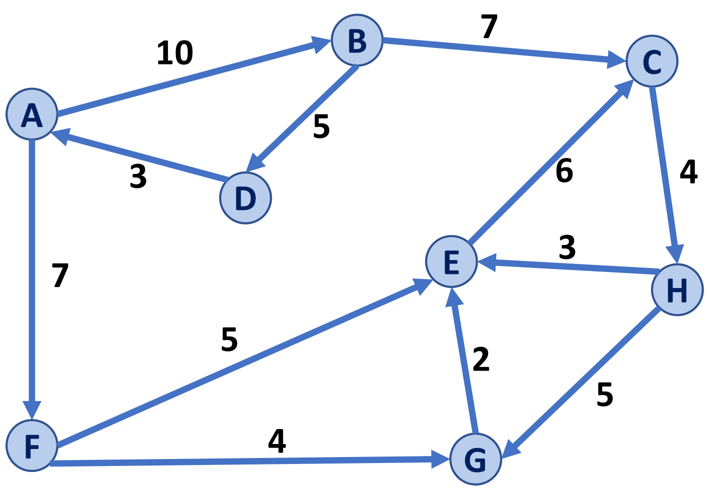
CS 225

Data Structures

November 29 – Dijkstra’s Algorithm Analysis

G Carl Evans

Dijkstra's Algorithm (SSSP)




```
PrimMST(G, s):
  foreach (Vertex v : G):
    d[v] = +inf
    p[v] = NULL
  d[s] = 0

  PriorityQueue Q // min distance, defined by d[v]
  Q.buildHeap(G.vertices())
  Graph T          // "labeled set"

  repeat n times:
    Vertex u = Q.removeMin()
    T.add(u)
    foreach (Vertex v : neighbors of u not in T):
      if cost(v, m) < d[v]:
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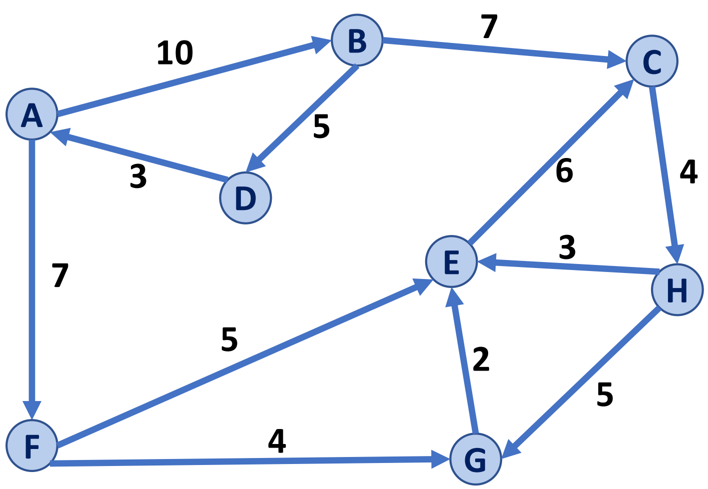
Dijkstra's Algorithm (SSSP)

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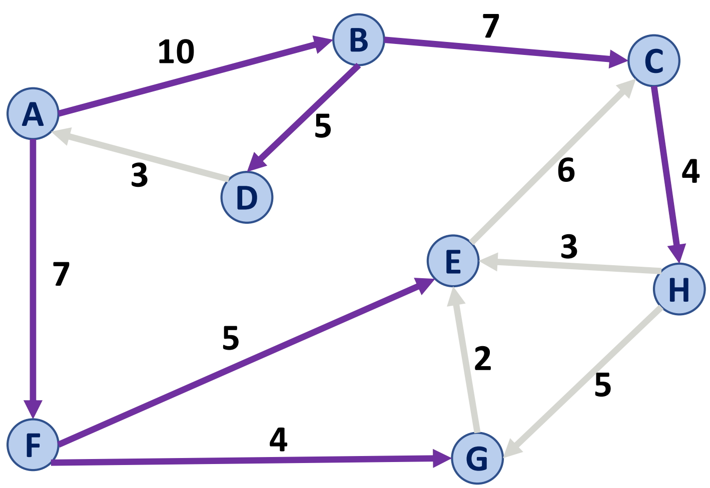


