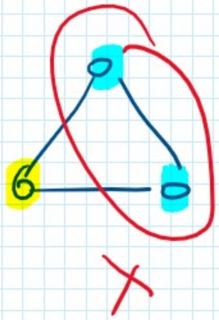
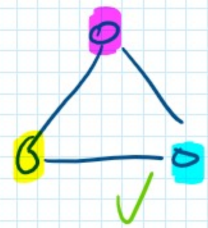
CS 173 Lecture 9b: Graph Coloring

Given a graph G, a proper coloring assigns to each vertex exactly one color such that no adjacent vertices have the same color.

oO (actually number)

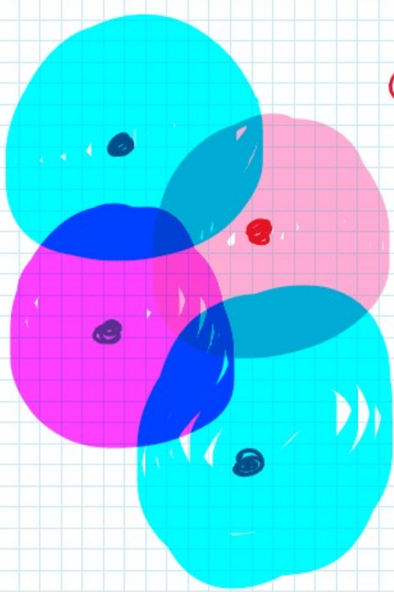




$\chi(\triangle) = 3$

Given graph G, the <u>chromatic number</u> $\chi(G)$ is the smallest number of colors needed to proper color G.

Application: Gov't buy out radio frequencies
stations w/ overlapping broadcast areas need different frequencies.



Q: fewest number of frequencies s.t. we can reassign all stations to these frequencies s.t. overlapping stations have different freq.s.

→ vertices are stations, edge between stations w/ overlapping areas.

Some bounds:

K_n. oO (complete graph on n vertices

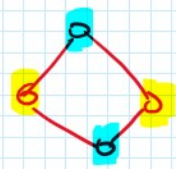
K_n : complete graph on n vertices

each vtx is adjacent to all other vertices : $n-1$

$$\chi(K_n) = n$$

C_n : cycle graph

C_3 $\chi(C_3)=3$



C_4

$\chi(C_4) \le 2$

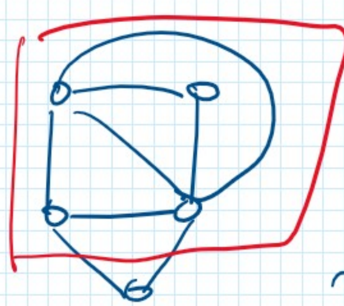
$\chi(C_4) \ge 2$

For $n \geq 3$ $\chi(C_n) = \begin{cases} 3 & \text{if } n \text{ is odd} \\ 2 & \text{if } n \text{ is even} \end{cases}$

Lower bounding of $\chi(G)$ by subgraphs.

Lemma. If H is a subgraph of G, then $\chi(H) \le \chi(G)$

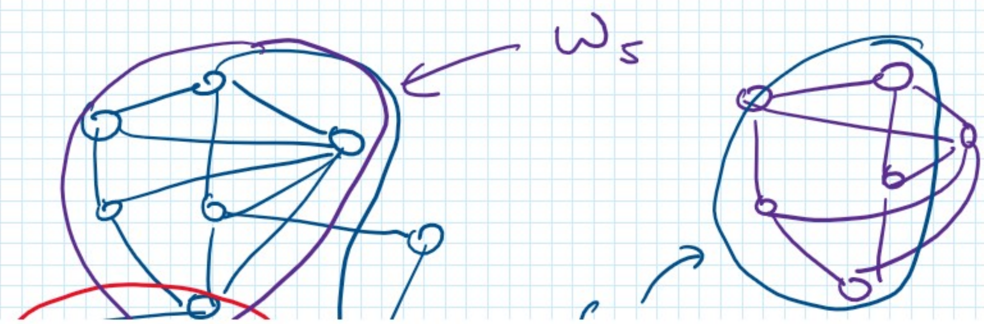
example:



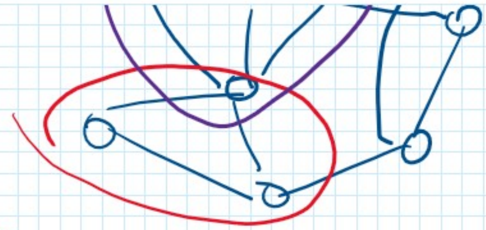
K_4

$\chi(G) \geq 4$.

example:



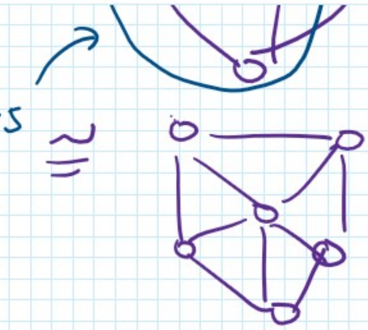
W_5



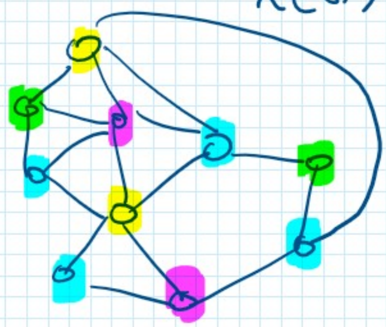
C_3

$\chi(G) \geq 3$

C_5 $\cong$



$\chi(G) \geq 4 \longleftarrow \chi(W_n) = \chi(C_n) + 1.$



$\chi(G) \leq 5$

Lemma (Greedy Coloring) Let $D = \max_{v \in V} \deg(v)$.

Then $\chi(G) \leq D+1$.

(often not the best upper bound).