



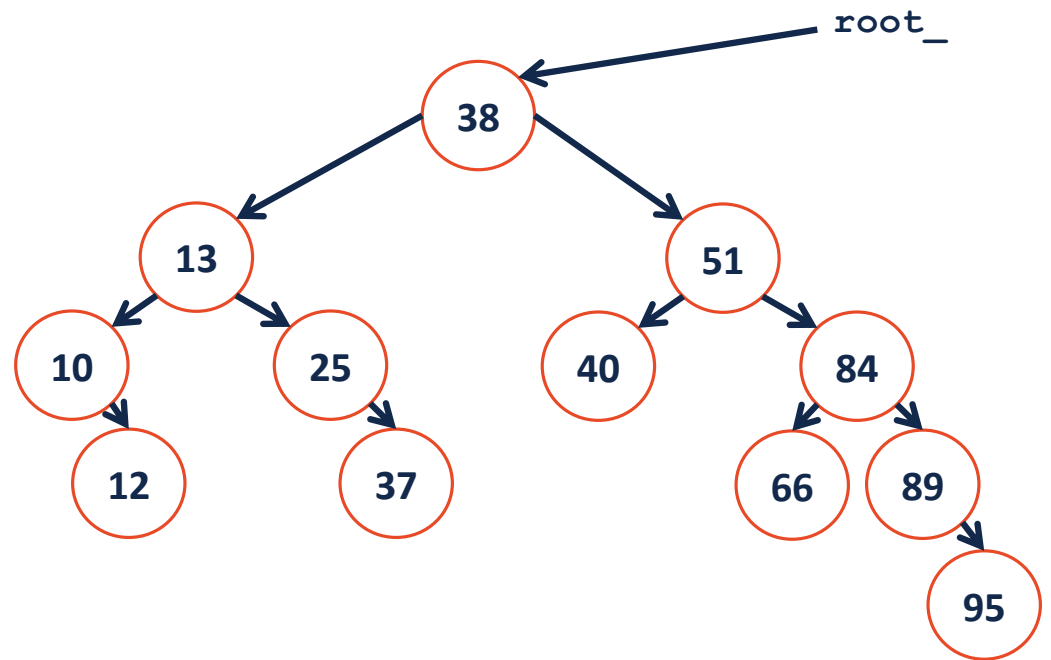
# CS 225

## Data Structures

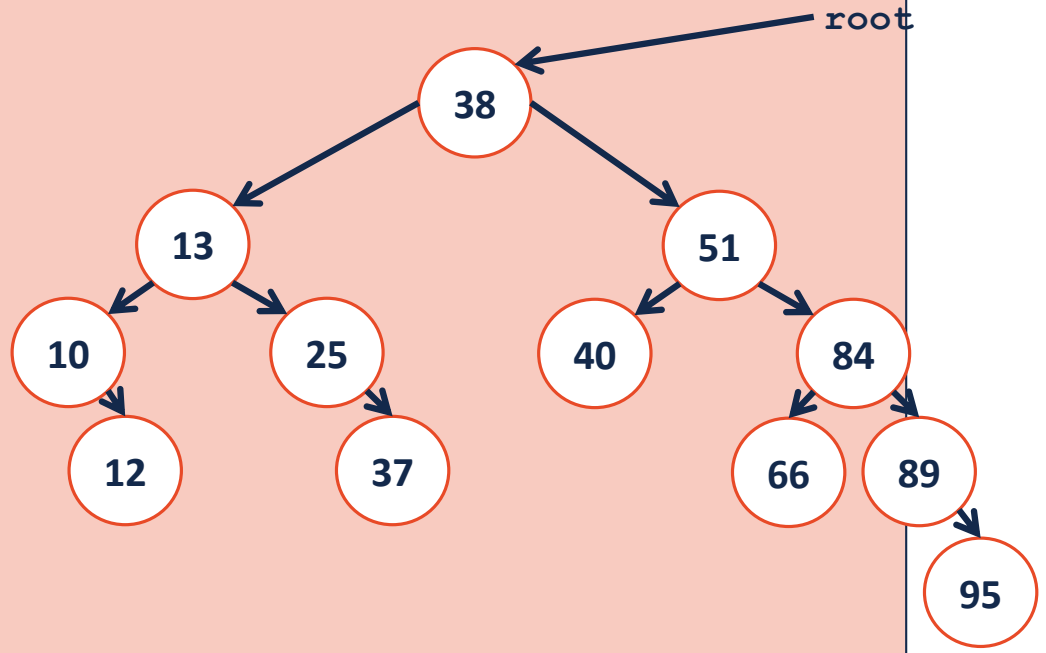
*October 7 – BST Remove*

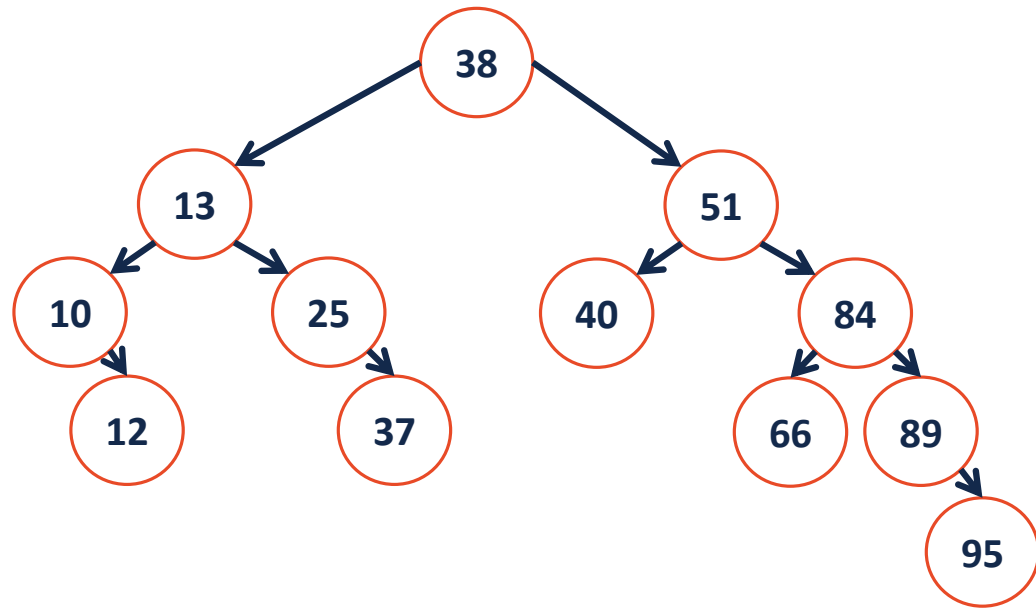
*G Carl Evans*

```
1 template<typename K, typename V>
2
3 void BST::_insert(TreeNode *& root, K & key, V & value) {
4     if(root == nullptr) {
5         TreeNode * t = _find(root, key);
6         t = new TreeNode(key, value);
7     }
8 }
```