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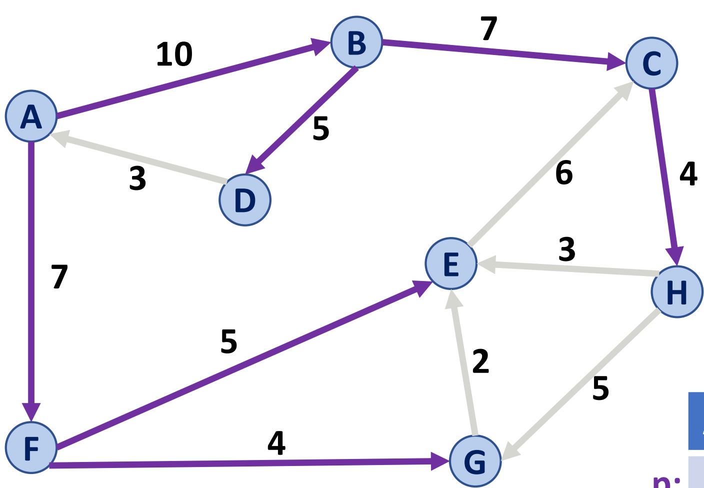
Dijkstra's Algorithm (SSSP)

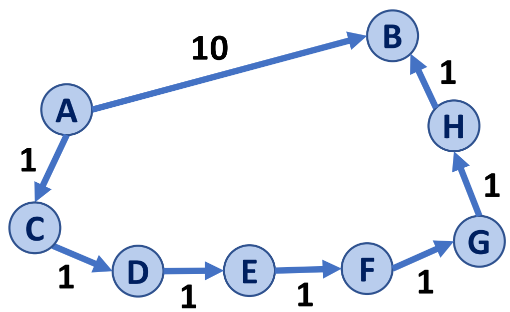
Dijkstra gives us the shortest path from our path (single source) to **every** connected vertex!



	A	B	C	D	E	F	G	H
p:	--	A	B	B	F	A	F	C
d:	0	10	17	15	12	7	11	21

Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle a single heavy-weight path vs. many light-weight paths?



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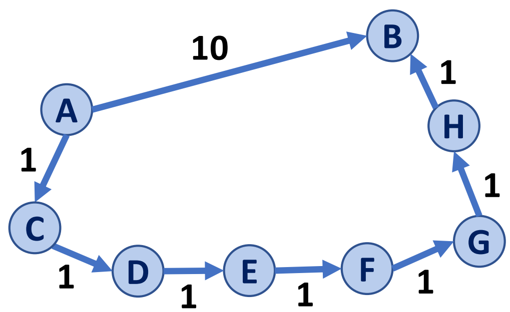

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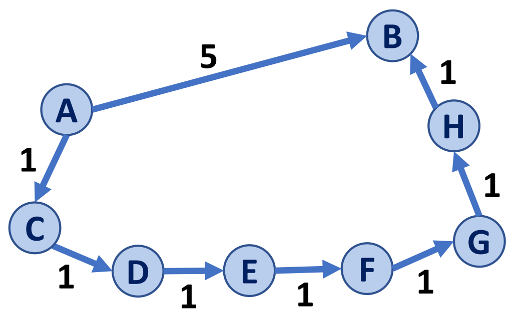
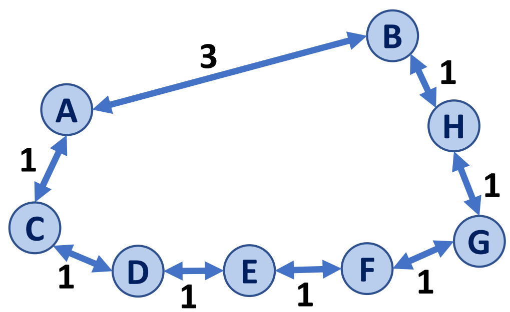


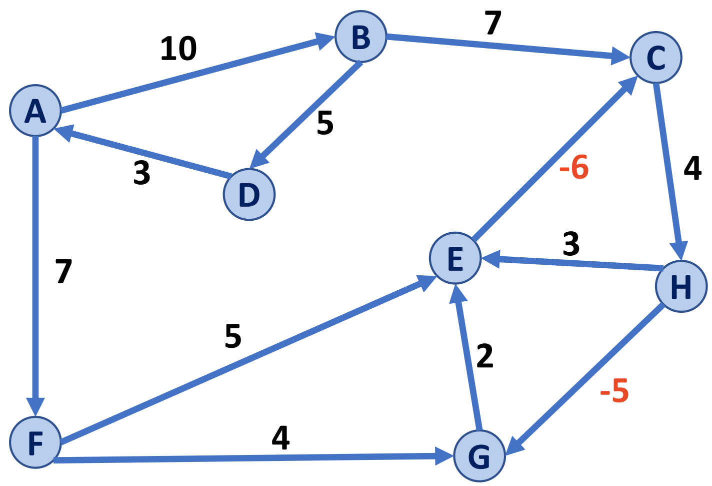
Dijkstra’s Algorithm (SSSP)

Q: How does Dijkstra handle undirected graphs?



