CS 225
Data Structures

Oct. 8 – BST Balance
Wade Fagen-Ulmschneider
BST Analysis

Therefore, for all BST:

Lower bound: \( h \geq O(\lg(n)) \)

Upper bound: \( h \leq O(n) \)
BST Analysis

The height of a BST depends on the order in which the data is inserted into it.

ex: 1 3 2 4 5 7 6 vs. 4 2 3 6 7 1 5

Q: How many different ways are there to insert keys into a BST?

Q: What is the average height of all the arrangements?
BST Analysis

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Q: What is the average height of all the arrangements?
# BST Analysis – Running Time

<table>
<thead>
<tr>
<th>Operation</th>
<th>BST Average case</th>
<th>BST Worst case</th>
<th>Sorted array</th>
<th>Sorted List</th>
</tr>
</thead>
<tbody>
<tr>
<td>find</td>
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<tr>
<td>insert</td>
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<tr>
<td>delete</td>
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<tr>
<td>traverse</td>
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</tbody>
</table>
Mattox Monday
Height-Balanced Tree

What tree makes you happier?

Height balance:  \( b = \text{height}(T_L) - \text{height}(T_R) \)

A tree is height balanced if:
BST Rotation

We will perform a rotation that maintains two properties:

1.

2.
The diagram shows a tree structure with the following nodes and values:

- Root node: 38
  - Child nodes: 13, 25
    - 13: Child node A
    - 25: Child node B
      - Child nodes: 10, 25
        - 10: Child node C
        - 25: Child node D
    - 25: Child node B
      - Child nodes: 10, 25
        - 10: Child node C
        - 25: Child node D

Another subtree with:
- Root node: 84
  - Child node: 51
  - Child node: 89
    - 51: Subtree A
    - 89: Subtree C
      - Child nodes: A, B, C, D
      - A: Child node A
      - B: Child node B
      - C: Child node C
      - D: Child node D

The diagram illustrates the hierarchical relationship among these nodes, with arrows indicating the parent-child connections.
BST Rotation Summary

- Four kinds of rotations (L, R, LR, RL)
- All rotations are local (subtrees are not impacted)
- All rotations are constant time: $O(1)$
- BST property maintained

GOAL:

We call these trees:
AVL Trees

Three issues for consideration:
- Rotations
- Maintaining Height
- Detecting Imbalance