



CS 225

Data Structures

November 18 – Graph Traversals

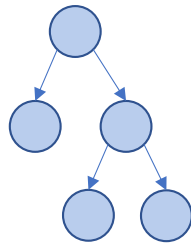
G Carl Evans

Traversal:

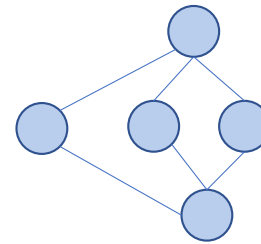
Objective: Visit every vertex and every edge in the graph.

Purpose: Search for interesting sub-structures in the graph.

We've seen traversal before ...but it's different:

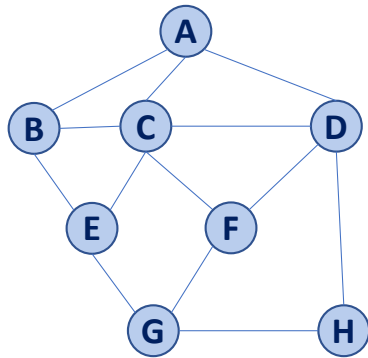


- Ordered
- Obvious Start
-

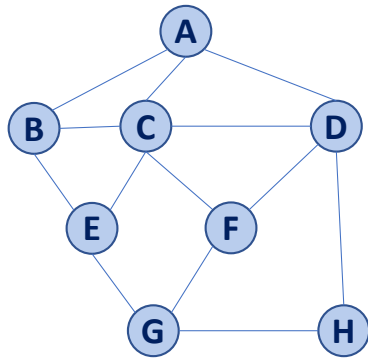


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Traversal: BFS



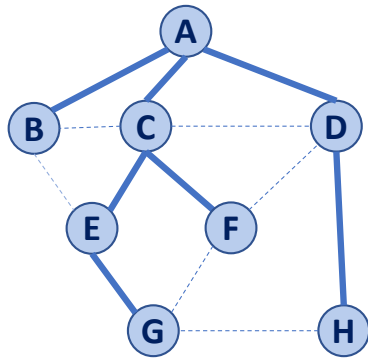
Traversal: BFS



v	d	P	Adjacent Edges
A			
B			
C			
D			
E			
F			
G			
H			



Traversal: BFS



v	d	P	Adjacent Edges
A	0	-	C B D
B	1	A	A C E
C	1	A	B A D E F
D	1	A	A C F H
E	2	C	B C G
F	2	C	C D G
G	3	E	E F H
H	2	D	D G

~~G H F E D B C A~~

```
1 BFS(G) :
2   Input: Graph, G
3   Output: A labeling of the edges on
4           G as discovery and cross edges
5
6   foreach (Vertex v : G.vertices()):
7     setLabel(v, UNEXPLORED)
8   foreach (Edge e : G.edges()):
9     setLabel(e, UNEXPLORED)
10  foreach (Vertex v : G.vertices()):
11    if getLabel(v) == UNEXPLORED:
12      BFS(G, v)
```

```
14 BFS(G, v) :
15   Queue q
16   setLabel(v, VISITED)
17   q.enqueue(v)
18
19   while !q.empty():
20     v = q.dequeue()
21     foreach (Vertex w : G.adjacent(v)):
22       if getLabel(w) == UNEXPLORED:
23         setLabel(v, w, DISCOVERY)
24         setLabel(w, VISITED)
25         q.enqueue(w)
26       elseif getLabel(v, w) == UNEXPLORED:
27         setLabel(v, w, CROSS)
```



BFS Analysis

Q: Does our implementation handle disjoint graphs?
If so, what code handles this?

- *How do we use this to count components?*

Q: Does our implementation detect a cycle?

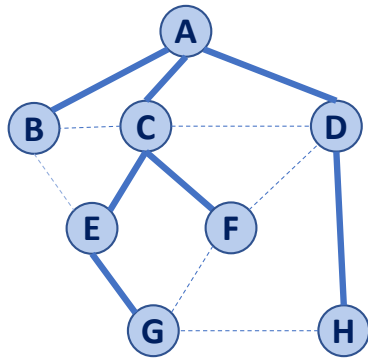
- *How do we update our code to detect a cycle?*

Q: What is the running time?


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Running time of BFS



While-loop at **:19**?

For-loop at **:21**?

v	d	P	Adjacent Edges
A	0	-	C B D
B	1	A	A C E
C	1	A	B A D E F
D	1	A	A C F H
E	2	C	B C G
F	2	C	C D G
G	3	E	E F H
H	2	D	D G



BFS Observations

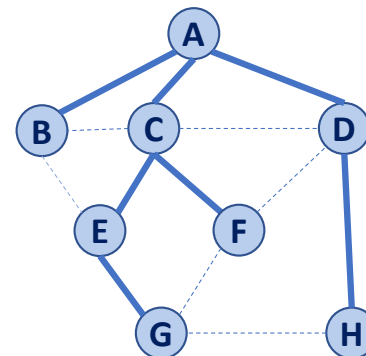
Q: What is a shortest path from **A** to **H**?

Q: What is a shortest path from **E** to **H**?

Q: How does a cross edge relate to **d**?

Q: What structure is made from discovery edges?

v	d	P	Adjacent Edges
A	0	-	C B D
B	1	A	A C E
C	1	A	B A D E F
D	1	A	A C F H
E	2	C	B C G
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G	3	E	E F H
H	2	D	D G





BFS Observations

Obs. 1: BFS can be used to count components.

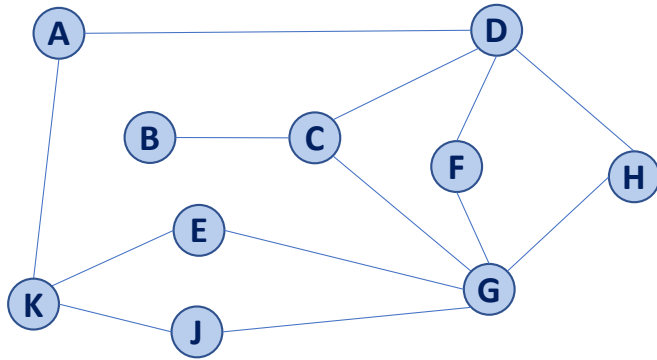
Obs. 2: BFS can be used to detect cycles.

Obs. 3: In BFS, d provides the shortest distance to every vertex.

Obs. 4: In BFS, the endpoints of a cross edge never differ in distance, d , by more than 1:

$$|d(u) - d(v)| = 1$$

Traversal: DFS



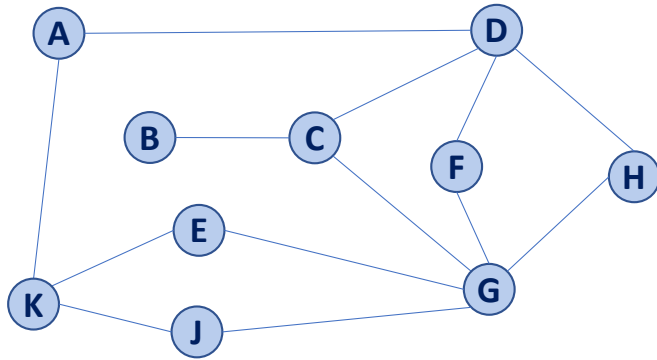
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25               q.enqueue(w)
26           elseif getLabel(v, w) == UNEXPLORED:
27               setLabel(v, w, CROSS)
```

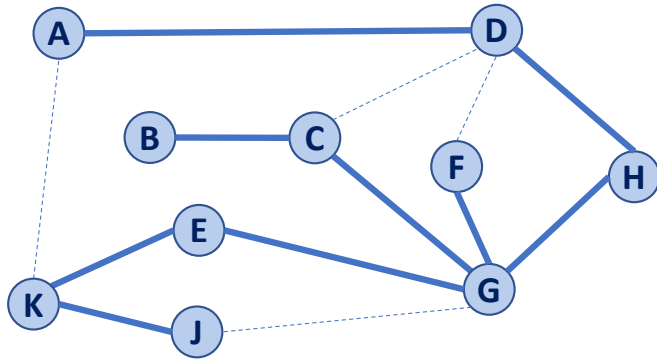
```
1 DFS(G) :
2   Input: Graph, G
3   Output: A labeling of the edges on
4           G as discovery and back edges
5
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8   foreach (Edge e : G.edges()):
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24       setLabel(w, VISITED)
25       DFS(G, w)
26     elseif getLabel(v, w) == UNEXPLORED:
27       setLabel(v, w, BACK)
```

Traversal: DFS



Traversal: DFS



————— Discovery Edge

----- Back Edge

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25       DFS(G, w)
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27       setLabel(v, w, BACK)
```

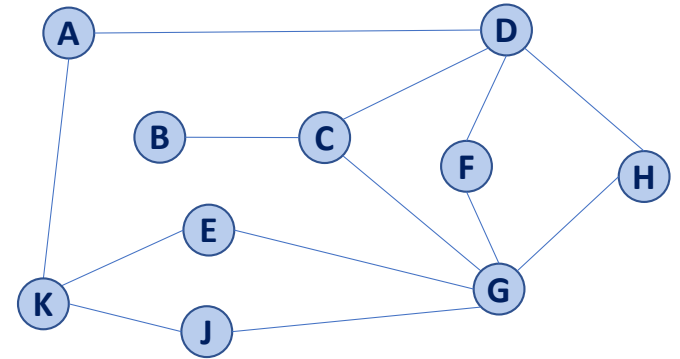
Running time of DFS

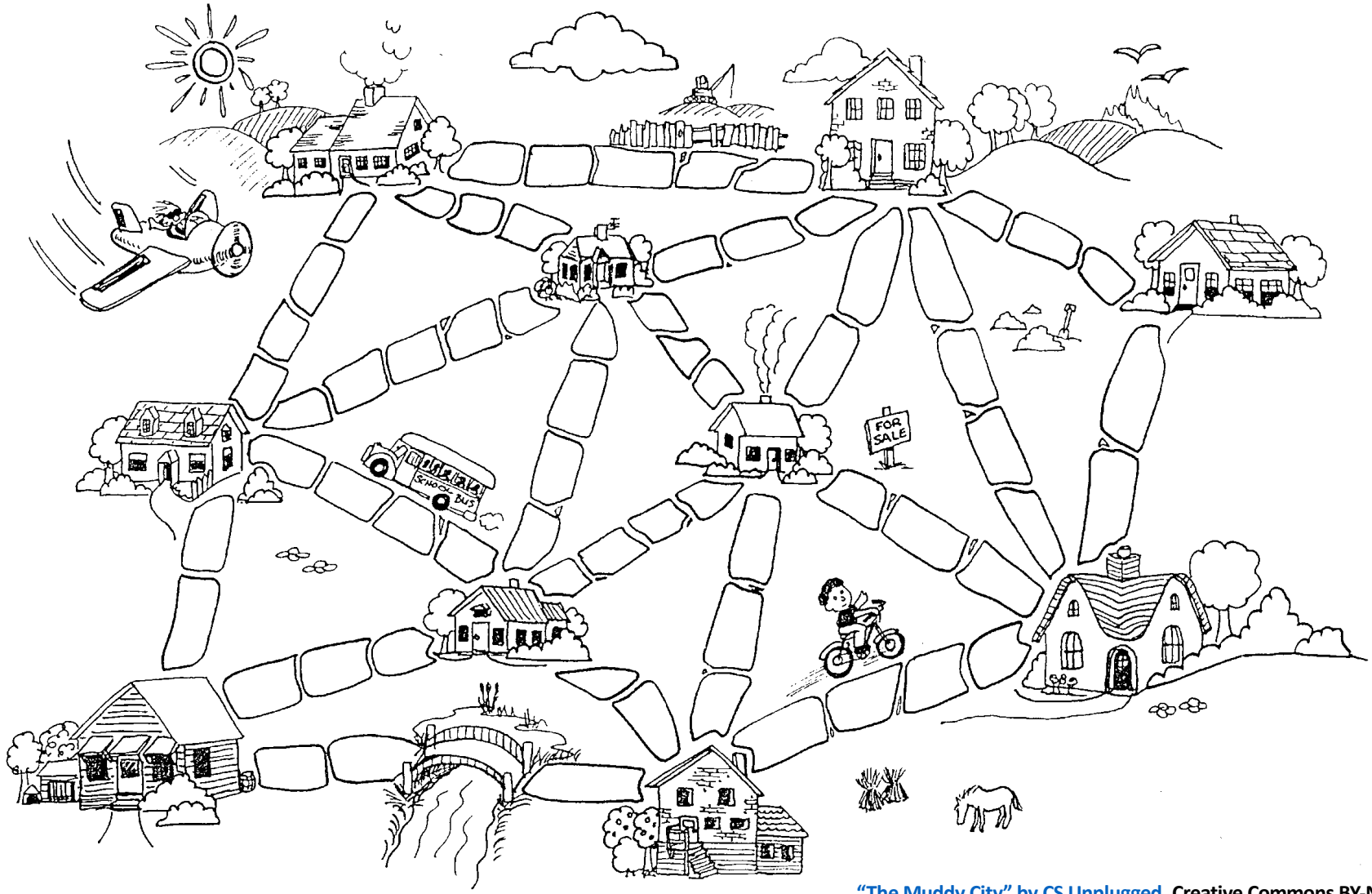
Labeling:

- Vertex:
- Edge:

Queries:

- Vertex:
- Edge:





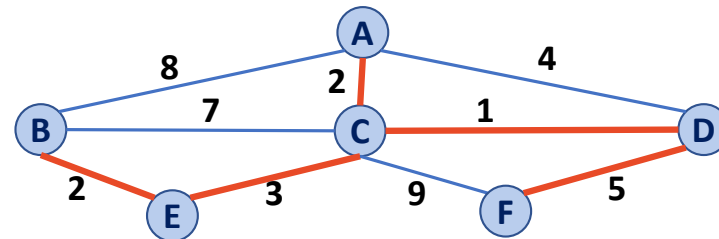
["The Muddy City"](#) by CS Unplugged, Creative Commons BY-NC-SA 4.0

Minimum Spanning Tree Algorithms

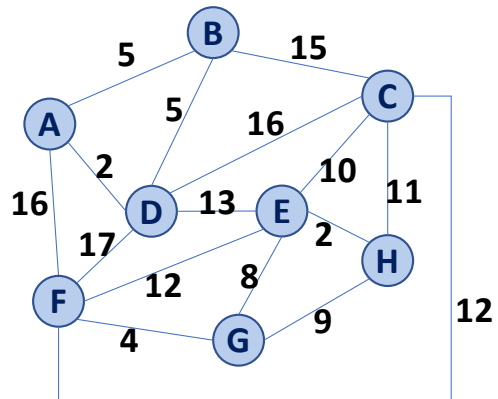
Input: Connected, undirected graph G with edge weights (unconstrained, but must be additive)

Output: A graph G' with the following properties:

- G' is a spanning graph of G
- G' is a tree (connected, acyclic)
- G' has a minimal total weight among all spanning trees

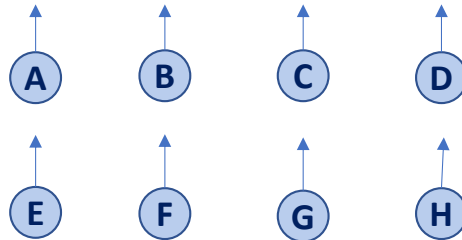
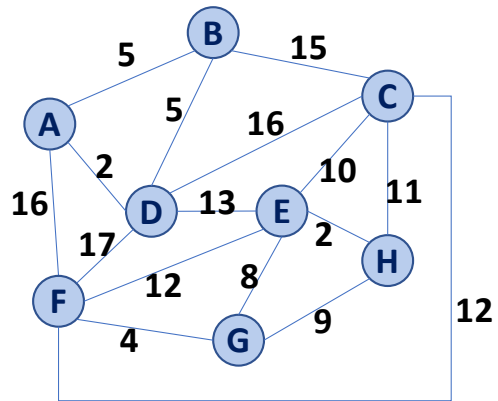


Kruskal's Algorithm



(A, D)
(E, H)
(F, G)
(A, B)
(B, D)
(G, E)
(G, H)
(E, C)
(C, H)
(E, F)
(F, C)
(D, E)
(B, C)
(C, D)
(A, F)
(D, F)

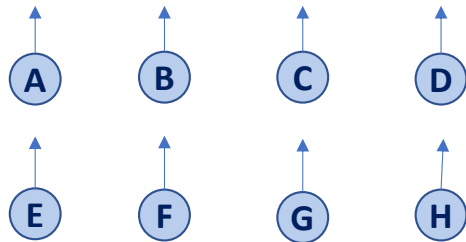
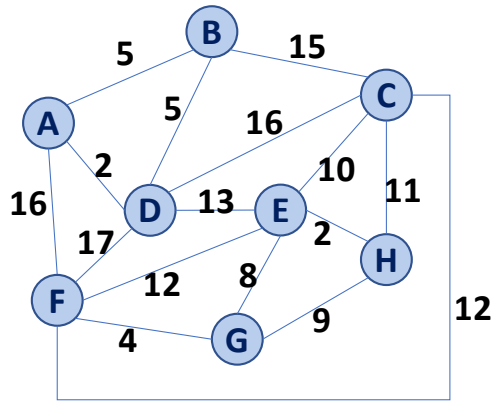
Kruskal's Algorithm



(A, D)
(E, H)
(F, G)
(A, B)
(B, D)
(G, E)
(G, H)
(E, C)
(C, H)
(E, F)
(F, C)
(D, E)
(B, C)
(C, D)
(A, F)
(D, F)

Kruskal's Algorithm

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(E, H)
(F, G)
(A, B)
(B, D)
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(G, H)
(E, C)
(C, H)
(E, F)
(F, C)
(D, E)
(B, C)
(C, D)
(A, F)
(D, F)



```

1 KruskalMST(G):
2   DisjointSets forest
3   foreach (Vertex v : G):
4     forest.makeSet(v)
5
6   PriorityQueue Q // min edge weight
7   foreach (Edge e : G):
8     Q.insert(e)
9
10  Graph T = (V, {})
11
12  while |T.edges()| < n-1:
13    Vertex (u, v) = Q.removeMin()
14    if forest.find(u) == forest.find(v):
15      T.addEdge(u, v)
16      forest.union( forest.find(u),
17                  forest.find(v) )
18
19  return T

```

Kruskal's Algorithm

Priority Queue:	Heap	Sorted Array
Building :7-9		
Each removeMin :13		

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Kruskal's Algorithm

Priority Queue:	Total Running Time
Heap	
Sorted Array	

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