

CS 225

Data Structures

Dec. 6 – Dijkstra's algorithm

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Prim's Algorithm

```
PrimMST(G, s):
6  foreach (Vertex v : G):
7      d[v] = +inf
8      p[v] = NULL
9  d[s] = 0
10
11  PriorityQueue Q // min distance, defined by d[v]
12  Q.buildHeap(G.vertices())
13  Graph T          // "labeled set"
14
15  repeat n times:
16      Vertex m = Q.removeMin()
17      T.add(m)
18      foreach (Vertex v : neighbors of m not in T):
19          if cost(v, m) < d[v]:
20              d[v] = cost(v, m)
21              p[v] = m
```

	Adj. Matrix	Adj. List
Heap		
Unsorted Array		

Prim's Algorithm

Sparse Graph:

Dense Graph:

```
PrimMST(G, s):
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20              d[v] = cost(v, m)
21              p[v] = m
```

	Adj. Matrix	Adj. List
Heap	$O(n^2 + m \lg(n))$	$O(n \lg(n) + m \lg(n))$
Unsorted Array	$O(n^2)$	$O(n^2)$

MST Algorithm Runtime:

- Kruskal's Algorithm:

$$O(n + m \lg(n))$$

- Prim's Algorithm:

$$O(n \lg(n) + m \lg(n))$$

- What must be true about the connectivity of a graph when running an MST algorithm?

- How does n and m relate?

MST Algorithm Runtime:

- Upper bound on MST Algorithm Runtime:
 $O(m \lg(n))$

Suppose I have a new heap:

	Binary Heap	Fibonacci Heap
Remove Min	$O(\lg(n))$	$O(\lg(n))$
Decrease Key	$O(\lg(n))$	$O(1)^*$

What's the updated running time?

```
PrimMST(G, s):
  6  foreach (Vertex v : G):
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19      if cost(v, m) < d[v]:
20        d[v] = cost(v, m)
21        p[v] = m
```

End of Semester Logistics

Lab: Your final CS 225 lab is released today!

- No lab sections next week (partial week).

Final Exam: Final exams start on Reading Day (Dec 14)

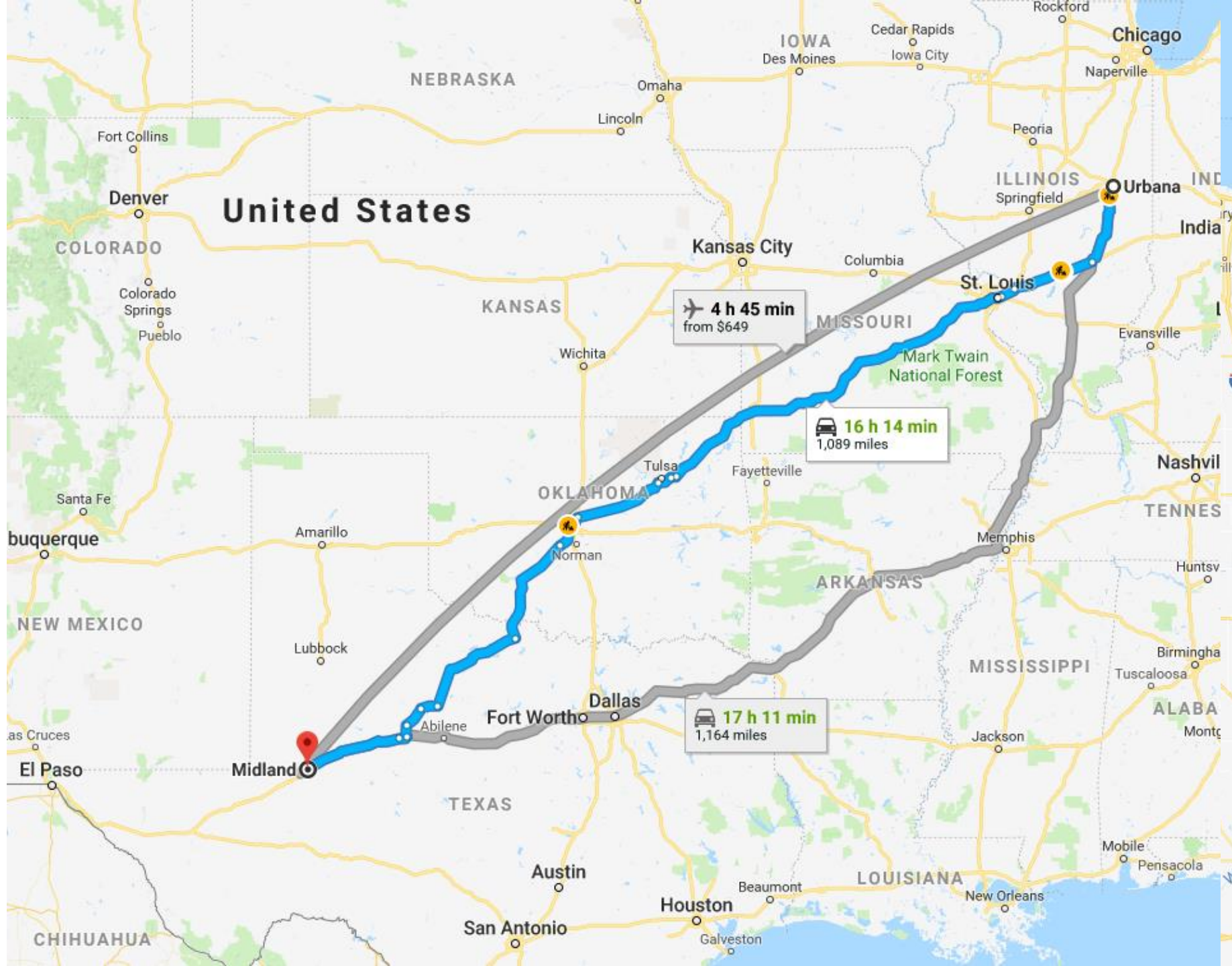
- Last day of office hours is Wednesday, Dec. 13.
- No office/lab hours once the first final exam is given.

Grades: We're working towards a "Pre-Final Update" including all current grades except your final exam.

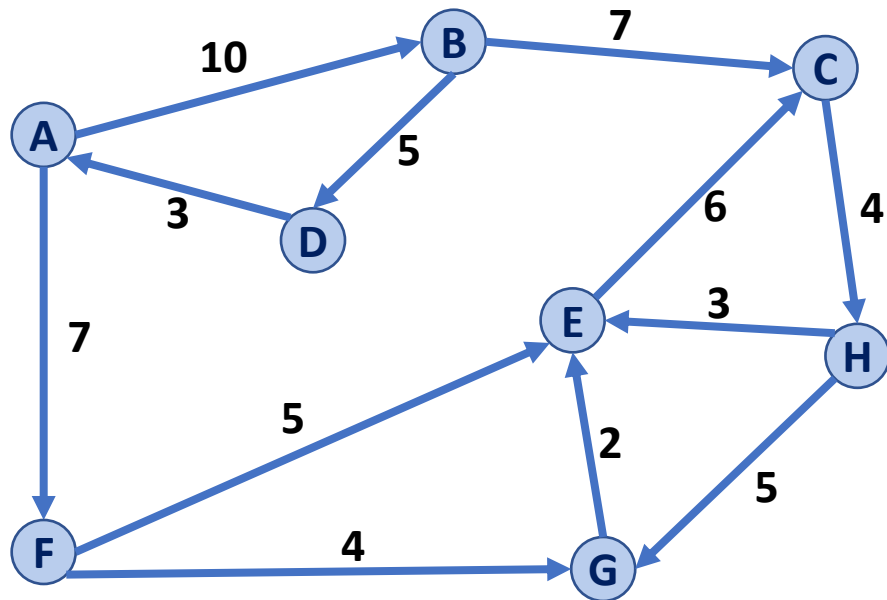
- Expect this update early next week.

Shortest Path





Dijkstra's Algorithm (SSSP)



```
DijkstraSSSP(G, s):
```

```
6  foreach (Vertex v : G):
```

```
7      d[v] = +inf
```

```
8      p[v] = NULL
```

```
9      d[s] = 0
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```
10
```

```
11  PriorityQueue Q // min distance, defined by d[v]
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12  Q.buildHeap(G.vertices())
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13  Graph T          // "labeled set"
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15  repeat n times:
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```
16      Vertex u = Q.removeMin()
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17      T.add(u)
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```
18      foreach (Vertex v : neighbors of u not in T):
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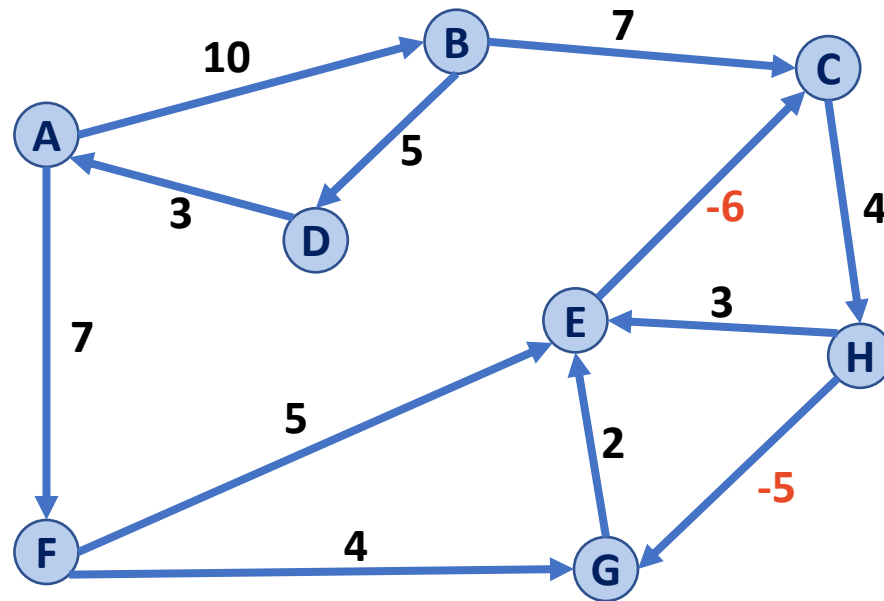
```
19          if _____ < d[v]:
```

```
20              d[v] = _____
```

```
21              p[v] = m
```

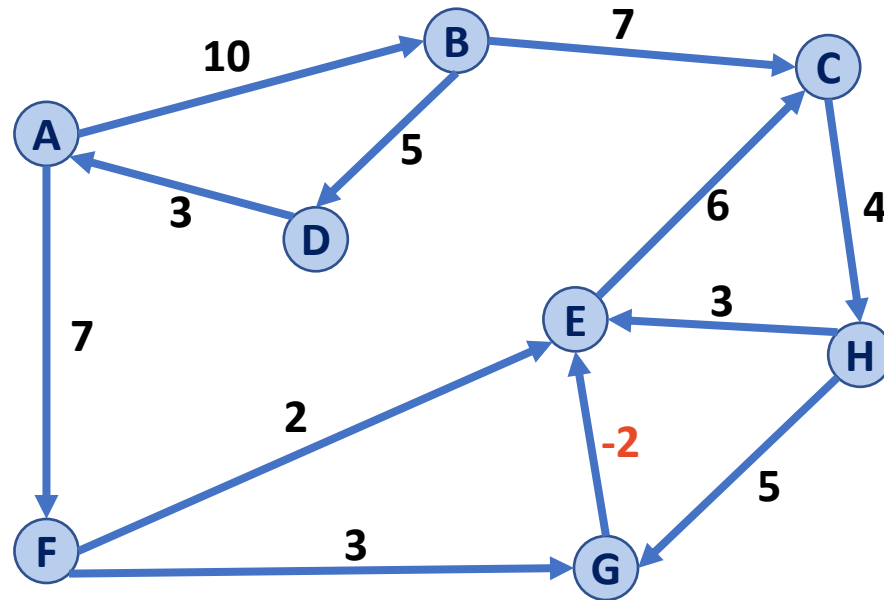
Dijkstra's Algorithm (SSSP)

What about negative weight cycles?



Dijkstra's Algorithm (SSSP)

What about negative weight edges, without negative weight cycles?



Dijkstra's Algorithm (SSSP)

What is the running time?

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19          if _____ < d[v]:
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```

CS 225 – Things To Be Doing

Exam 12 (programming) is ongoing

More Info: <https://courses.engr.illinois.edu/cs225/fa2017/exams/>

MP7: The final MP!

Due: Monday, Dec. 11 at 11:59pm

Lab: lab_ml released today

lab_ml: Due Sunday @ 11:59pm

New POTDs every M/W/F

Worth +1 Extra Credit /problem (up to +40 total)