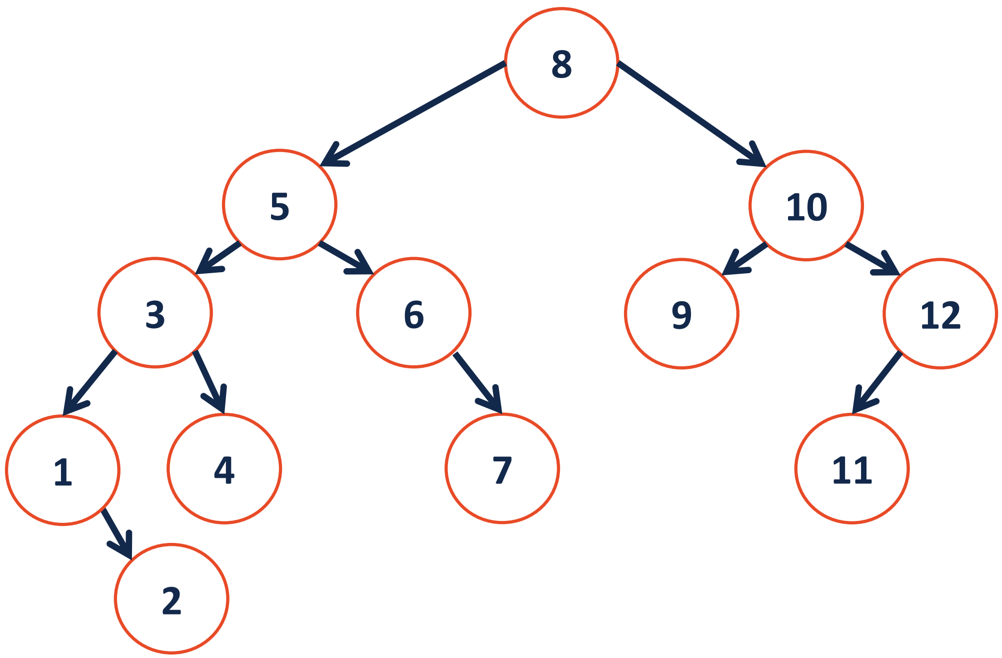
# CS 225

**Data Structures**

*October 11 – AVL Analysis*

*G Carl Evans*


8
5
10
3
6
9
12
1
4
7
11
2

# AVL Tree Analysis

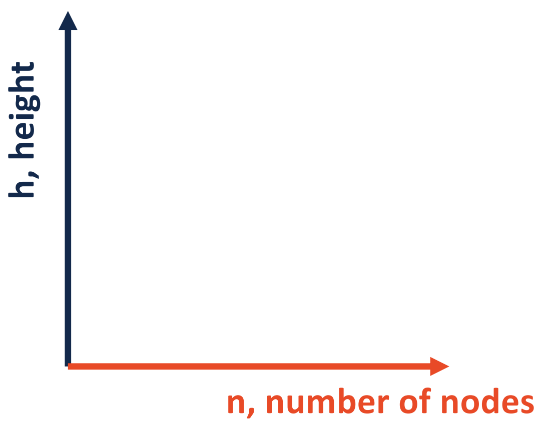
**We know:** insert, remove and find runs in: ___________.

**We will argue that: h** is __________.

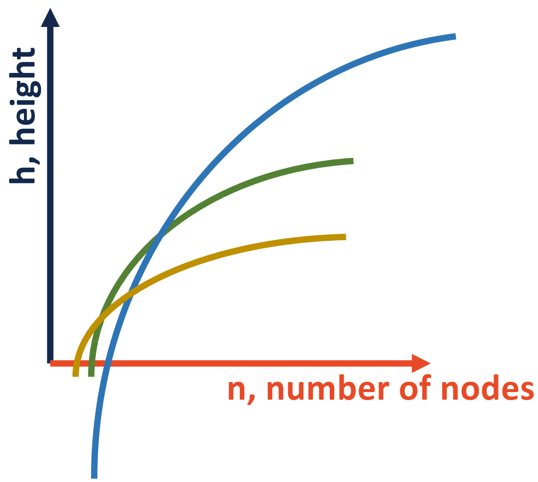
# AVL Tree Analysis

Definition of big-O:

…or, with pictures:

