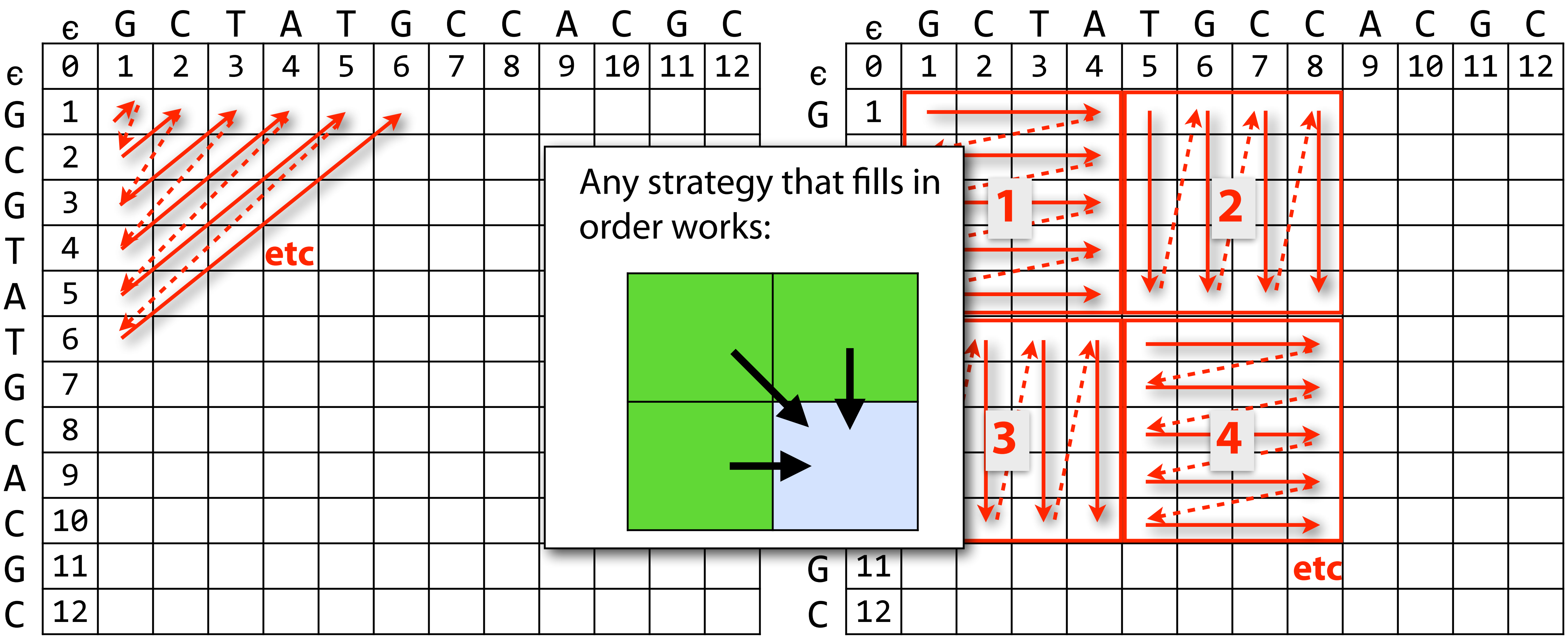
| | ϵ | C | A | A | T |
|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 |
| C | 1 | 0 | 1 | 2 | 3 |
| A | 2 | 1 | 0 | 1 | 2 |
| T | 3 | 2 | 1 | 1 | 1 |

# Edit Distance: dynamic programming



# Edit Distance: dynamic programming



# Assignment 11: a_edist

Learning Objective:

Use dynamic programming to build an edit distance matrix

Construct an optimal edit string from the edit matrix

**Consider:** Does substitution, insertion, and deletion need to have the same 'weight' as a penalty? How could you modify the code to take a user-specified input for each?

# Edit Distance

| | ϵ | C | A | A | T |
|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 |
| C | 1 | 0 | 1 | 2 | 3 |
| A | 2 | 1 | 0 | 1 | 2 |
| T | 3 | 2 | 1 | 1 | 1 |

Edit distance for x, y

But where and what is the edit?

# Edit Distance

**Traceback** corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?

| | ϵ | C | A | A | T |
|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 |
| C | 1 | 0 | 1 | 2 | 3 |
| A | 2 | 1 | 0 | 1 | 2 |
| T | 3 | 2 | 1 | 1 | 1 |

Q: How did I get here?

# Edit Distance

**Traceback** corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor ($\nwarrow$, $\Leftarrow$ or $\Uparrow$) gave the minimum?

| | ϵ | C | A | A | T |
|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 |
| C | 1 | 0 | 1 | 2 | 3 |
| A | 2 | 1 | 0 | 1 | 2 |
| T | 3 | 2 | 1 | 1 | 1 |

$D[2, \;\; 4] =$ ____________

Q: How did I get here?

# Edit Distance

**Traceback** corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?

| | ϵ | C | A | A | T |
|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 |
| C | 1 | 0 | 1 | 2 | 3 |
| A | 2 | 1 | 0 | 1 | 2 |
| T | 3 | 2 | 1 | 1 | 1 |

$D[3, \ 3] =$ ____________

Q: How did I get here?

# Edit Distance

**Traceback** corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor ($\nwarrow$, $\Leftarrow$ or $\Uparrow$) gave the minimum?

| | ϵ | C | A | A | T |
|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 |
| C | 1 | 0 | 1 | 2 | 3 |
| A | 2 | 1 | 0 | 1 | 2 |
| T | 3 | 2 | 1 | 1 | 1 |

$D[2, \ 3] =$ __________

Q: How did I get here?

# Edit Distance

**Traceback** corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor ($\Leftarrow\!\!\!\nwarrow$, $\Leftarrow$ or $\Uparrow$) gave the minimum?

| | ϵ | C | A | A | T |
|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 |
| C | 1 | 0 | 1 | 2 | 3 |
| A | 2 | 1 | 0 | **1** | 2 |
| T | 3 | 2 | 1 | 1 | 1 |

$D[1,\ \ 3] =$ ______

$D[1,\ \ 2] =$ ______

$D[2,\ \ 2] =$ ______

Q: How did I get here?

# Edit Distance

**Traceback** corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor ($\nwarrow$, $\Leftarrow$ or $\Uparrow$) gave the minimum?

| | ϵ | C | A | A | T |
|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 |
| C | 1 | 0 | **1** | 2 | [illegible] |
| A | 2 | 1 | **0** | **1** | 2 |
| T | 3 | 2 | 1 | 1 | 1 |

Tie!

$$D[1,\ \ 3] = 2 + 1$$

$$D[1,\ \ 2] = 1 + 0$$

$$D[2,\ \ 2] = 0 + 1$$

Q: How did I get here?

# Edit Distance

**Traceback** corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?

| | ϵ | C | A | A | T |
|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 |
| C | 1 | 0 | 1 | 2 | 3 |
| A | 2 | 1 | 0 | 1 | 2 |
| T | 3 | 2 | 1 | 1 | 1 |

# Edit Distance

**Traceback** corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?

| | ϵ | C | A | A | T |
|---|---|---|---|---|---|
| **ϵ** | 0 | 1 | 2 | 3 | 4 |
| **C** | 1 | 0 | 1 | 2 | 3 |
| **A** | 2 | 1 | 0 | 1 | 2 |
| **T** | 3 | 2 | 1 | 1 | 1 |

Path 1

**MIMM**

```
C - A T
|   | |
C A A T
```

# Edit Distance

**Traceback** corresponds to an optimal alignment / edit transcript

At each step, ask: which neighbor (⇖, ⇐ or ⇑) gave the minimum?