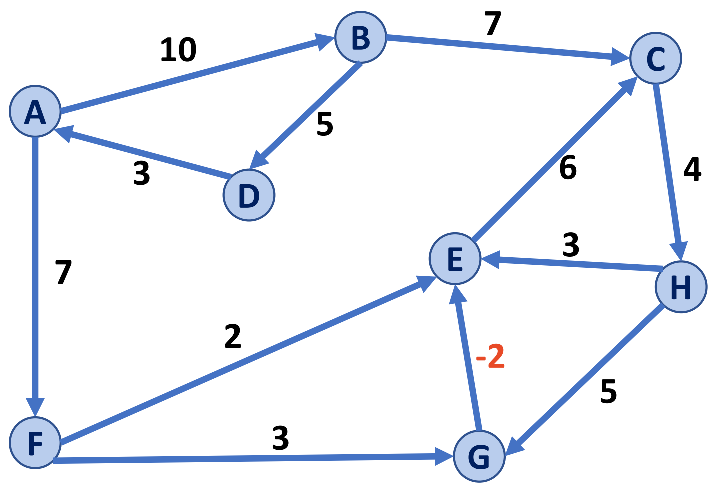
Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle negative weight cycles?



Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle negative weight edges, without a negative weight cycle?



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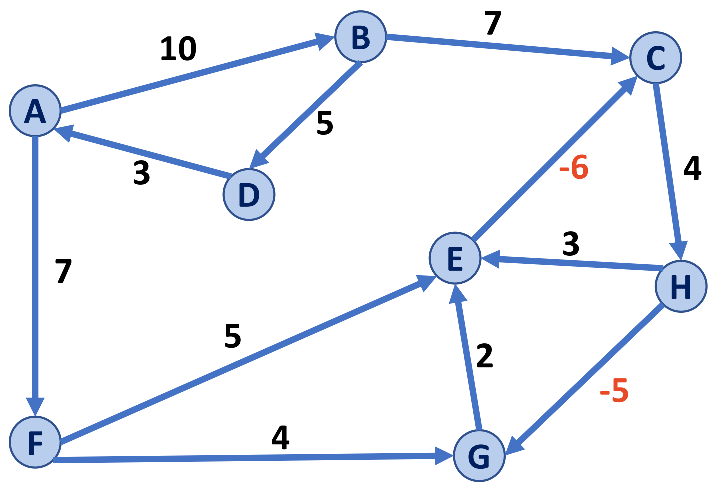

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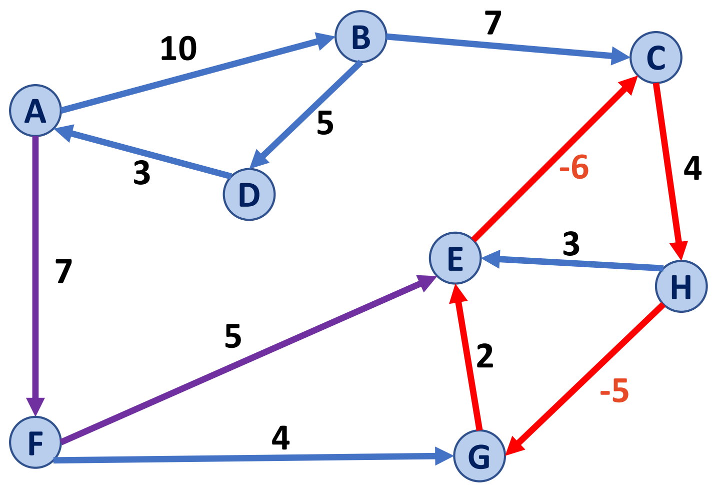

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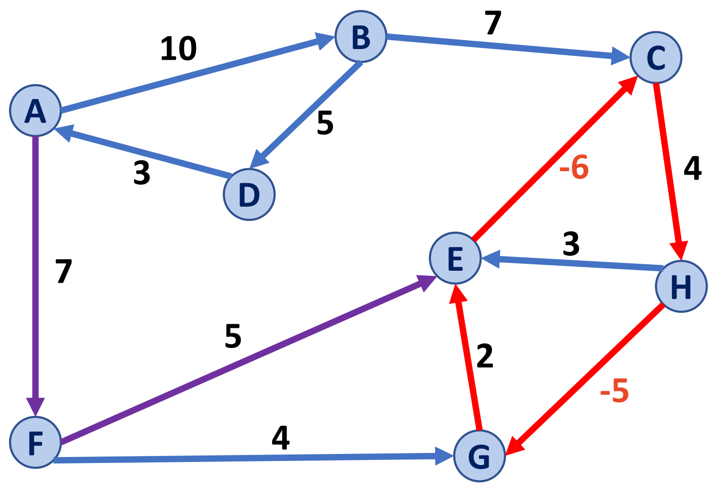

Shortest Path (A ➔ E): A → F → E → (C → H → G → E)*

Length: 12 Length: -5 (repeatable)

Dijkstra's Algorithm (SSSP)

