BTree Properties
For a BTree of order \( m \):
1. All keys within a node are ordered.
2. All leaves contain no more than \( m-1 \) nodes.
3. All internal nodes have exactly one more children than keys.
4. Root nodes can be a leaf or have \([2, m]\) children.
5. All non-root, internal nodes have \([\text{ceil}(m/2), m]\) children.
6. All leaves are on the same level.

BTree Analysis
The height of the BTree determines maximum number of ______________ possible in search data.

...and the height of our structure:

Therefore, the number of seeks is no more than: ____________.

...suppose we want to prove this!

BTree Proof #1
In our AVL Analysis, we saw finding an upper bound on the height \((h \text{ given } n, \text{ aka } h = f(n))\) is the same as finding a lower bound on the keys \((n \text{ given } h, \text{ aka } f^{-1}(h))\).

Goal: We want to find a relationship for BTrees between the number of keys \((n)\) and the height \((h)\).

BTree Strategy:
1. Define a function that counts the minimum number of nodes in a BTree of a given order.
   a. Account for the minimum number of keys per node.

2. Proving a minimum number of nodes provides us with an upper-bound for the maximum possible height.

Proof:
1a. The minimum number of nodes for a BTree of order \( m \) at each level is as follows:
   - root:
   - level 1:
   - level 2:
   - level 3:
   - level \( h \):

1b. The minimum total number of nodes is the sum of all levels:

2. The minimum number of keys:

3. Finally, we show an upper-bound on height:
So, how good are BTrees?
Given a BTree of order 101, how much can we store in a tree of height=4?

Minimum:

Maximum:

Range-based Searches:
Q: Consider points in 1D: \( p = \{ p_1, p_2, ..., p_n \} \).

...what points fall in \([11, 42]\)?

Tree Construction:

kd-Tree Construction:
How many dimensions exist in our input space?

How do we want to “order” our dimensions?

Range-based Searches:

Running Time:

Extending to k-dimensions:
Consider points in 2D: \( p = \{ p_1, p_2, ..., p_n \} \):

kd-Tree Motivation:
First, let’s try and divide our space up:

<table>
<thead>
<tr>
<th>CS 225 – Things To Be Doing:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.  Mp_traversals due today</td>
</tr>
<tr>
<td>2.  Potsds ongoing</td>
</tr>
<tr>
<td>3.  Exam 2 practice releases on Tuesday</td>
</tr>
</tbody>
</table>