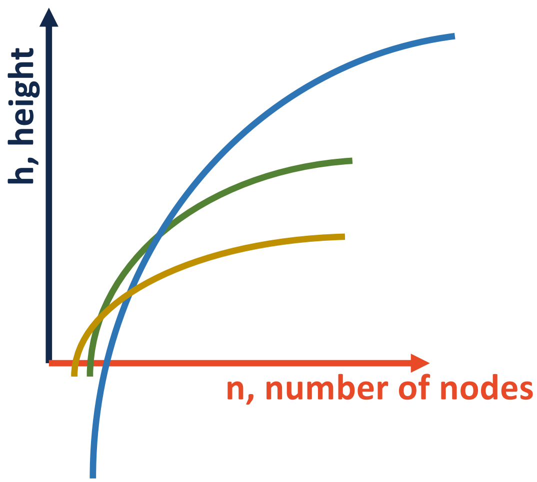


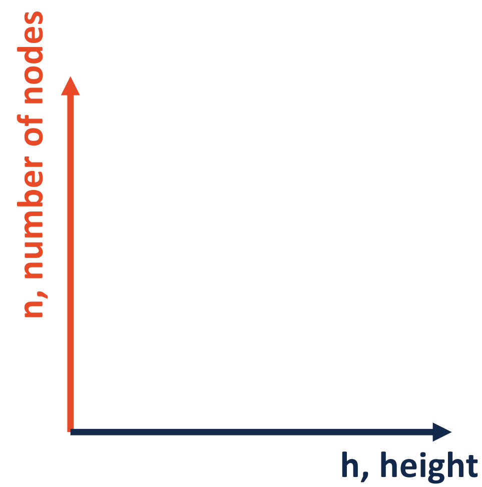
# AVL Tree Analysis



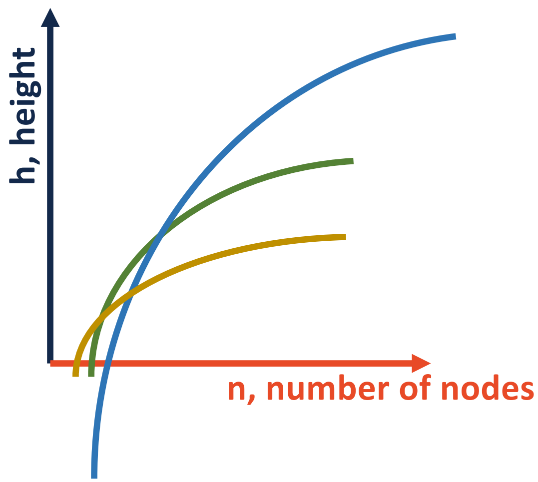

- The height of the tree, **f(n)**, will always be <u>less than</u> **c × g(n)** for all values where **n > k**.

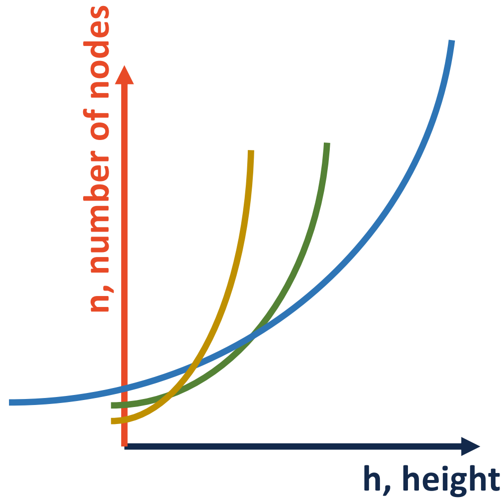
# AVL Tree Analysis



# AVL Tree Analysis




- The number of nodes in the tree, $\mathbf{f^{-1}(h)}$, will always be <u>greater than</u> $\mathbf{c \times g^{-1}(h)}$ for all values where $\mathbf{n > k}$.

# Plan of Action

Since our goal is to find the lower bound on **n** given **h**, we can begin by defining a function given **h** which describes the smallest number of nodes in an AVL tree of height **h**:

# Simplify the Recurrence

**N(h)** = 1 + N(h - 1) + N(h - 2)

# State a Theorem

**Theorem:** An AVL tree of height h has at least ___________.

**Proof:**

I. Consider an AVL tree and let **h** denote its height.

II. Case: _______________

An AVL tree of height ____ has at least ____ nodes.

# Prove a Theorem

III. Case: ________________

An AVL tree of height ____ has at least ____ nodes.

# Prove a Theorem

By an Inductive Hypothesis (IH):

We will show that:

An AVL tree of height ____ has at least ____ nodes.

# Prove a Theorem

V. Using a proof by induction, we have shown that:

…and inverting:

# Summary of Balanced BST

**Red-Black Trees**

- Max height: 2 * lg(n)
- Constant number of rotations on insert, remove, and find

**AVL Trees**

- Max height: 1.44 * lg(n)
- Rotations:

# Summary of Balanced BST

**Pros:**

- Running Time:

  - Improvement Over:

- Great for specific applications:

# Summary of Balanced BST

**Cons:**

- Running Time:

- In-memory Requirement:

# Red-Black Trees in C++

C++ provides us a balanced BST as part of the standard library:

```
std::map<K, V> map;
```

# Red-Black Trees in C++

```
V & std::map<K, V>::operator[]( const K & )
```

# Red-Black Trees in C++

```
V & std::map<K, V>::operator[]( const K & )
```

```
std::map<K, V>::erase( const K & )
```

# Red-Black Trees in C++

```
iterator std::map<K, V>::lower_bound( const K & );
iterator std::map<K, V>::upper_bound( const K & );
```

# Iterators

## Why do we care?

```
DFS dfs(...);
for ( ImageTraversal::Iterator it = dfs.begin(); it != dfs.end(); ++it ) {
  std::cout << (*it) << std::endl;
}
```

# Iterators

## Why do we care?

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DFS dfs(...);
for ( ImageTraversal::Iterator it = dfs.begin(); it != dfs.end(); ++it ) {
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```
DFS dfs(...);
for ( const Point & p : dfs ) {
  std::cout << p << std::endl;
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```

# Iterators

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DFS dfs(...);
for ( ImageTraversal::Iterator it = dfs.begin(); it != dfs.end(); ++it ) {
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}
```

```
DFS dfs(...);
for ( const Point & p : dfs ) {
  std::cout << p << std::endl;
}
```

```
ImageTraversal & traversal = /* ... */;
for ( const Point & p : traversal ) {
  std::cout << p << std::endl;
}
```

# Every Data Structure So Far

| | Unsorted Array | Sorted Array | Unsorted List | Sorted List | Binary Tree | BST | AVL |
|---|---|---|---|---|---|---|---|
| **Find** | | | | | | | |
| **Insert** | | | | | | | |
| **Remove** | | | | | | | |
| **Traverse** | | | | | | | |