



CS 225

Data Structures

October 4 – BST Remove

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MP Redo

- There will be two redos of MPs this semester. The redo will be done on the code in your repo on the last day of class December 8th. The redo will change your grade to the following.

$$\text{Max}(\text{old_grade}, .90 * \text{new_grade})$$

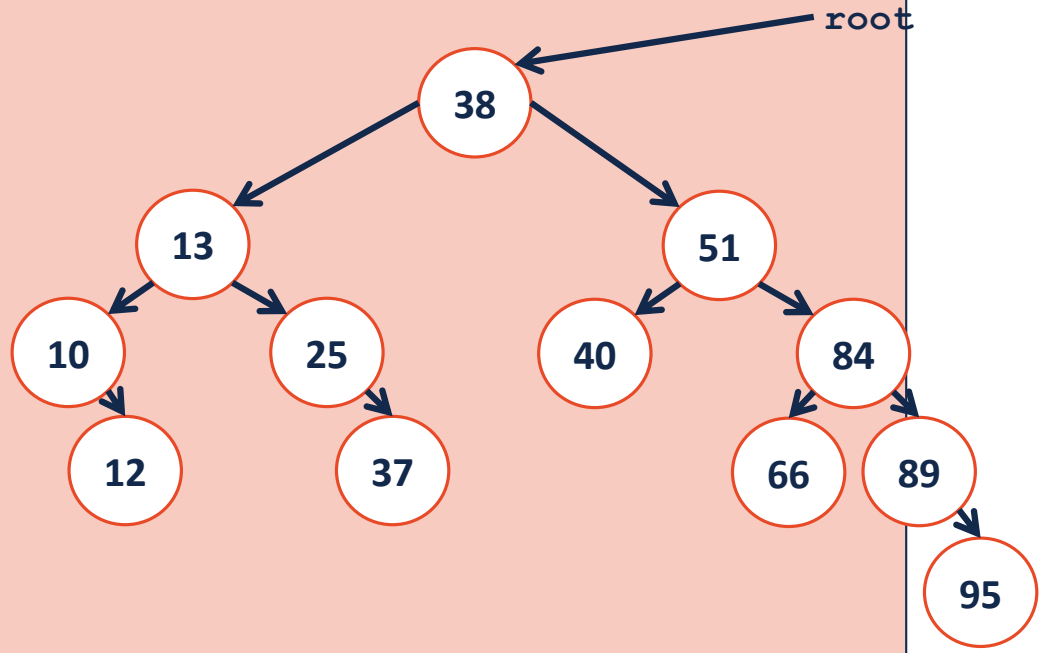


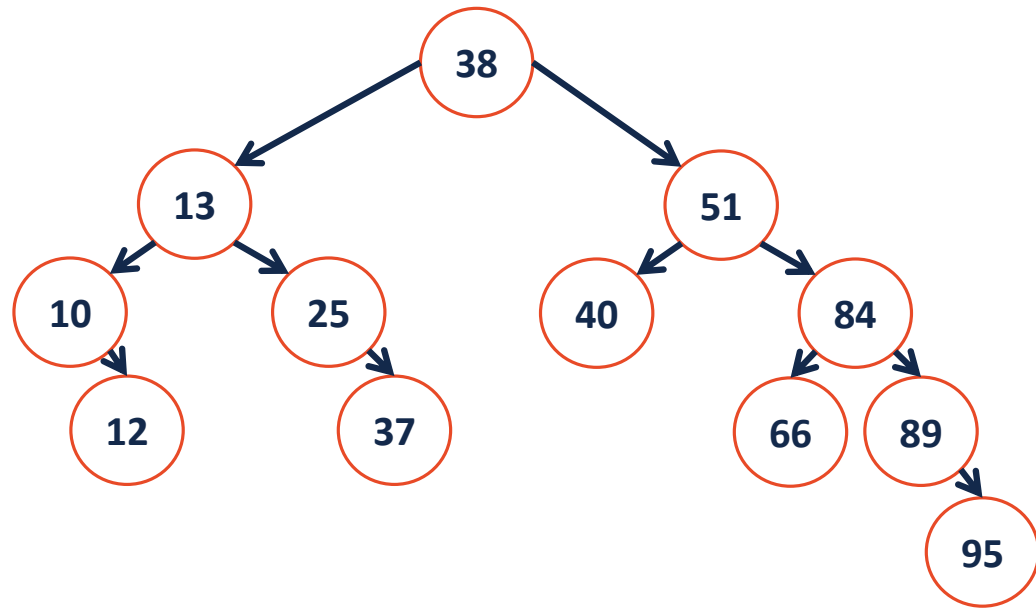
Honors Section

Functional Data Structures in Clojure

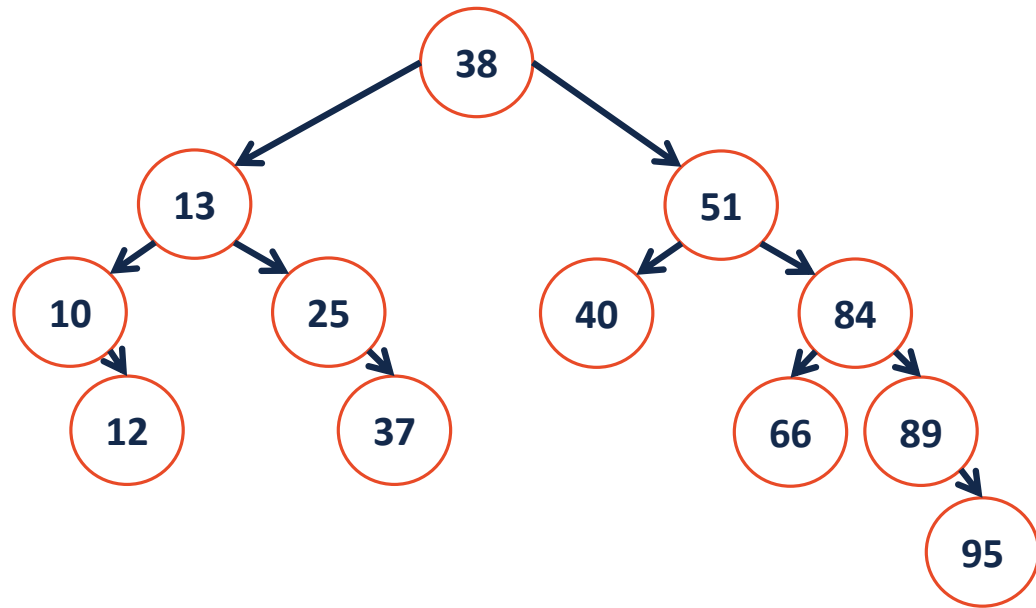
- Website <https://uiuc-cs199-225-fa21.netlify.app/>
- Tonight, is the first lecture for the honors section zoom info <https://uiuc-cs199-225-fa21.netlify.app/docs/>

```
1  template<typename K, typename V>
2  _____ _remove(TreeNode *& root, const K & key) {
3
4
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26 }
```

