CS 225
Data Structures

October 15 – AVL Applications
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On Friday, we proved an upper-bound on the height of an AVL tree is $2 \times \log(n)$ or $\mathcal{O}(\log(n))$:

$$N(h) := \text{Minimum \# of nodes in an AVL tree of height } h$$
$$N(h) = 1 + N(h-1) + N(h-2)$$
$$> N(h-1) + N(h-2)$$
$$> 2 \times N(h-2)$$
$$> 2^{h/2}$$

**Theorem #1:**
Every AVL tree of height $h$ has at least $2^{h/2}$ nodes.
AVL Runtime Proof

On Friday, we proved an upper-bound on the height of an AVL tree is $2 \times \lg(n)$ or $O(\lg(n))$:

\[
\begin{align*}
\# \text{ of nodes } (n) & \geq N(h) > 2^{h/2} \\
n & > 2^{h/2} \\
\lg(n) & > h/2 \\
2 \times \lg(n) & > h \\
h & < 2 \times \lg(n), \text{ for } h \geq 1
\end{align*}
\]

Proved: The maximum number of nodes in an AVL tree of height $h$ is less than $2 \times \lg(n)$. 
Summary of Balanced BST

AVL Trees
- Max height: $1.44 \times \lg(n)$
- Rotations:
Summary of Balanced BST

**AVL Trees**
- Max height: $1.44 \times \lg(n)$
- Rotations:
  - Zero rotations on find
  - One rotation on insert
  - $O(h) = O(\lg(n))$ rotations on remove

**Red-Black Trees**
- Max height: $2 \times \lg(n)$
- Constant number of rotations on insert (max 2), remove (max 3).
Why AVL?
Summary of Balanced BST

Pros:
- Running Time:
  - Improvement Over:

- Great for specific applications:
Summary of Balanced BST

Cons:
- Running Time:

- In-memory Requirement:
Red-Black Trees in C++

C++ provides us a balanced BST as part of the standard library:

```cpp
std::map<K, V> map;
```
Red-Black Trees in C++

V & std::map<K, V>::operator[]( const K & )
Red-Black Trees in C++

V & std::map<K, V>::operator[]( const K & )

std::map<K, V>::erase( const K & )
Red-Black Trees in C++

iterator std::map<K, V>::lower_bound( const K & );
iterator std::map<K, V>::upper_bound( const K & );
Mattox Monday
CS 225 -- Course Update

Over the next two days, your grades will be uploaded into Compass for our first “grade update” where we will calculate your current course grade for you.

We will discuss the grades for the course as a whole (ex: average, etc) in lecture on Wednesday.
Iterators

Why do we care?

```cpp
DFS dfs(...);
for ( ImageTraversal::Iterator it = dfs.begin(); it != dfs.end(); ++it ) {
    std::cout << (*it) << std::endl;
}
```
Iterators

Why do we care?

```
1    DFS dfs(...);
2    for ( ImageTraversal::Iterator it = dfs.begin(); it != dfs.end(); ++it ) {
3        std::cout << (*it) << std::endl;
4    }
```

```
1    DFS dfs(...);
2    for ( const Point & p : dfs ) {
3        std::cout << p << std::endl;
4    }
```
Iterators

Why do we care?

```cpp
DFS dfs(...);
for ( ImageTraversal::Iterator it = dfs.begin(); it != dfs.end(); ++it ) {
    std::cout << (*it) << std::endl;
}
```

```cpp
DFS dfs(...);
for ( const Point & p : dfs ) {
    std::cout << p << std::endl;
}
```

```cpp
ImageTraversal & traversal = /* ... */;
for ( const Point & p : traversal ) {
    std::cout << p << std::endl;
}
```
Iterators

ImageTraversal *traversal = /* ... */;
for ( const Point & p : traversal ) {
    std::cout << p << std::endl;
}
<table>
<thead>
<tr>
<th></th>
<th>Unsorted Array</th>
<th>Sorted Array</th>
<th>Unsorted List</th>
<th>Sorted List</th>
<th>Binary Tree</th>
<th>BST</th>
<th>AVL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Find</td>
<td></td>
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<tr>
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<tr>
<td>Remove</td>
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<td></td>
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</tr>
<tr>
<td>Traverse</td>
<td></td>
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</tr>
</tbody>
</table>
Range-based Searches

**Q:** Consider points in 1D: \( p = \{p_1, p_2, \ldots, p_n\} \).
   ...what points fall in \([11, 42]\)?

**Tree construction:**
Range-based Searches

Balanced BSTs are useful structures for range-based and nearest-neighbor searches.

Q: Consider points in 1D: \( p = \{p_1, p_2, ..., p_n\} \).
...what points fall in \([11, 42]\)?

Ex:
Range-based Searches

Q: Consider points in 1D: $p = \{p_1, p_2, ..., p_n\}$. What points fall in $[11, 42]$?

Ex:

```
3   6   11   33   41   44   55
```
Range-based Searches

Q: Consider points in 1D: $p = \{p_1, p_2, ..., p_n\}$. ...what points fall in [11, 42]?

Tree construction:
Range-based Searches
Range-based Searches

Diagram showing a tree structure with nodes labeled from 3 to 55.
Range-based Searches

Q: Consider points in 1D: \( p = \{p_1, p_2, ..., p_n\} \).
...what points fall in \([11, 42]\)?
Range-based Searches
Range-based Searches

Q: Consider points in 1D: $p = \{p_1, p_2, ..., p_n\}$. ...what points fall in [11, 42]?

Ex:

3 6 11 33 41 44 55
Range-based Searches

Consider points in 2D: \( p = \{p_1, p_2, \ldots, p_n\} \).

**Q:** What points are in the rectangle: 
\[ (x_1, y_1), (x_2, y_2) \]? 

**Q:** What is the nearest point to \((x_1, y_1)\)?
Range-based Searches

Consider points in 2D: $p = \{p_1, p_2, ..., p_n\}$.

Tree construction:
Range-based Searches
kD-Trees
kD-Trees