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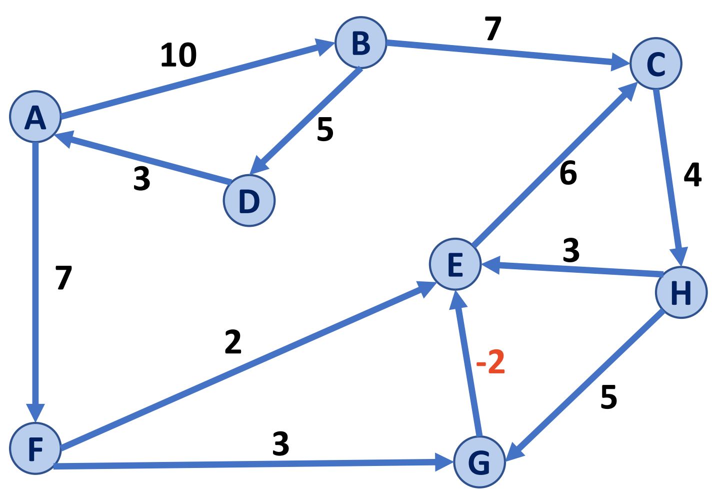

Dijkstra's Algorithm (SSSP)

What is Dijkstra's running time?

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DijkstraSSSP(G, s):
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      if cost(u, v) + d[u] < d[v]:
        d[v] = cost(u, v) + d[u]
        p[v] = m

  return T
```

Suppose I have a new heap:

	Binary Heap	Fibonacci Heap
Remove Min	O(lg(n))	O(lg(n))
Decrease Key	O(lg(n))	O(1)*

What's the updated running time?

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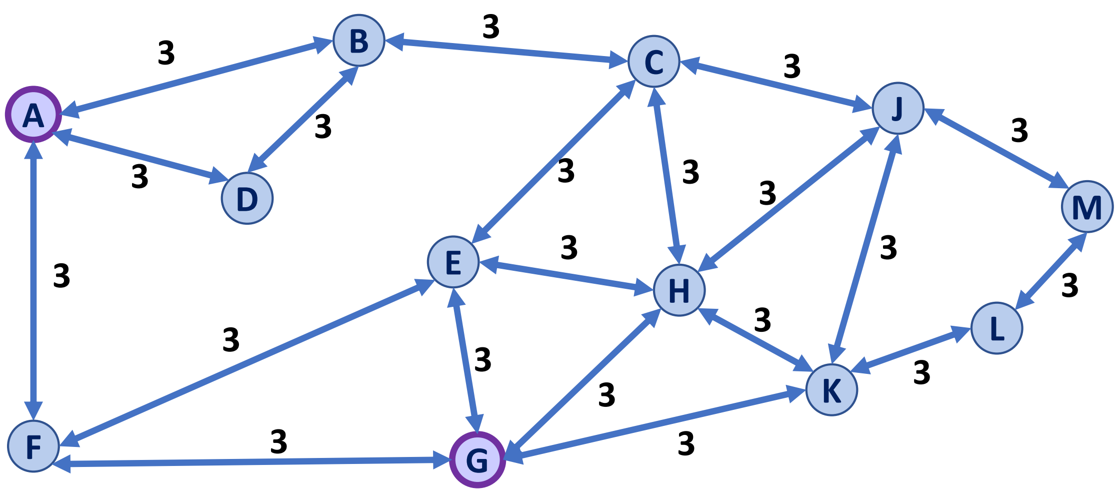
```

Landmark Path Problem

Suppose you want to travel from **A** to **G**.