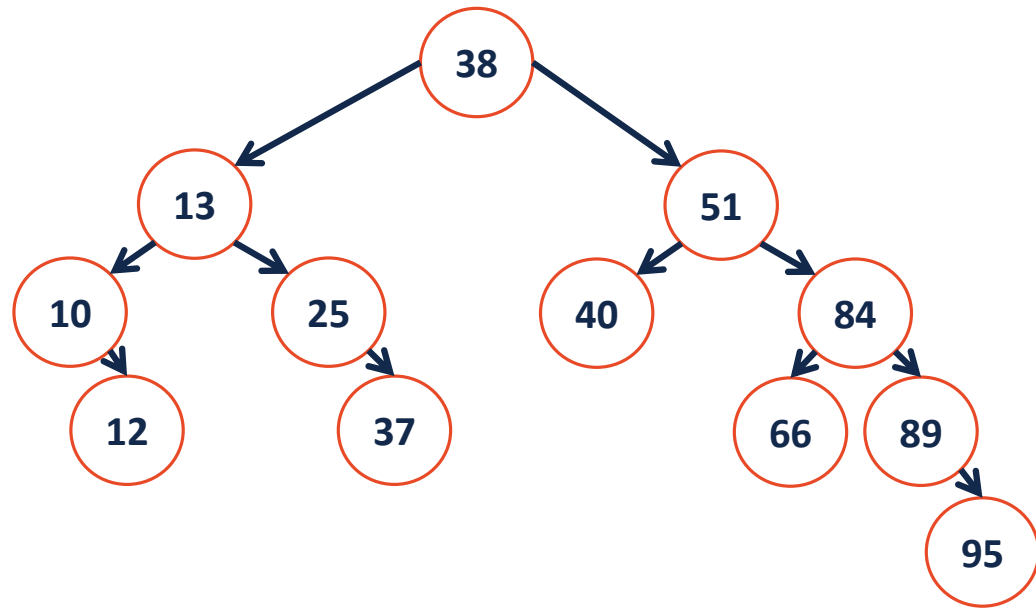


```
1 template<typename K, typename V>
2 _____ _remove(TreeNode *& root, const K & key) {
3
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26 }
```