| | ϵ | C | A | A | T |
|---|---|---|---|---|---|
| **ϵ** | 0 | 1 | 2 | 3 | 4 |
| **C** | 1 | 0 | 1 | 2 | 3 |
| **A** | 2 | 1 | 0 | 1 | 2 |
| **T** | | | 1 | 1 | 1 |

Path 2

**MIMM**

```
C - A T
|   | |
C A A T
```

**MMIM**

```
C A - T
| |   |
C A A T
```

# Edit Distance

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| T | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| G | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| C | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 |
| A | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | 2 | 3 | 4 | 5 |
| C | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 2 | 3 | 4 |
| G | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 | 3 | 2 | 3 |
| C | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 3 | 3 | 2 |

Q: How did I get here?

# Edit Distance

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| T | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| G | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| C | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 |
| A | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | 2 | 3 | 4 | 5 |
| C | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 2 | 3 | 4 |
| G | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 | 3 | 2 | 3 |
| C | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 3 | 3 | 2 |

$$D[11,\quad 12] = 3 + 1$$

$$D[11,\quad 11] = 2 + 0$$

$$D[12,\quad 11] = 3 + 1$$

A: From here

Q: How did I get here?

# Edit Distance

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| T | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| G | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| C | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 |
| A | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | 2 | 3 | 4 | 5 |
| C | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 2 | 3 | 4 |
| G | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 | 3 | 2 | 3 |
| C | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 3 | 3 | 2 |

Q: How did I get here?

# Edit Distance

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| T | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| G | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| C | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 |
| A | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | 2 | 3 | 4 | 5 |
| C | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 2 | 3 | 4 |
| G | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 | 3 | 2 | 3 |
| C | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 3 | 3 | 2 |

A: From here

Q: How did I get here?

$$D[10,\quad 11] = 3 + 1$$

$$D[10,\quad 10] = 2 + 0$$

$$D[11,\quad 10] = 3 + 1$$

# Edit Distance

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| T | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| G | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| C | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 |
| A | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | 2 | 3 | 4 | 5 |
| C | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 2 | 3 | 4 |
| G | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 | 3 | 2 | 3 |
| C | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 3 | 3 | 2 |

Q: How did I get here?

# Edit Distance

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| T | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| G | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| C | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 |
| A | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | 2 | 3 | 4 | 5 |
| C | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 2 | 3 | 4 |
| G | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 | 3 | 2 | 3 |
| C | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 3 | 3 | 2 |

A: From here

Q: How did I get here?

$$D[9,\ \ 10] = 3 + 1$$

$$D[9,\ \ 9] = 2 + 0$$

$$D[10,\ \ 9] = 3 + 1$$

# Edit Distance

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| T | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| G | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| C | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 |
| A | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | 2 | 3 | 4 | 5 |
| C | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 2 | 3 | 4 |
| G | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 | 3 | 2 | 3 |
| C | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 3 | 3 | 2 |

Alignment:

```
GCGTATG-CACGC
|| |||| |||||
GC-TATGCCACGC
```

MMDMMMMIMMMMM

# Edit Distance

**Dynamic Programming** fills our table with optimal distances

**Traceback** identifies the optimal *edit string* to convert $X$ to $Y$

What is our efficiency? ($|X| = m, |Y| = n$ )

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| T | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| G | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| C | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 |
| A | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | 2 | 3 | 4 | 5 |
| C | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 2 | 3 | 4 |
| G | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 | 3 | 2 | 3 |
| C | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 3 | 3 | 2 |

# Edit Distance

**Dynamic Programming** fills our table with optimal distances

**Traceback** identifies the optimal *edit string* to convert $X$ to $Y$

| | ϵ | G | C | T | A | T | G | C | C | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| G | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| C | 2 | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| G | 3 | 2 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| T | 4 | 3 | 2 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| T | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| G | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| C | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 |
| A | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 2 | 2 | 3 | 4 | 5 |
| C | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 2 | 3 | 4 |
| G | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 | 3 | 2 | 3 |
| C | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 3 | 3 | 2 |

