Overview:
In this lab you will learn how to teach computer how to learn to win a game. You will use a graph to represent a state space.

Using a graph as a state space:
Before an AI problem can be solved it must be represented as a state space. The state space is then searched to find a solution to the problem. A state space essentially consists of a set of nodes representing each state of the problem, arcs between nodes representing the legal moves from one state to another, an initial state and a goal state. Each state space takes the form of a tree or a graph. For visualization take a look partial state space for tic-tac-toe:

The Game of Nim
A game starts with k tokens. Players alternate turns with Player 1 starting the game. Each turn, a player may pick up 1 or 2 tokens. The player who picks up last token wins.

Exercise 1.1: How would you represent each state in this game?
HINT: What do we need to keep track of in each state?
Represent each of the states using the player number and the tokens.

Exercise 1.2: Connect the states in the following state space graph for a game with starting tokens k = 3: Nim(3)

Reinforcement learning:
Finally, we need to apply reinforcement learning. In reinforcement learning, an algorithm is rewarded for making a good decision and punished for making a poor decision. We will define a good decision as all decisions made by the player who won. Therefore, if Player 1 took the last token, all choices made by Player 1 are rewarded. The reward is captured in our algorithm as the edge weight. When we consider a path through the graph, we can find that all edges along a path that has Player 1 winning (eg: the last vertex in the path goes to Player 2 with no tokens remaining, or "p2-0", meaning that Player 1 took the last token), then all choices made by Player 1 (edges where Player 1 is the source vertex) are rewarded by increasing the edge weight by +1 and all choices made by Player 2 are punished by changing the edge weight by -1.
Exercise 2.1:
Let’s label the state “Player 1 - 5 tokens available” as p1-5. What is the label of the state where p1 wins? What about where p2 wins?
When p1 wins - p2-0
When p2 wins - p1-0

Exercise 2.2: Given initial edge weights as 0, what will be updated edge weights after the next two games:
1. p1-5 -> p2-4 -> p1-2 -> p2-1 -> p1-0
2. p1-5 -> p2-3 -> p1-2 -> p2-0

Exercise 2.3: Given the following edge weights for a game Nim(5), find how the trained players would play. Give the path they will follow.
Remember the start state is p1-5:

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<th>p1-5</th>
<th>p1-4</th>
<th>p1-3</th>
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p1-5 -> p2-3 -> p1-2 -> p2-0

AFTER YOU’RE DONE WITH LAB CODING:
Exercise 2.4: Would you prefer to go first or second in Nim(10)?
I would prefer to be the first player and take one token to end up in state p2-9.

In the programming part of this lab, you will:
- Using a graph as a state space
- Reinforcement learning
- How to teach a computer how to learn to win the game of Nim
- Implement next functions:
  - NimLearner constructor - which creates the vertices and edges for the state space of a game of Nim;
  - playRandomGame - which returns a random path through the graph of the state space as a vector<Edge>.
  - updateEdgeWeights - which updates the edge weights along a given path on the graph of the state space.

As your TA and CAs, we’re here to help with your programming for the rest of this lab section! 😊