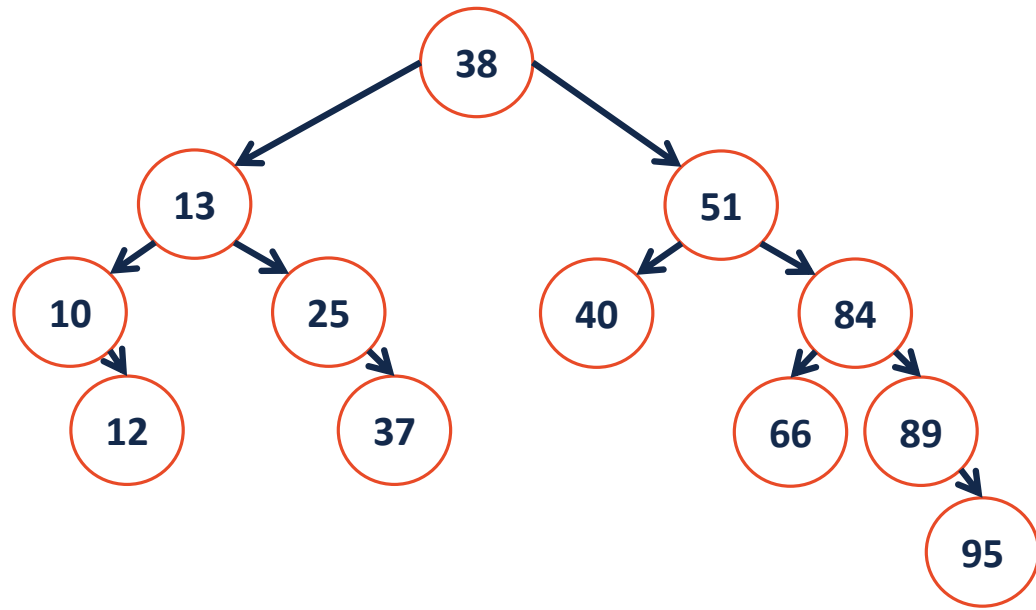




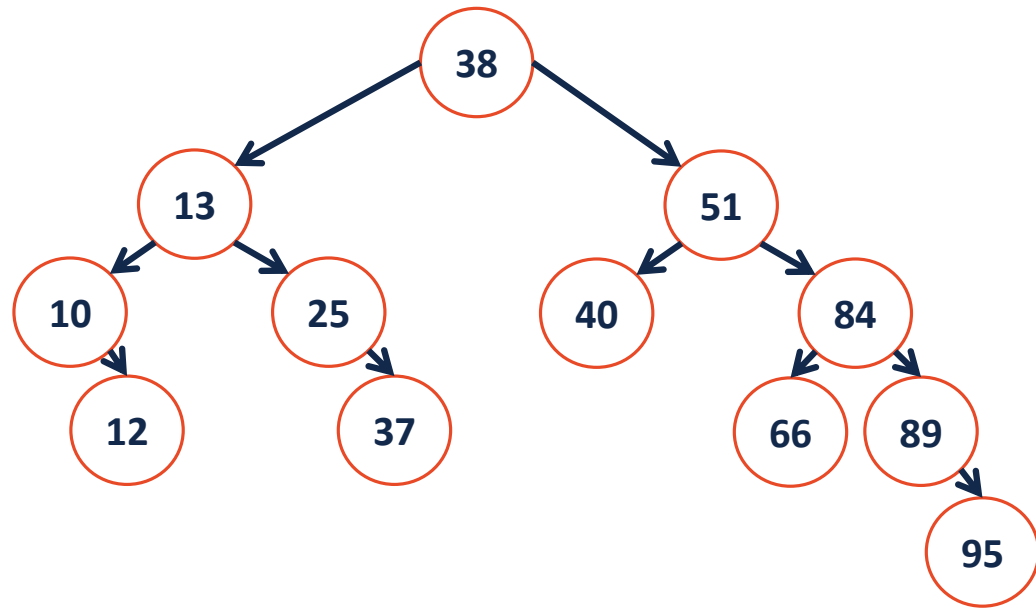
`remove (40) ;`



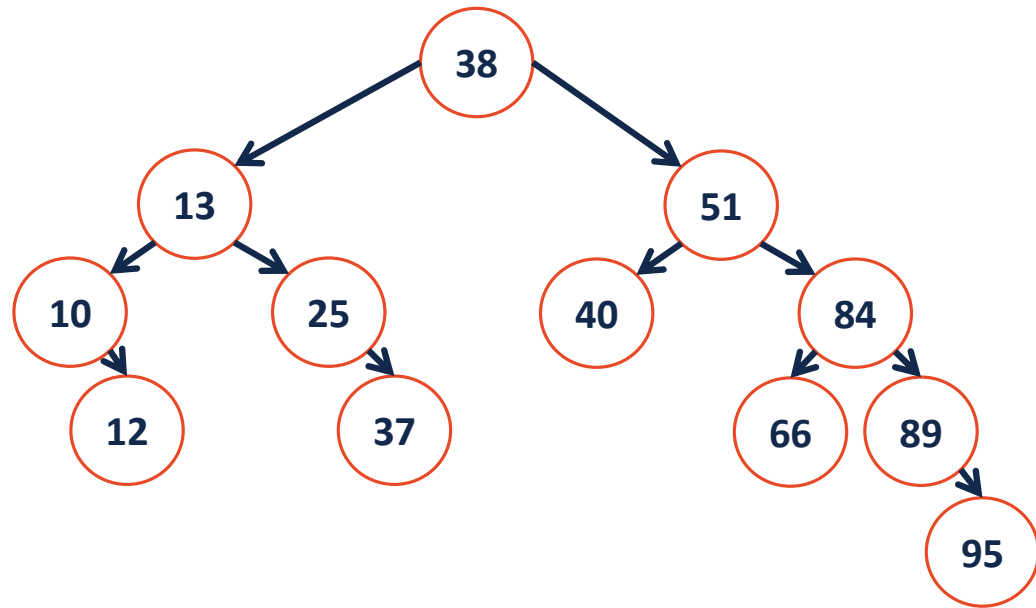
**remove (25) ;**



`remove(10);`



`remove (13) ;`



`remove (38) ;`



# BST Analysis – Running Time

Operation	BST Worst Case
find	
insert	
delete	
traverse	



## BST Analysis

Every operation that we have studied on a BST depends on the height of the tree:  **$O(h)$** .

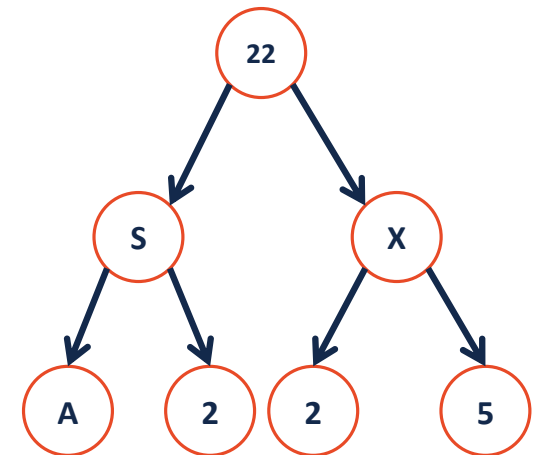
...what is this in terms of  **$n$** , the amount of data?

We need a relationship between  **$h$**  and  **$n$** :

$$f(h) \leq n \leq g(h)$$

# BST Analysis

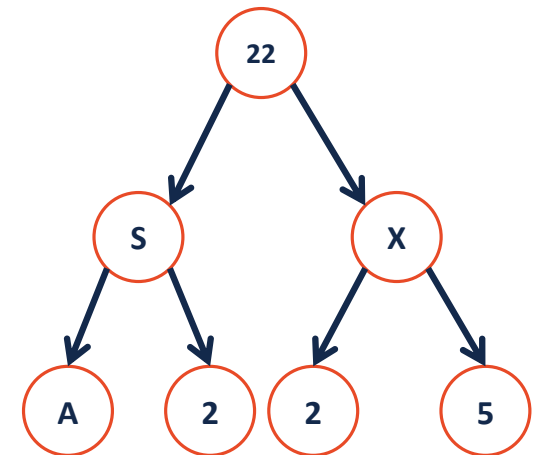
**Q:** What is the maximum number of nodes in a tree of height  $h$ ?



# BST Analysis

**Q:** What is the minimum number of nodes in a tree of height  $h$ ?

What is the maximum height for a tree of  $n$  nodes?





# BST Analysis

Therefore, for all BST:

**Lower bound:**

**Upper bound:**



## BST Analysis

The height of a BST depends on the order in which the data is inserted into it.

**ex: 1 3 2 4 5 7 6**

**vs.**

**4 2 3 6 7 1 5**



## BST Analysis

**Q:** How many different ways are there to insert keys into a BST?

**Q:** What is the average height of all the arrangements?

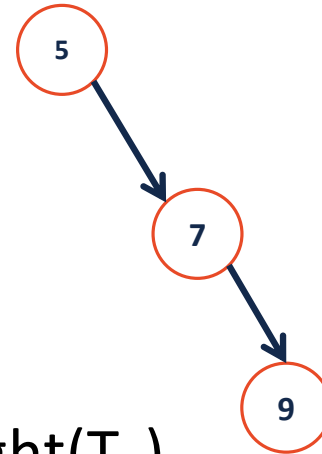
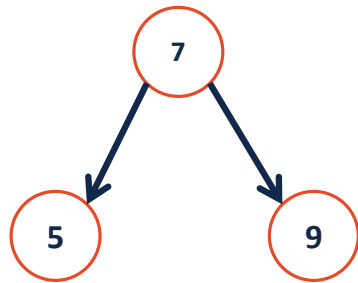
# BST Analysis – Running Time

Operation	BST Average case	BST Worst case	Sorted array	Sorted List
<b>find</b>				
<b>insert</b>				
<b>delete</b>				
<b>traverse</b>				



# Height-Balanced Tree

What tree makes you happier?



Height balance:  $b = \text{height}(T_L) - \text{height}(T_R)$

A tree is height balanced if: