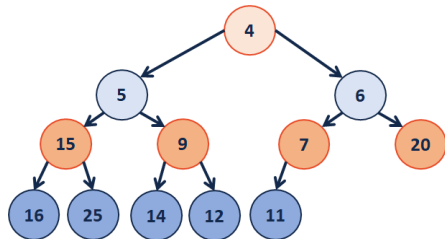


A Heap Data Structure

(specifically a minHeap in this example, as the minimum element is at the root)



-	4	5	6	15	9	7	20	16	25	14	12	11				
---	---	---	---	----	---	---	----	----	----	----	----	----	--	--	--	--

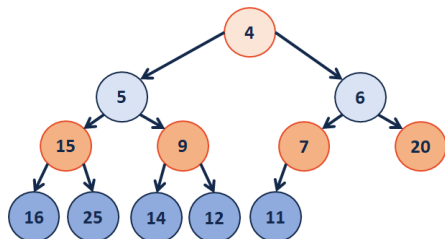
Given an index i , its parent and children can be reached in $O(1)$ time:

- $leftChild := 2i$
- $rightChild := 2i + 1$
- $parent := floor(i / 2)$

Formally, a complete binary tree T is a minHeap if:

- $T = \{ \}$ or
- $T = \{r, T_L, T_R\}$ and r is less than the roots of T_L, T_R and T_L, T_R are minHeaps

Inserting into a Heap



-	4	5	6	15	9	7	20	16	25	14	12	11				
---	---	---	---	----	---	---	----	----	----	----	----	----	--	--	--	--

Heap.hpp (partial)

```

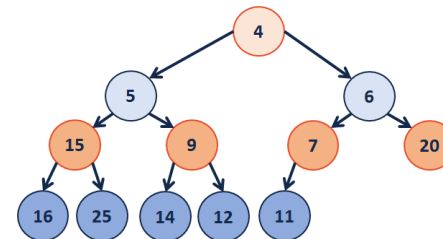
1  template <class T>
2  void Heap<T>::_insert(const T & key) {
3      // Check to ensure there's space to insert an element
4      // ...if not, grow the array
5      if ( size_ == capacity_ ) { _growArray(); }
6
7      // Insert the new element at the end of the array
8      item_[++size] = key;
9
10     // Restore the heap property
11     _heapifyUp(size);
12 }
31 template <class T>
32 void Heap<T>::_heapifyUp( _____ ) {
33     if ( index > _____ ) {
34         if ( item_[index] < item_[ parent(index) ] ) {
35             std::swap( item_[index], item_[ parent(index) ] )
36         };
37         _heapifyUp( _____ );
38     }
39 }

```

How do we complete this code?

Running time of insert?

Heap Operation: removeMin / heapifyDown:



-	4	5	6	15	9	7	20	16	25	14	12	11				
---	---	---	---	----	---	---	----	----	----	----	----	----	--	--	--	--

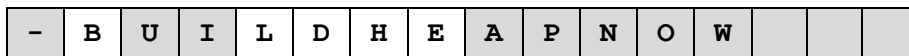
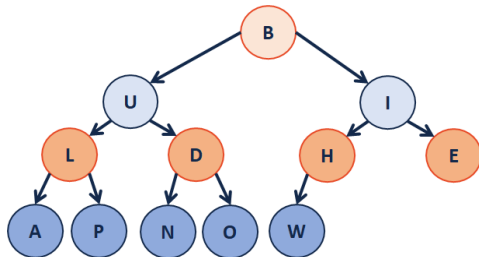
```

Heap.hpp (partial)
1  template <class T>
2  void Heap<T>::_removeMin() {
3      // Swap with the last value
4      T minValue = item_[1];
5      item_[1] = item_[size_];
6      size--;
7
8      // Restore the heap property
9      heapifyDown();
10
11     // Return the minimum value
12     return minValue;
13 }

51  template <class T>
52  void Heap<T>::_heapifyDown(int index) {
53     if ( !_isLeaf(index) ) {
54         T minChildIndex = _minChild(index);
55         if ( item_[index] > item_[minChildIndex] ) {
56             std::swap( item_[index], item_[minChildIndex] );
57             _heapifyDown( minChildIndex );
58         }
59     }
60 }

```

Q: How do we construct a heap given data?



```

Heap.cpp (partial)
1  template <class T>
2  void Heap<T>::buildHeap() {
3      for (unsigned i = parent(size); i > 0; i--) {
4          heapifyDown(i);
5      }
6  }

```

Running Time?

Theorem: The running time of buildHeap on array of size n is:

Strategy:

Define S(h):

Let **S(h)** denote the sum of the heights of all nodes in a complete tree of height **h**.

S(0) =

S(1) =

S(h) =

Proof of S(h) by Induction:

Finally, finding the running time:

- | CS 225 – Things To Be Doing: | |
|------------------------------|--|
| 1. | Theory Exam 3 starts on Thursday (topic list online, more soon!) |
| 2. | MP5 du date: Monday, November 5 th |
| 3. | lab_hash is due Sunday, November 4 th |
| 4. | Daily POTDs are ongoing! |