Smart Union Options:
- Union by Height (root := -h - 1)
- Union by Size (root := -n)
- Union by Rank (root := #union ops)

In all smart unions:
....height of UpTree: ____________.

How do we improve this?

Running Time:
- Worst case running time of find(k):
- Worst case running time of union(r1, r2), given roots:
- New function: “Iterated Log”:
  \[ \log^*(n) := \]
- Overall running time:
  o A total of \( m \) union/find operation runs in:

A Review of Major Data Structures so Far

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<th>Array-based</th>
<th>List/Pointer-based</th>
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</table>

DisjointSets.cpp (partial)

```cpp
int DisjointSets::find(int i) {
    if (arr_[i] < 0) { return i; }
    else { return _find(arr_[i]); }  
}
```

DisjointSets.cpp (partial)

```cpp
void DisjointSets::unionBySize(int root1, int root2) {
    int newSize = arr_[root1] + arr_[root2];
    // If arr_[root1] is less than (more negative), it is the
    // larger set; we union the smaller set, root2, with root1.
    if (arr_[root1] < arr_[root2]) {
        arr_[root1] = root2;
        arr_[root2] = newSize;
    }
    // Otherwise, do the opposite:
    else {
        arr_[root1] = root1;
        arr_[root2] = newSize;
    }
}
```

An Introduction to Graphs
Motivation:
Graphs are awesome data structures that allow us to represent an enormous range of problems. To study these problems, we need:
1. A common vocabulary to talk about graphs
2. Implementation(s) of a graph
3. Traversals on graphs
4. Algorithms on graphs

CS 225 – Things To Be Doing:
1. Theory Exam 3 is ongoing!
2. lab_heaps due Sunday, November 11th
3. MP6 released; Extra Credit +7 deadline November 12th
4. Daily POTDs are ongoing!