What is our efficiency? ($|X| = m$, $|Y| = n$ )

**Table filling:** Filling (m + 1) (n + 1) cells, each requiring constant work, so $O(mn)$

**Traceback:** Each step goes ⇖, ⇐ or ⇑. Worst case: traceback never moves diagonally, requiring $m$ ⇑'s and $n$ ⇐'s, so $O(m + n)$

# `string buildEditString(X, Y)`

Input: **`string X`**: Input string X (edits with respect to X)

**`string Y`**: Input string Y (edits turn X into Y)

Output: **`string`**: An optimal edit string produced by the matrix

| | ϵ | C | A | A | T |
|---|---|---|---|---|---|
| ϵ | 0 | 1 | 2 | 3 | 4 |
| C | 1 | 0 | 1 | 2 | 3 |
| A | 2 | 1 | 0 | 1 | 2 |
| T | 3 | 2 | 1 | 1 | 1 |

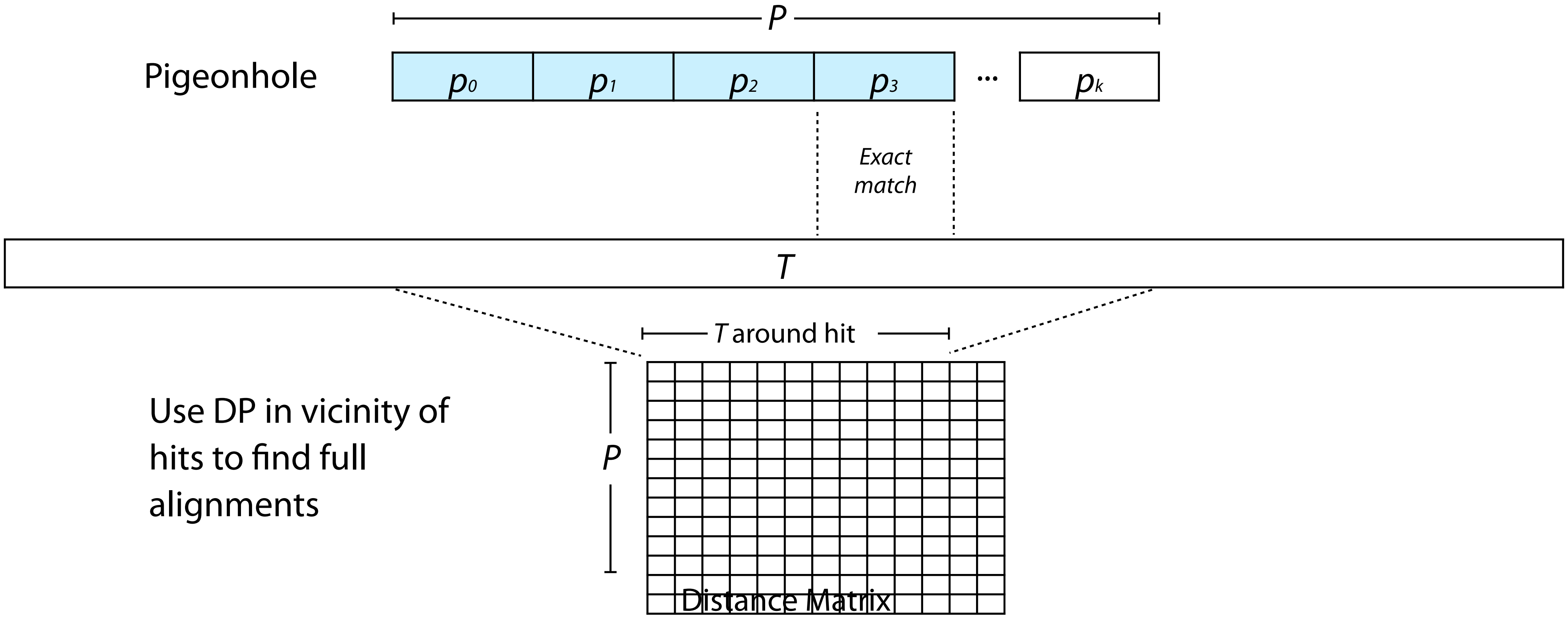
On tie: prioritize diagonal, then vertical, then horizontal

**MIMM**

```
C - A T
|   | |
C A A T
```

# Approximate Pattern Matching

"Seed and extend" works for edit distance too!




Use DP in vicinity of hits to find full alignments

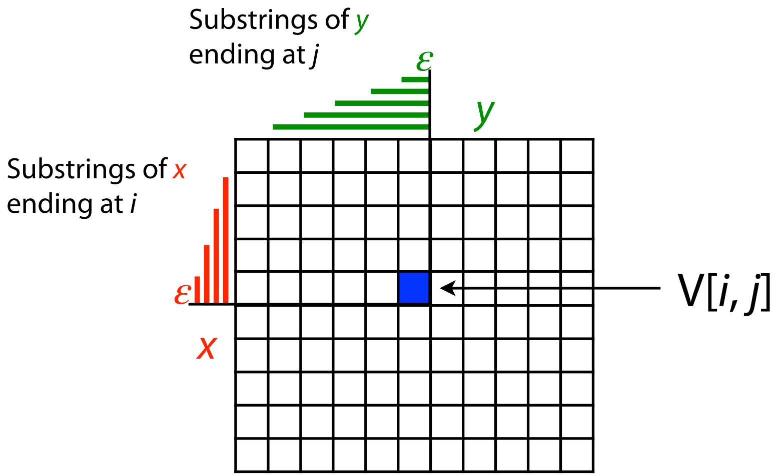
# Bonus Slide

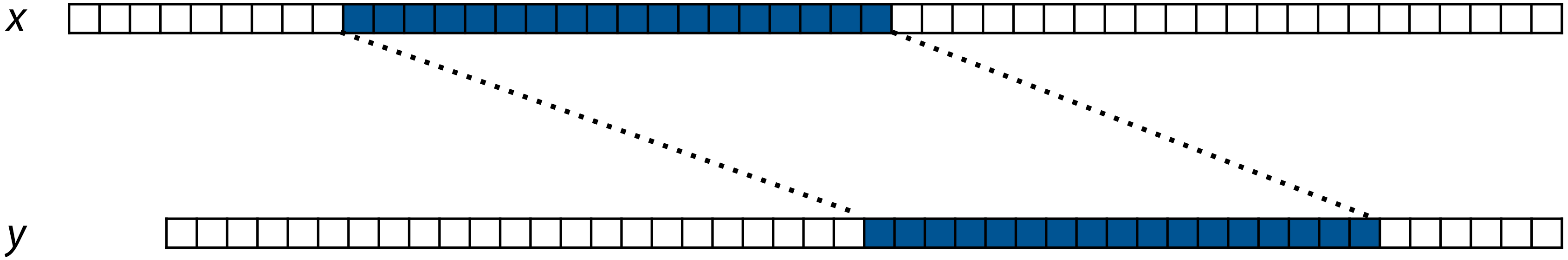
$s(a, b)$

| | A | C | G | T | - |
|---|---|---|---|---|---|
| A | 0 | 4 | 2 | 4 | 8 |
| C | 4 | 0 | 4 | 2 | 8 |
| G | 2 | 4 | 0 | 4 | 8 |
| T | 4 | 2 | 4 | 0 | 8 |
| - | 8 | 8 | 8 | 8 | |

$s(a, b)$

| | A | C | G | T | - |
|---|---|---|---|---|---|
| A | 1 | -1 | -1 | -1 | -1 |
| C | -1 | 1 | -1 | -1 | -1 |
| G | -1 | -1 | 1 | -1 | -1 |
| T | -1 | -1 | -1 | 1 | -1 |
| - | -1 | -1 | -1 | -1 | |




… and much more!