October 1 – Binary Search Tree (BST)

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Binary Tree as a Search Structure
Binary Tree (BST)

A BST is a binary tree $T$ such that:
#pragma once

template <typename K, typename V>
class BST {
public:
    BST();
    void insert(const K key, V value);
    V remove(const K & key);
    V find(const K & key) const;
    TreeIterator traverse() const;

private:
    struct TreeNode {
        TreeNode *left, *right;
        K & key;
        V & value;
        TreeNode(K & k, V & v) : key(k), value(v), left(NULL),
        right(NULL) { }
        
    };
    TreeNode *head_;
template<typename K, typename V>

find(const K & key) const {
}
template<typename K, typename V>

TreeNode ** find(TreeNode ** root, const K & key) const
template<typename K, typename V>
void BST::_insert(TreeNode * & root, K & key, V & value) {
    TreeNode * t = _find(root, key);
    t = new TreeNode(key, value);
}
template<
typename K, 
typename V>

________________________ remove(TreeNode * & root, const K & key) {

}
remove(40);
remove(25);
remove(10);
remove(13);
<table>
<thead>
<tr>
<th>Operation</th>
<th>BST Worst Case</th>
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</thead>
<tbody>
<tr>
<td>find</td>
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<tr>
<td>insert</td>
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<tr>
<td>delete</td>
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<tr>
<td>traverse</td>
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BST Analysis

Every operation that we have studied on a BST depends on the height of the tree: $O(h)$.

...what is this in terms of $n$, the amount of data?

We need a relationship between $h$ and $n$:

$f(h) \leq n \leq g(h)$
BST Analysis

Q: What is the maximum number of nodes in a tree of height $h$?
BST Analysis

Q: What is the minimum number of nodes in a tree of height $h$?

What is the maximum height for a tree of $n$ nodes?
BST Analysis

Therefore, for all BST:

**Lower bound:**

**Upper bound:**
BST Analysis

The height of a BST depends on the order in which the data is inserted into it.

ex: 1 3 2 4 5 7 6       vs.       4 2 3 6 7 1 5

Q: How many different ways are there to insert keys into a BST?

Q: What is the average height of all the arrangements?
BST Analysis

Q: How many different ways are there to insert keys into a BST?

Q: What is the average height of all the arrangements?
## BST Analysis – Running Time

<table>
<thead>
<tr>
<th>Operation</th>
<th>BST Average case</th>
<th>BST Worst case</th>
<th>Sorted array</th>
<th>Sorted List</th>
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</thead>
<tbody>
<tr>
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