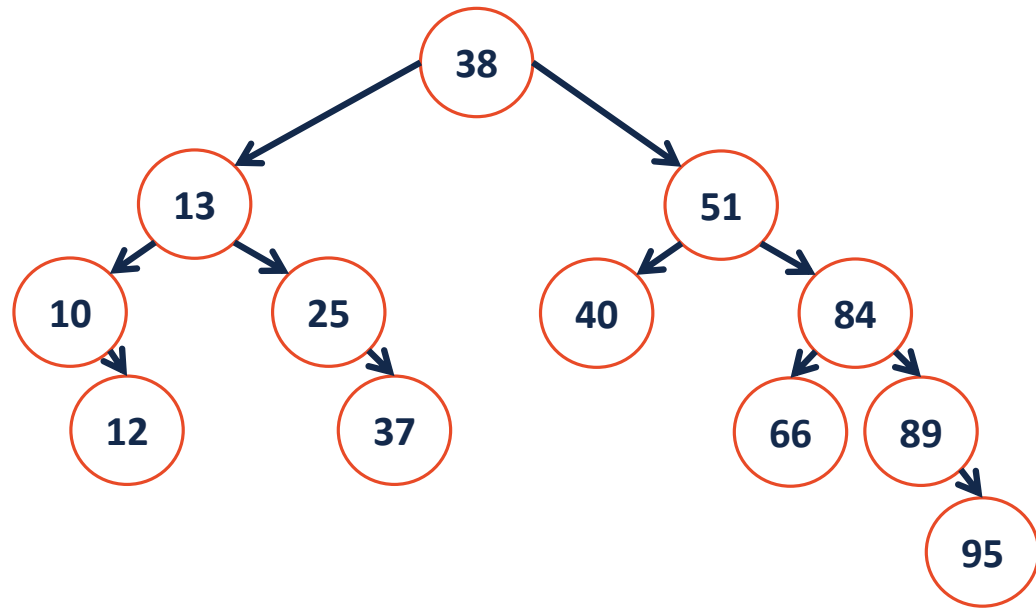




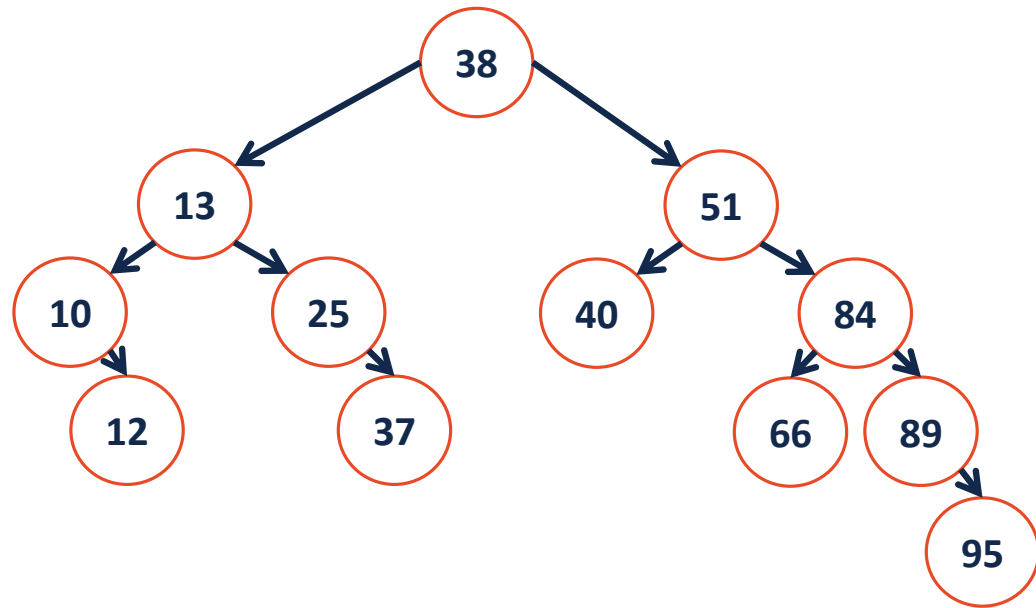
`remove (40) ;`



remove (25) ;



`remove(10);`



`remove (13) ;`

BST Analysis – Running Time

Operation	BST Worst Case
find	
insert	
delete	
traverse	



BST Analysis

Every operation that we have studied on a BST depends on the height of the tree: **$O(h)$** .

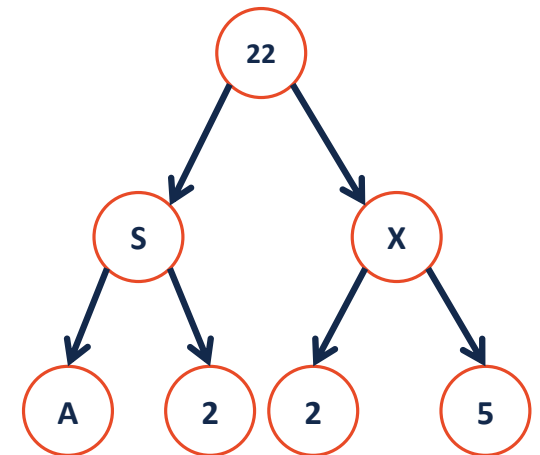
...what is this in terms of **n** , the amount of data?

We need a relationship between **h** and **n** :

$$f(h) \leq n \leq g(h)$$

BST Analysis

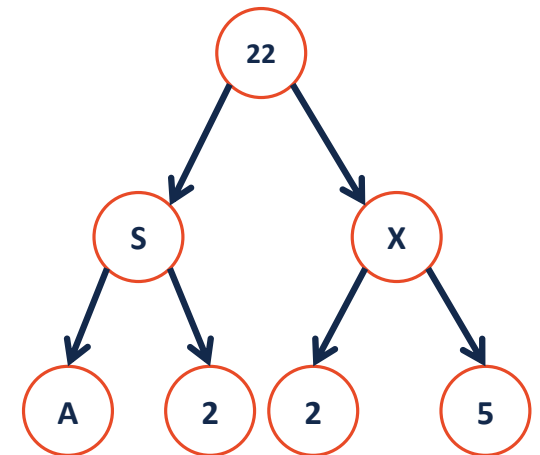
Q: What is the maximum number of nodes in a tree of height **h** ?



BST Analysis

Q: What is the minimum number of nodes in a tree of height h ?

What is the maximum height for a tree of n nodes?





BST Analysis

Therefore, for all BST:

Lower bound:

Upper bound:



BST Analysis

The height of a BST depends on the order in which the data is inserted into it.

ex: **1 3 2 4 5 7 6**

vs.

4 2 3 6 7 1 5

Q: How many different ways are there to insert keys into a BST?

Q: What is the average height of all the arrangements?



BST Analysis

Q: How many different ways are there to insert keys into a BST?

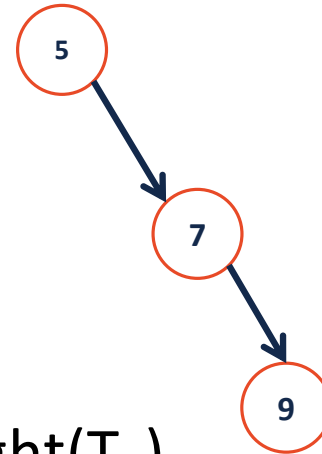
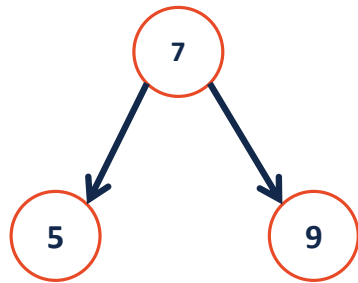
Q: What is the average height of all the arrangements?

BST Analysis – Running Time

Operation	BST Average case	BST Worst case	Sorted array	Sorted List
find				
insert				
delete				
traverse				

Height-Balanced Tree

What tree makes you happier?



Height balance: $b = \text{height}(T_L) - \text{height}(T_R)$

A tree is height balanced if: