CS 225

Data Structures

October 16 – kd-Tree and Btrees Intro

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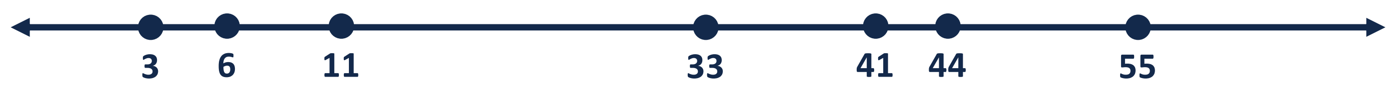
Range-based Searches

Balanced BSTs are useful structures for range-based and nearest-neighbor searches.

Q: Consider points in 1D: $\mathbf{p = \{p_1, p_2, \ldots, p_n\}}$.

…what points fall in [11, 42]?

Ex:



Range-based Searches

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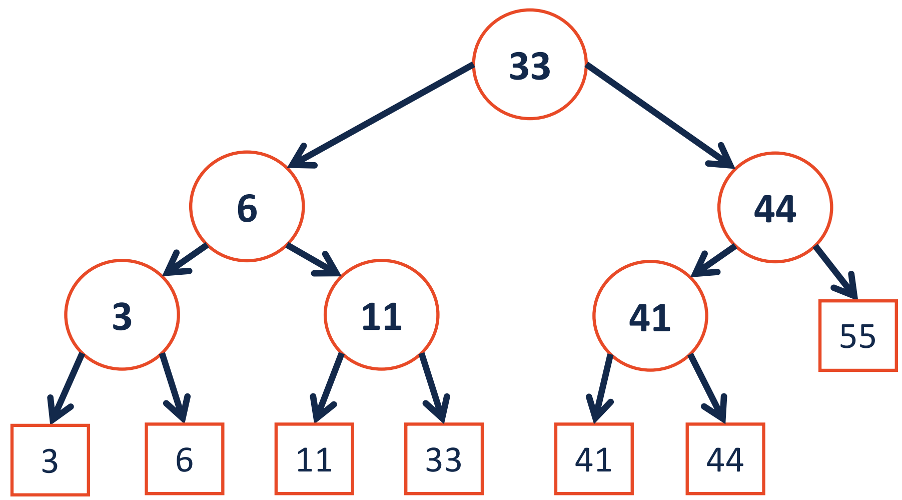
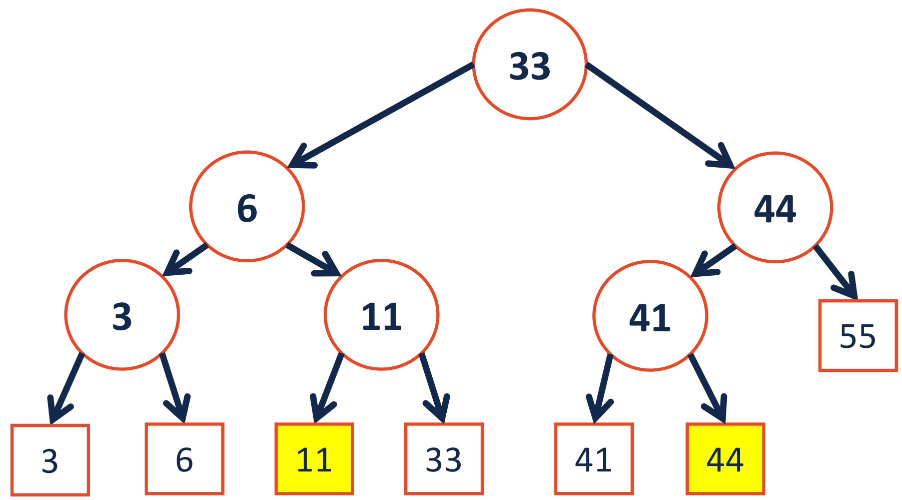
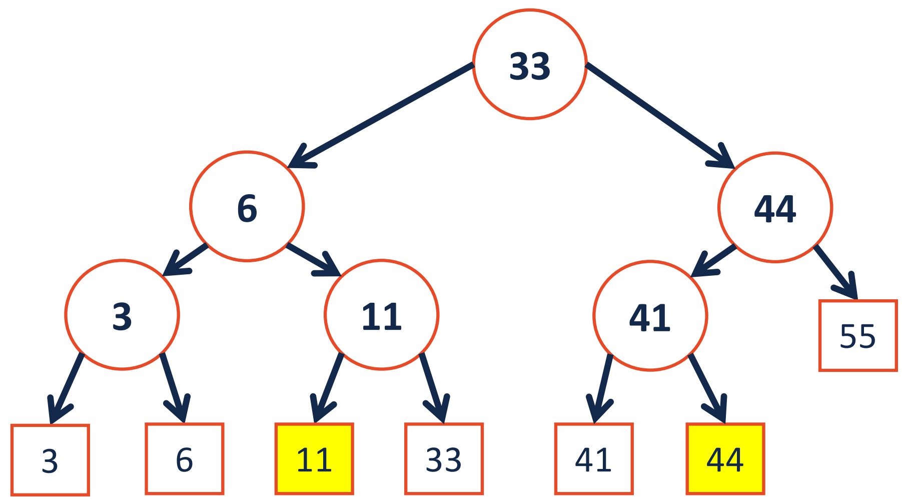
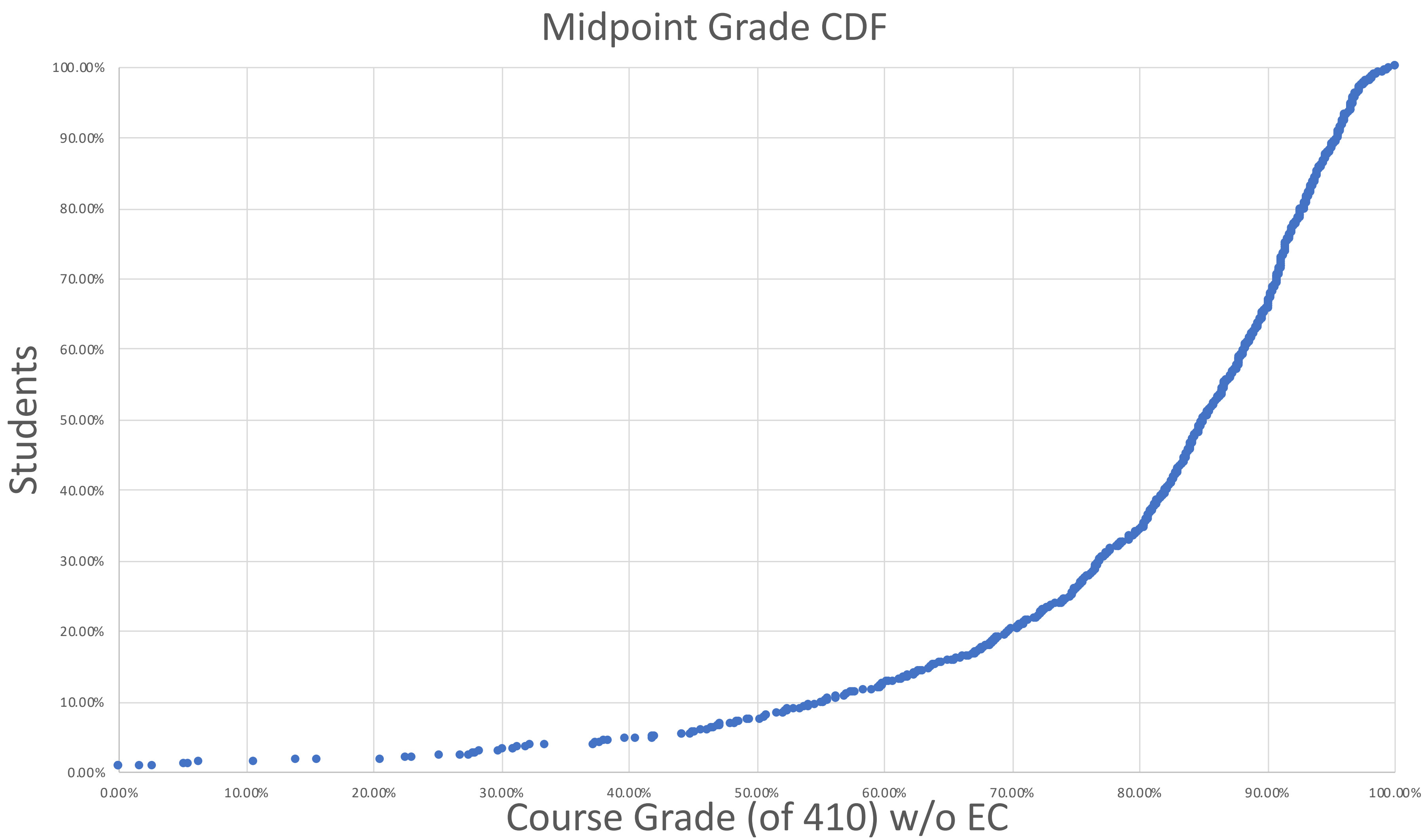


Range-based Searches



Running Time




Midpoint Grade CDF
Students
100.00%
90.00%
80.00%
70.00%
60.00%
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40.00%
50.00%
60.00%
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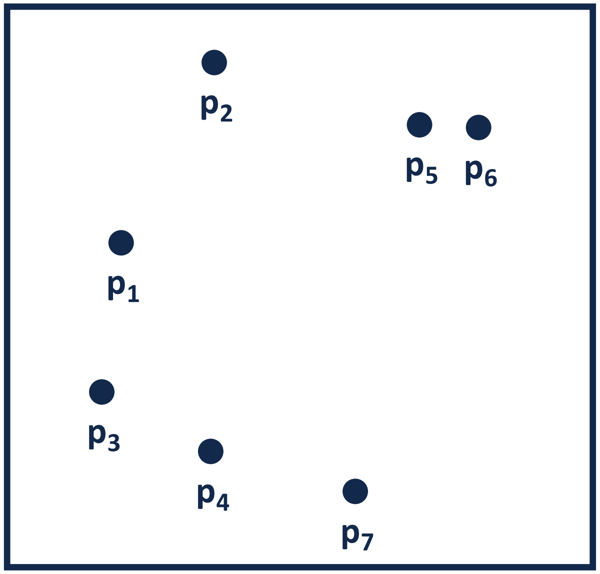
80.00%
90.00%
100.00%
Course Grade (of 410) w/o EC

Range-based Searches

Consider points in 2D: $\mathbf{p = \{p_1, p_2, \ldots, p_n\}}$.

Q: What points are in the rectangle:
[(x_1, y_1), (x_2, y_2)]?

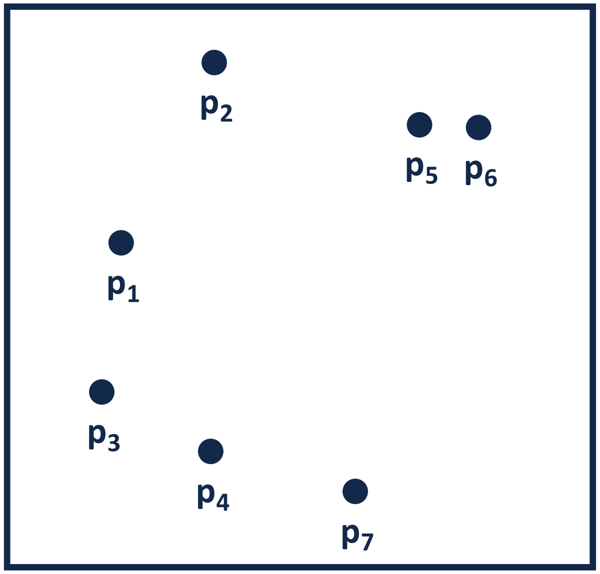


Q: What is the nearest point to $\mathbf{(x_1, y_1)}$?

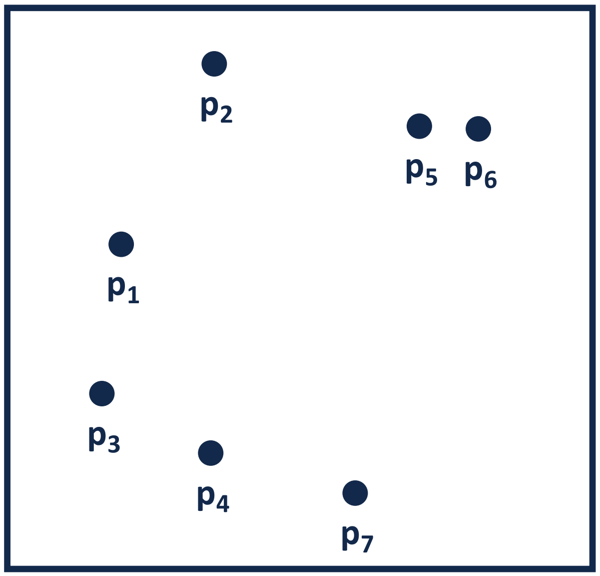
Range-based Searches

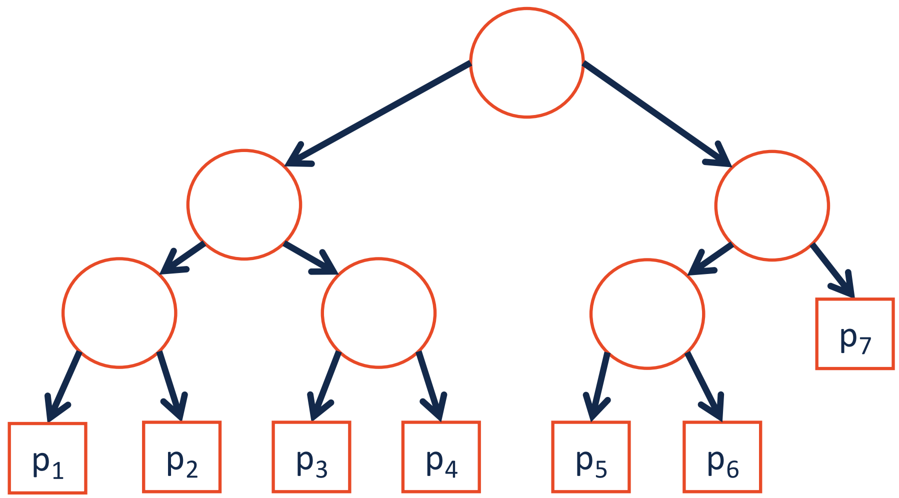
Consider points in 2D: $\mathbf{p} = \{\mathbf{p_1}, \mathbf{p_2}, \ldots, \mathbf{p_n}\}$.

Space divisions:

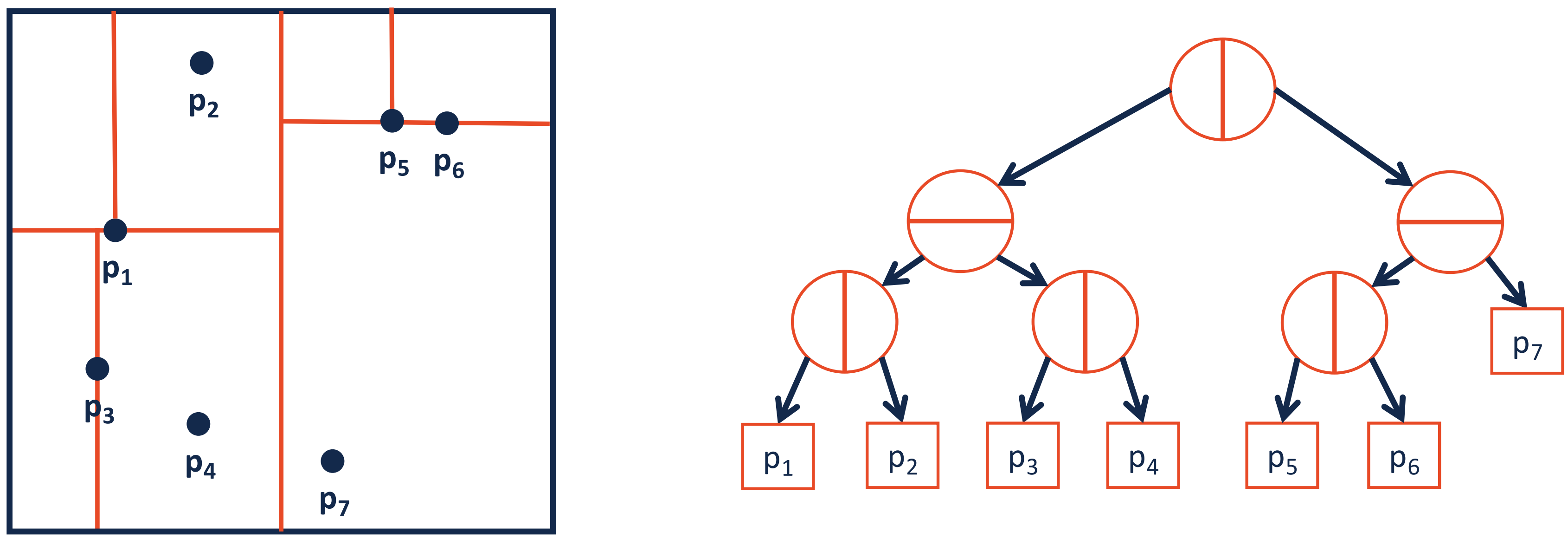


Range-based Searches

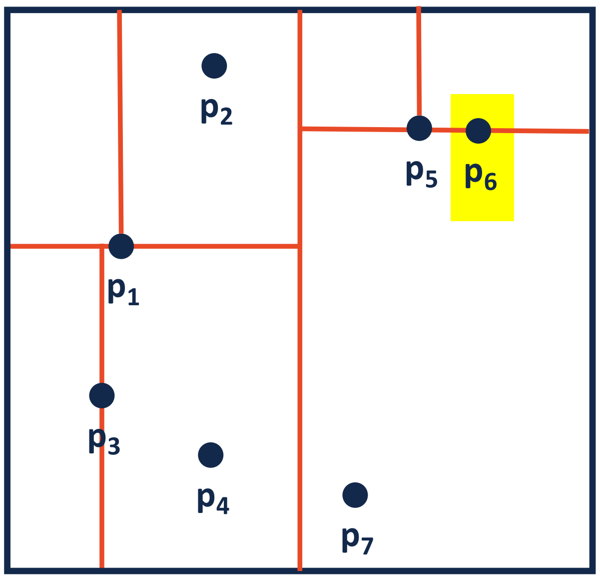


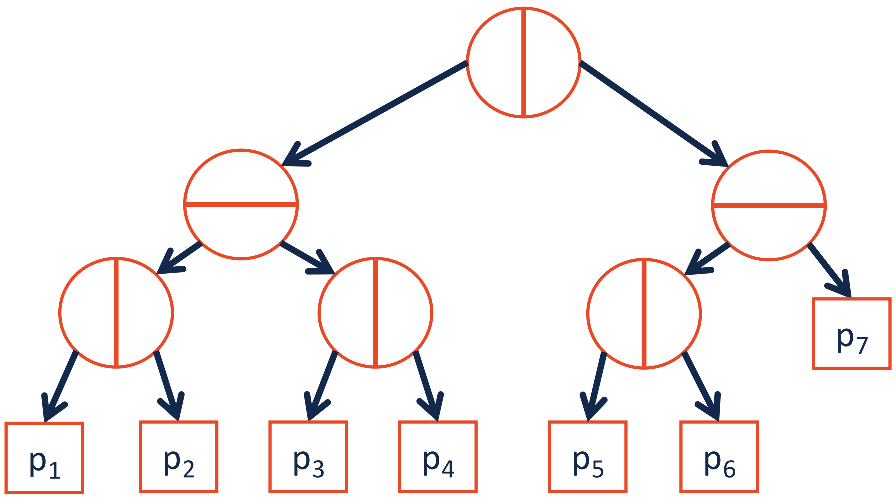


kD-Trees

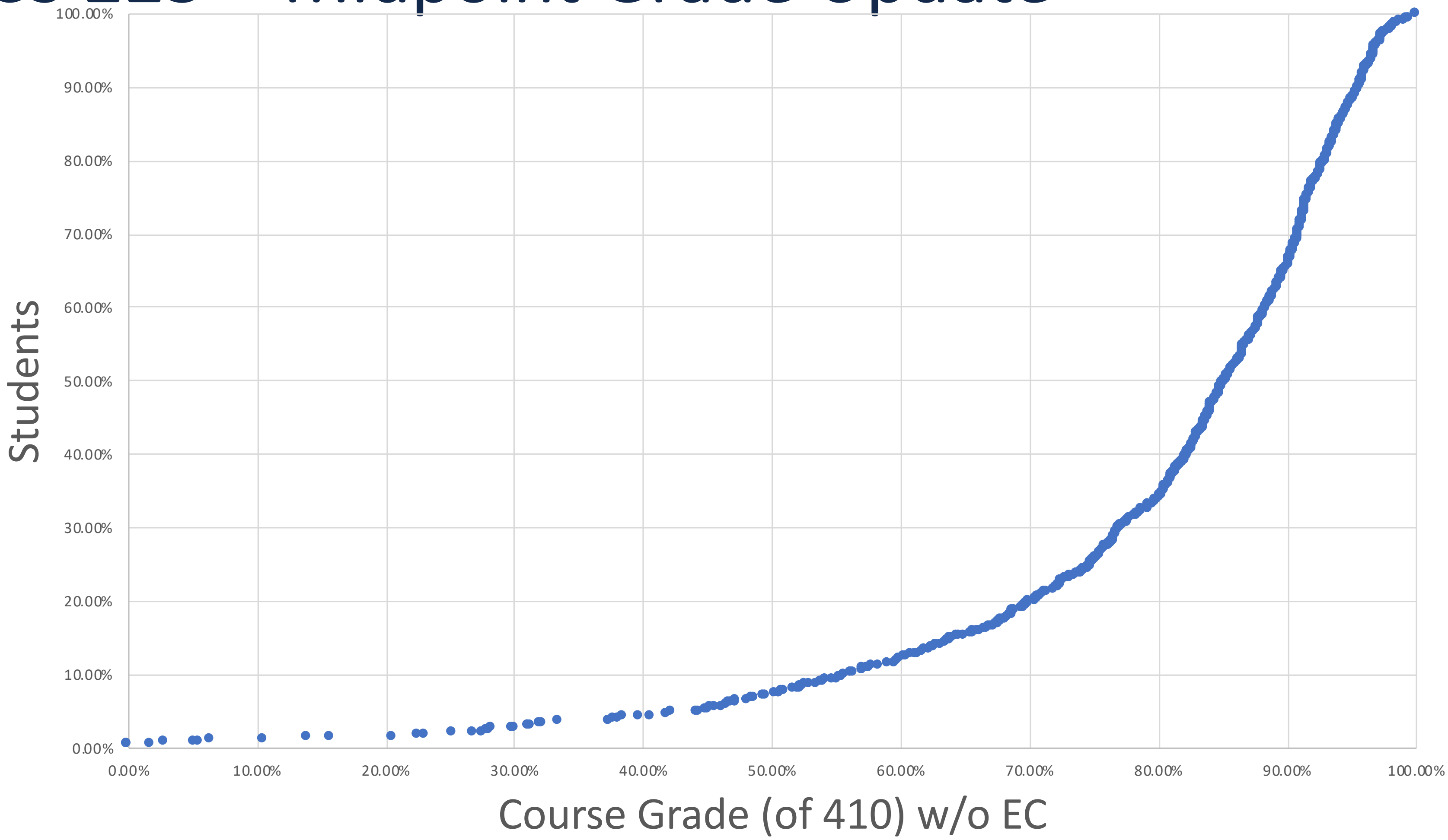


kD-Trees





CS 225 – Midpoint Grade Update



B-Trees

B-Trees

Q: Can we always fit our data in main memory?

Q: Where else can we keep our data?

However, Our big-O has assumed uniform time for all operations.

Vast Differences in Time

A **3GHz** CPU performs 3m operations in _______.

Old Argument: "Disk Storage is Slow"

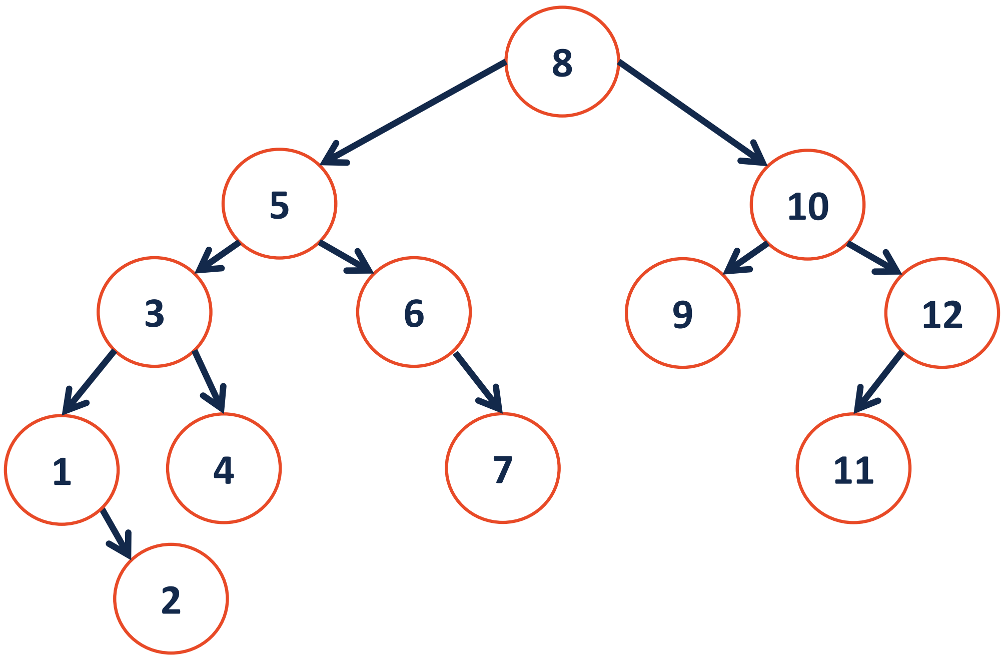
- Bleeding-edge storage is pretty fast:

    SSD

- Large Disks (25 TB+) still have slow throughout:

New Argument: "The Cloud is Slow!"

AVLs on Disk



Real Application

Imagine storing driving records for everyone in the US:

How many records?

How much data in total?

How deep is the AVL tree?

BTree Motivations

Knowing that we have large seek times for data, we want to:

BTree (of order m)

-3	8	23	25	31	42	43	55

m=9

Goal: Minimize the number of reads!

Build a tree that uses ______________________ / node

[1 network packet]

[1 disk block]

BTree Insertion

A **BTrees** of order **m** is an m-way tree:

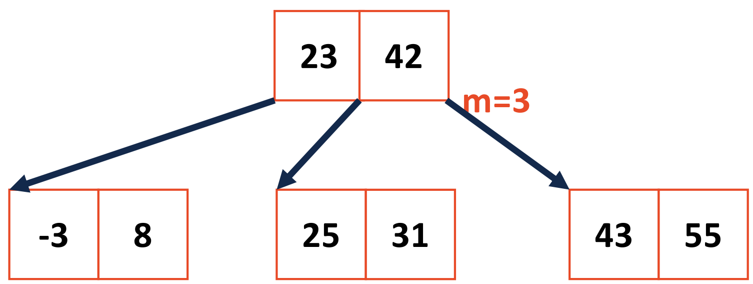
- All keys within a node are ordered
- All leaves contain hold no more than **m-1** nodes.

BTree Insertion

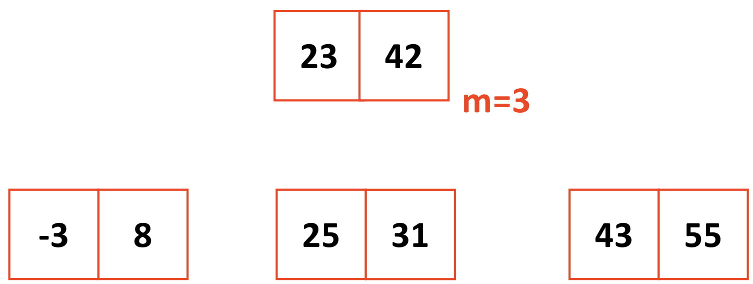
When a BTree node reaches **m** keys:

m=5

BTree Recursive Insert



BTree Recursive Insert



BTree Visualization/Tool

https://www.cs.usfca.edu/~galles/visualization/BTree.html

Btree Properties

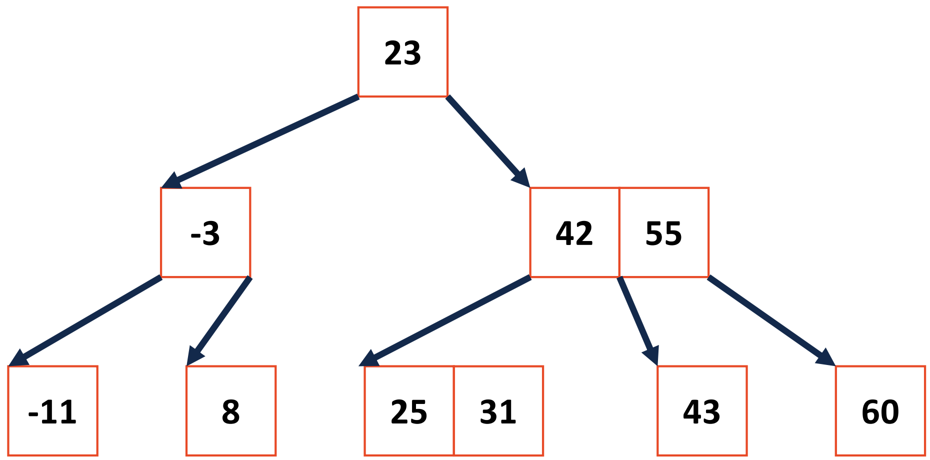
A **BTrees** of order **m** is an m-way tree:

- All keys within a node are ordered
- All leaves contain hold no more than **m-1** nodes.

- All internal nodes have exactly **one more key than children**
- Root nodes can be a leaf or have **[2, m]** children.
- All non-root, internal nodes have **[ceil(m/2), m]** children.

- All leaves are on the same level

BTree Search



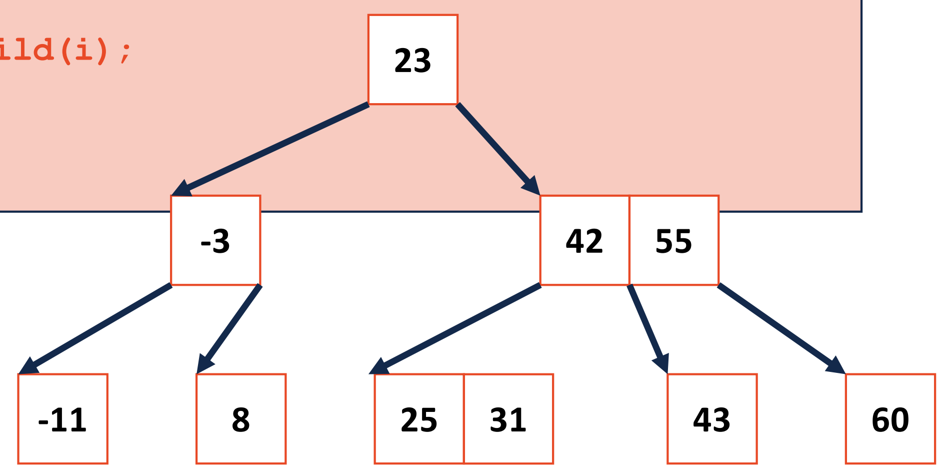
BTree Search

```
bool Btree::_exists(BTreeNode & node, const K & key) {

  unsigned i;
  for ( i = 0; i < node.keys_ct_ && key < node.keys_[i]; i++) { }

  if ( i < node.keys_ct_ && key == node.keys_[i] ) {
    return true;
  }

  if ( node.isLeaf() ) {
    return false;
  } else {
    BTreeNode nextChild = node._fetchChild(i);
    return _exists(nextChild, key);
  }
}
```



BTree Analysis

The height of the BTree determines maximum number of _____________ possible in search data.

…and the height of the structure is: _______________.

Therefore: The number of seeks is no more than ___________.

…suppose we want to prove this!

BTree Analysis

In our AVL Analysis, we saw finding an upper bound on the height (given **n**) is the same as finding a lower bound on the nodes (given **h**).

We want to find a relationship for BTrees between the number of keys (**n**) and the height (**h**).