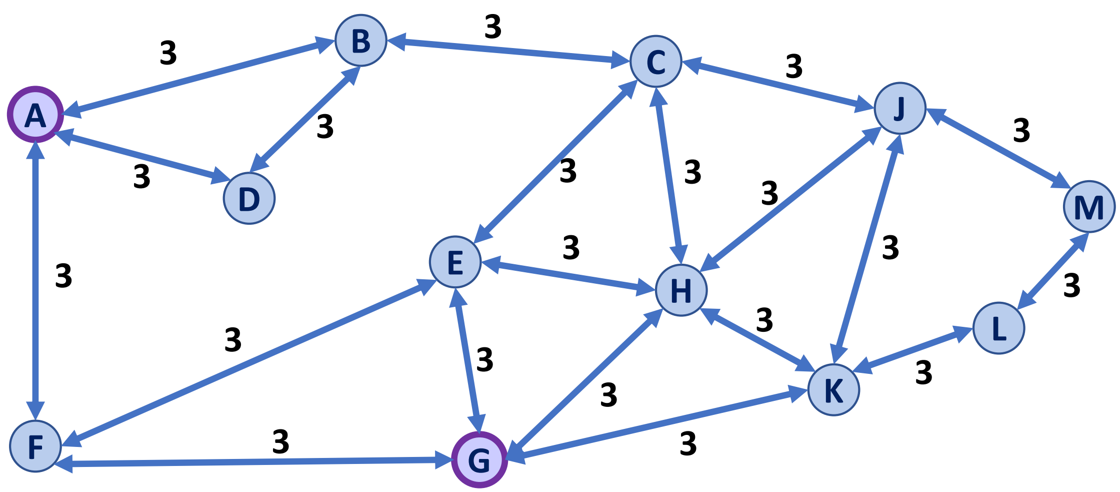
Q1: What is the shortest path from **A** to **G**?



Landmark Path Problem

Suppose you want to travel from **A** to **G**.

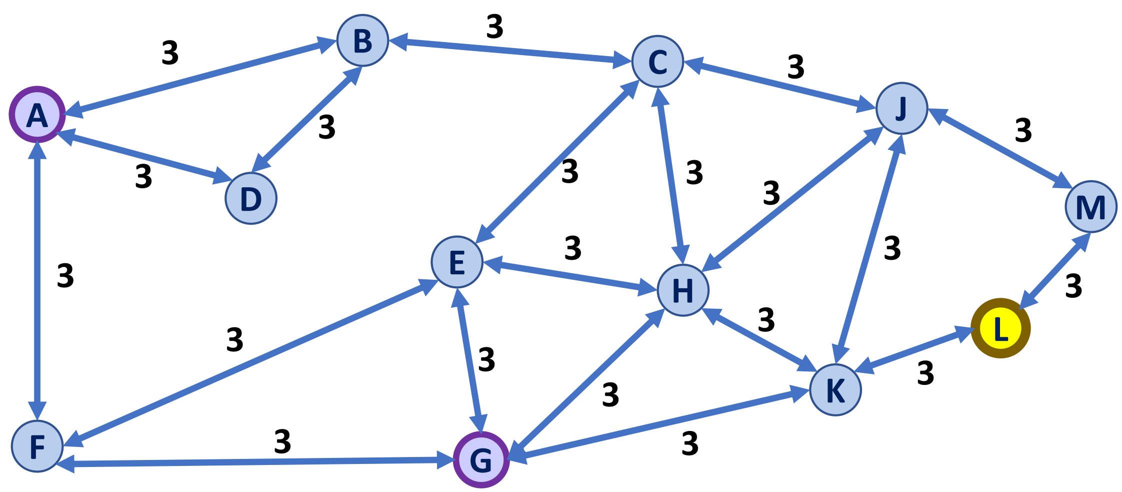
Q2: What is the fastest algorithm to use to find the shortest path?



Landmark Path Problem

In your journey between **A** and **G**, you also want to visit the landmark **L**.

Q3: What is the shortest path from **A** to **G** that visits **L**?



Landmark Path Problem

In your journey between **A** and **G**, you also want to visit the landmark **L**.

Q4: What is the fastest algorithm to find this path?

Q5: What are the specific call(s) to this algorithm?