#24: BTree Analysis

October 21, 2020 · *G Carl Evans*

BTree$_m$

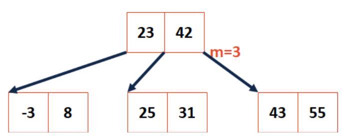
Goal: Build a tree that uses ____________________ /node!

…optimize the algorithm for your platform!

A **BTree of order m** is an m-way tree where:

1. All keys within a node are ordered.

BTree Insert, m=3:



Great interactive visualization of BTrees:

https://www.cs.usfca.edu/~galles/visualization/BTree.html

BTree Properties

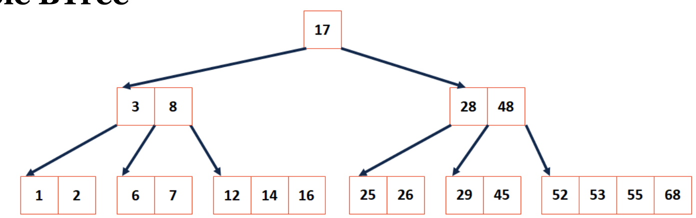
For a BTree of order **m**:

- All keys within a node are ordered.
- All leaves contain no more than **m-1** nodes.

- All internal nodes have exactly **one more child than keys**.
- Root nodes can be a leaf or have **[2, m]** children.
- All non-root, internal nodes have **[ceil(m/2), m]** children.

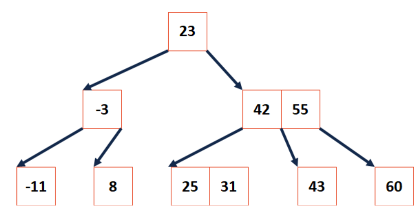
- All leaves are on the same level.

Example BTree



What properties do we know about this BTree?

BTree Search



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BTree Analysis

The height of the BTree determines maximum number of ____________ possible in search data.

...and the height of our structure:

Therefore, the number of seeks is no more than: ___________.

...suppose we want to prove this!

BTree Proof #1

In our AVL Analysis, we saw finding an **upper bound** on the height (**h** given **n,** aka **h = f(n)**) is the same as finding a **lower bound** on the keys (**n** given **h,** aka $\mathbf{f^{-1}(h)}$).

Goal: We want to find a relationship for BTrees between the number of keys (**n**) and the height (**h**).

BTree Strategy:

1. Define a function that counts the minimum number of nodes in a BTree of a given order.
   a. Account for the minimum number of keys per node.
2. Proving a minimum number of nodes provides us with an upper-bound for the maximum possible height.

Proof:

1a. The minimum number of <u>nodes</u> for a BTree of order **m** at each level is as follows:

root:

level 1:

level 2:

level 3:

level h:

1b. The minimum total number of <u>nodes</u> is the sum of all levels:

2. The minimum number of keys:

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3. Finally, we show an upper-bound on height:

CS 225 – Things To Be Doing:
1. mp_mosaic ec deadline Monday **2.** Daily POTDs stopped for break **3.** No class Friday for Exam 3