Heap Operation: insert / heapifyUp:

```
-                  4                  5
  15               16                  25
  9  7             14  12             11
```

Running Time?

Heap Operation: removeMin / heapifyDown:

```
-                  4                  5
  15               16                  25
  9  7             14  12             11
```

Q: How do we construct a heap given data?
An O(n) approach to buildHeap:

```
template <class T>
void Heap<T>::buildHeap() {
    for (unsigned i = parent(size); i > 0; i--) {
        heapifyDown(i);
    }
}
```

**Theorem:** The running time of buildHeap on array of size n is: __________.

**Strategy:**
- We know that constant work is done based on the distance a node is away from the root (e.g., its height).
- Therefore, the running time is proportional to the sum of the heights of the heights of all the nodes.
- We will work towards creating a proof around the sum of the heights of all the nodes.

**Define S(h):**
Let \( S(h) \) denote the sum of the heights of all nodes in a complete tree of height \( h \).

\[
S(0) = \\
S(1) = \\
S(h) =
\]

**Proof of S(h) by Induction:**

- **Base Case(s):**
- **General Case:**

Finally, finding the running time:

**Heap Sort**

Algorithm:

1. 
2. 
3. 

Running time?

Why do we care about another sort?

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**CS 225 – Things To Be Doing:**

1. Register for CS 225's Final Exam!
2. Exam #9 starts today
3. MP5 due tonight (grace period until tomorrow)
4. New lab on Wednesday
5. Daily POTDs